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YÖS

YABANCI ÖĞRENCİ SINAVLARI
FOREIGN STUDENT EXAMINATIONS

MATEMATİK
MATHEMATICS

KONU ÖZETLİ SORU BANKASI
QUESTION BANK WITH SUBJECT SUMMARIES

221 Konu Testi
Subject Tests

Test Sorusu 3461
Test Questions

Türkiye'de ve Dünyada!
In Turkey and Around the World!

METROPOL YAYINLARI

5. Baskı - 5th Edition

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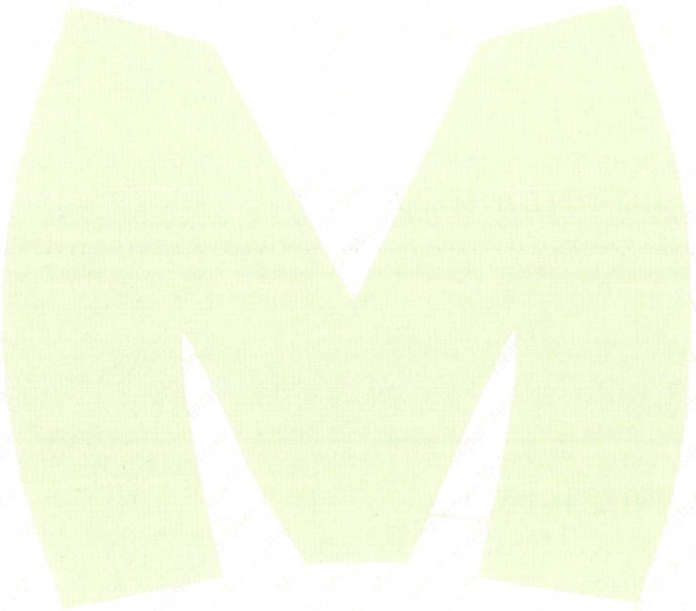

BÖLÜM CHAPTER

1

TARİHSEL NOT / HISTORICAL NOTE

Johann Heinrich Lambert [1728 – 1777]

Johann Lambert, ilk defa π nin bir irrasyonel sayı olduğunu kesin olarak ispatlayan ünlü Fransız matematikçisidir.

Johann Lambert was the first to provide a rigorous proof that π is an irrational number.

SAYILAR / NUMBERS

Bu bölüm, 163 test sorusu içermektedir.

This chapter includes 163 test questions.

BÖLÜM / CHAPTER 1

SAYILAR / NUMBERS

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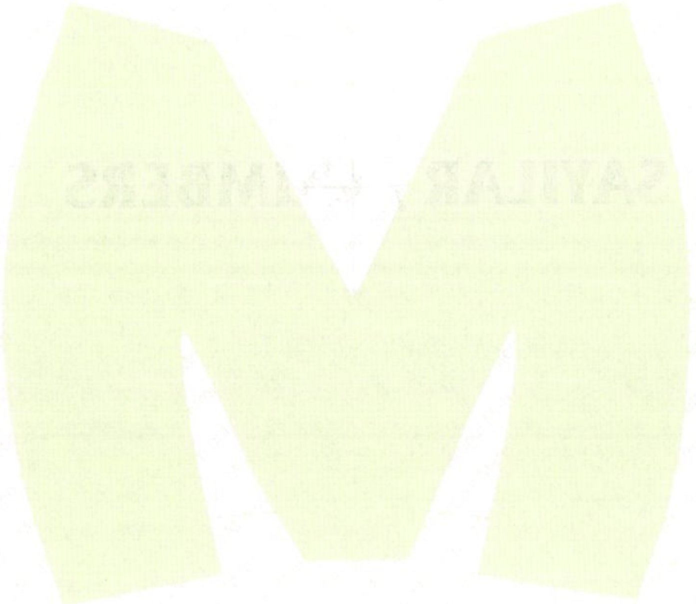


BÖLÜM

01

CHAPTER

SAYILAR
NUMBERS

TEMEL KAVRAMLAR / BASIC CONCEPTS

Sayı: Çoklukları gösterebilmek için kullanılan sembollere ***sayı*** denir.

Rakam: Sayıları yazmak için kullanılan sembollerden her birine ***rakam*** denir.

Rakamlar kümesi; { 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 }

Örneğin; 716 sayısının rakamları 7, 1 ve 6 dır.

Number: Symbols used in order to express quantities.

Digit: Each symbol used to expression of the numbers is called a *digit.*

Set of digits: {0, 1, 2, 3, 4, 5, 6, 7, 8, 9}

For example; digits of the number 716 are 7, 1, and 6.

Önemli / Important

- Aritmetikte kullanılan çeşitli işlemleri iyi anlayabilmek için, ortak sayı sistemine gereksinim vardır.
- Sayı sisteminde, rakam olarak adlandırılan 10 sembol kullanılır. Bunlar; 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 dur. Bu on sembolü kullanarak sayıları yazabiliriz.
- Bu sayıların her biri, nesnelerin miktarlarını belirtmek için kullanılır. Semboller ilk olarak, sayma amacıyla kullanılmıştır.
- In order to get a good understanding of the various operations used in arithmetic, one should have a clear idea of the common number system.
- The system of numbers makes use of only ten symbols called digits: 0,1,2,3,4,5,6,7,8,9. By means of these ten symbols we can write a number of any size.
- Each of these symbols is used to denote the number of objects. The symbols were first used for counting.

İŞLEM ÖNCELİĞİ / THE ORDER of OPERATION

Herhangi bir sayı kümesinde toplama, çıkarma, çarpma ve bölme işlemleri (yani, dört işlem) yapılırken aşağıdaki sıra uygulanır:

▼ Parantez, köşeli parantez, mutlak değer ve kesir çizgisi gibi grup sembolleri içeren işlemler yapılır.

▼ Üslü ifadeler sadeleştirilir.

▼ Soldan sağa doğru çarpma ve bölme yapılır.

▼ Soldan sağa doğru toplama ve çıkarma yapılır.

In any set of numbers addition, subtraction, multiplication and division must be done as the following order.

▼ Perform operations inside grouping symbols. Grouping symbols include parantheses ( ), brackets [ ], braces { }, absolute value symbols | |, and the fraction bar.

▼ Simplify exponential expressions.

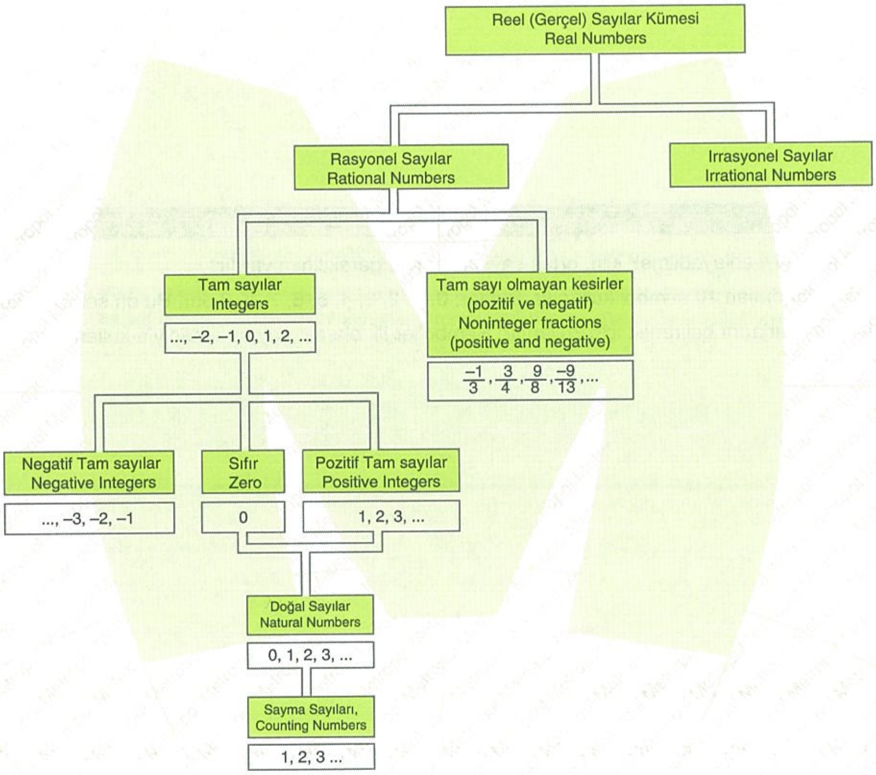
▼ Do multiplication and division as they occur from left to right.

▼ Do addition and subtraction as they occur from left to right.

SAYI KÜMELERİ / NUMBER SETS

- **Sayma sayıları kümesi /** Set of counting numbers: $C = \{1, 2, 3, 4, \ldots\}$
- **Doğal sayılar kümesi /** Set of natural number: $N = \{0, 1, 2, 3, 4, \ldots\}$
- **Tam sayılar kümesi /** Set of integers: $Z = \{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}$
- **Negatif tam sayılar kümesi /** Set of negative integers: $Z^- = \{\ldots, -3, -2, -1\}$
- **Pozitif tam sayılar kümesi /** Set of positive integers: $Z^+ = \{1, 2, 3, \ldots\}$
- **Rasyonel sayılar kümesi /** Set of rational numbers: $Q = \{\frac{a}{b} : a,b \in Z,\ b \neq 0\}$
- **İrrasyonel sayılar kümesi /** Set of irrational numbers: Rasyonel olmayan sayıların kümesine irrasyonel sayılar kümesi denir. Q' ile gösterilir.

  The set of all the numbers which are non-rational is called the set of irrational numbers, and its shown by Q'
- **Reel sayılar kümesi /** Set of real numbers: $R = Q \cup Q'$


Reel (Gerçel) Sayılar Kümesi
Real Numbers
Rasyonel Sayılar
Rational Numbers
Irrasyonel Sayılar
Irrational Numbers
Tam sayılar
Integers
..., –2, –1, 0, 1, 2, ...
Tam sayı olmayan kesirler
(pozitif ve negatif)
Noninteger fractions
(positive and negative)
–1/3, 3/4, 9/8, –9/13, ...
Negatif Tam sayılar
Negative Integers
..., –3, –2, –1
Sıfır
Zero
0
Pozitif Tam sayılar
Positive Integers
1, 2, 3, ...
Doğal Sayılar
Natural Numbers
0, 1, 2, 3, ...
Sayma Sayıları,
Counting Numbers
1, 2, 3 ...

Önemli / Important

- $C \subset N \subset Z \subset Q \subset R$
- $Z = Z^- \cup \{0\} \cup Z^+$

DOĞAL SAYILAR / NATURAL NUMBERS

▼ N = {0, 1, 2, 3, ...} kümesine **doğal sayılar kümesi**, N^+ = {1, 2, 3, ...} kümesine de **pozitif doğal sayılar kümesi** denir.

123 ∈ N, 25 ∈ N, 4572 ∈ N

▼ **Doğal sayılarda basamak kavramı:** Bir doğal sayıyı oluşturan rakamların yazıldıkları yere **basamak** denir.

▼ **Rakamın Sayı Değeri:** Bir veya birden çok basamaklı bir sayıdaki, herhangi bir rakamın bulunduğu basamağa bağlı olmadan tek başına gösterdiği sayıya, bu rakamın **sayı değeri** denir. Rakamın sayı değeri hiç değişmez.

▼ **Rakamın Basamak Değeri:** Bir veya birden çok basamaklı bir sayıdaki herhangi bir rakamın bulunduğu basamağa göre aldığı değere, bu rakamın **basamak değeri** denir.

▼ N = {0, 1, 2, 3, ...} is called set of natural numbers, and N^+ = {1, 2, 3, ...} is called set of pozitive natural numbers.
123 ∈ N, 25 ∈ N, 4572 ∈ N

▼ **The concept of place value in natural numbers:** The position of each digit written in a natural number is called the place or position of that digit.

▼ **The number value of a digit:** In a number consisting of one or more than one digits, each one of these digits, regardless of it's place in the number, has a value; this value is called the number value of that digit. The number value of a digit never changes.

▼ **The place value of a digit:** In a number consisting of one or more than one digits, the value of each of these digits considering its place in the number is called its place value.

a b c d e

	Sayı değeri The value of the number	Basamak değeri The value of the place
e → Birler basamağı (Ones place) ⟶	e	1.e
d → Onlar basamağı (Tens place) ⟶	d	10.d
c → Yüzler basamağı (Hundreds place) ⟶	c	100.c
b → Binler basamağı (Thousands place) ⟶	b	1000.b
a → Onbinler basamağı (Ten thousands place) ⟶	a	10000.a

Önemli / Important

- Doğal sayılar onluk sayı sisteminde yazıldıkları için basamak isimleri 10'un kuvvetleri olarak isimlendirilir.
- As the natural numbers are written in the decimal number system, the names of the places or positions in these numbers are named using powers of ten.

Önemli / Important

- Toplamları sabit olan iki doğal sayının çarpımlarının en büyük olabilmesi için sayılar arasındaki farkın en küçük olması gerekir.
- Two natural numbers with a constant sum should have minimum difference between them in order to maximize their product.

Önemli / Important

- Toplamları sabit olan iki doğal sayının çarpımlarının en küçük olabilmesi için sayılar arasındaki farkın en büyük olması gerekir.
- Two natural numbers with a constant sum should have maximum difference between them in order to minimize their product.

DOĞAL SAYILARDA ÇÖZÜMLEME / ANALYSIS of The NATURAL NUMBERS

Tanım: Doğal sayının rakamlarının basamak değerlerinin toplamı şeklinde yazılmasına bu sayının **çözümlenmesi** denir.

Definition: The application of writing a natural number as summation of the place values of its digits is called the **analysis** of that natural number.

▼ 34 sayısı çözümlenirse,

$34 = 30 + 4 = 10.3 + 4$ bulunur.

O halde, ab sayısının çözümlenişi aşağıdaki gibi tanımlanır.

$\mathbf{ab = 10 \cdot a + b}$

▼ 235 sayısı çözümlenirse,

$235 = 200 + 30 + 5 = 100 \cdot 2 + 10 \cdot 3 + 5$ bulunur.

O halde, abc sayısının çözümlenişi aşağıdaki gibi tanımlanır.

$\mathbf{abc = 100 \cdot a + 10 \cdot b + c}$

▼ If we analyze 34,

$34 = 30 + 4 = 10.3 + 4$ is obtained.

Therefore, analysis of the number ab is defined as the following.

$\mathbf{ab = 10 \cdot a + b}$

▼ If we analyze 235,

$235 = 200 + 30 + 5 = 100 \cdot 2 + 10 \cdot 3 + 5$ is obtained.

Therefore, analysis of the number abc is defined as the following.

$\mathbf{abc = 100 \cdot a + 10 \cdot b + c}$

Önemli / Important

- ab ve ba iki basamaklı birer doğal sayı olmak üzere aşağıdaki eşitlikler vardır:
- If ab and ba two-place natural numbers, there are following equalities:

I. $ab + ba = (10 \cdot a + b) + (10 \cdot b + a)$

$= 11 \cdot a + 11 \cdot b = 11 \cdot (a + b)$

II. $ab - ba = (10 \cdot a + b) - (10 \cdot b + a)$

$= 9 \cdot a - 9 \cdot b$

$= 9 \cdot (a - b)$

TABAN ARİTMETİĞİ / BASE ARITHMETICS

Tanım: Sayının tanımlandığı sayma sistemi o sayının tabanını oluşturur. Bu tabanda yapılan işlemlere de **taban aritmetiği** adı verilir.

Definition: The countion system on which the number is defined forms the base. Operations that are being done in the base are called **base arithmetics.**

FARKLI BİR TABANDA VERİLEN SAYININ ONLUK TABANDA YAZILMASI
CHANGING a GIVEN NUMBER in a BASE OTHER THAN TEN to The BASE of TEN

▼ a, b, c, d birer rakam / are digit
$t > a, t > b, t > c, t > d$

$t \in N^+$, $t > 1$ olmak üzere / if $t \in N^+$, $t > 1$

$(abcd)_t = x$

$(a\,b\,c\,d)_t = a.t^3 + b.t^2 + c.t + d$

$d.t^0$
$c.t^1$
$b.t^2$
$a.t^3$

$\Rightarrow x = d{\cdot}t^0 + c{\cdot}t^1 + b{\cdot}t^2 + a{\cdot}t^3$

$(ab,cd)_t = x$

$(a\,b,c\,d)_t = a.t^1 + b.t^0 + c.t^{-1} + d.t^{-2}$

$d.t^{-2}$
$c.t^{-1}$
$b.t^0$
$a.t^1$

$\Rightarrow x = d{\cdot}t^{-2} + c{\cdot}t^{-1} + b{\cdot}t^0 + a{\cdot}t^1$

10'LUK TABANDA VERİLEN BİR SAYIYI BAŞKA BİR TABANDA YAZMA
CHANGING a GIVEN NUMBER in BASE TEN to a NUMBER in ANOTHER BASE

▼ 10 tabanında verilen sayı, hangi tabanda yazılacaksa o tabana bölünür. Sıfır dahil olmak üzere tüm kalanlar yazılır. Son kalandan başlanarak yazılan rakamlar yeni sayıyı oluşturur.

▼ Divide the number (base 10) by the new base as many times as possible and write down all remainders, including zeros. The remainders are the digits of the number written in the desired base, starting from the last remainder.

TABAN ARİTMETİĞİNDE İŞLEMLER
OPERATIONS in The BASE ARITHMETICS

TOPLAMA / ADDITION

▼ Taban aritmetiğinde toplama işlemi yapılırken rakamların toplamı verilen tabandan büyük veya eşit olduğunda toplamın sonucu tabana bölünür, kalan sonuç kısmına yazılır bölüm bir sonraki basamağa eklenir.

▼ As doing the operation of addition in base arithmetics, whenever the value of the added digits becomes bigger than or equal to the base, the result of the addition needs to be devided to the base and remainder becomes the new result and the quotient will add to one column to the left.

ÇIKARMA / SUBTRACTION

▼ Taban aritmetiğinde çıkarma işlemi yapılırken 10'luk sistemdekine benzer biçimde bir soldaki basamaktan 1(bir) almak gerektiğinde aktarıldığı basamağa katkısı tabanın sayı değeri kadardır. Fakat alındığı basamak 1 azalır.

▼ When doing the operation of subtraction in base arithmetics, whenever we need to take a 1 (one) from one column to the left as we do in the base of ten, the added value to the right column will equal to the base and the digit-value of the left column will decrease by 1 unit.

ÇARPMA / MULTIPLICATION

▼ Taban aritmetiğinde çarpma işlemi yapılırken rakamların çarpımı tabanın sayı değerinden büyük ise sonuç tabandaki sayıya bölünür.

▼ When doing the operation of multiplication in base arithmetics, if the product of integers exeeds the number value of the base, the result will be devided to the base

TAM SAYILAR / INTEGERS

Tanım: {..., –3, –2, –1, 0, 1, 2, 3, ...} kümesinin her elemanına **tam sayı** denir. Tam sayılar kümesi **Z** ile gösterilir.

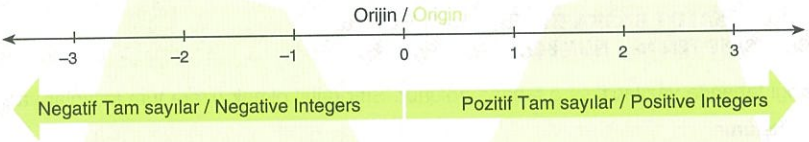
$Z = Z^- \cup \{0\} \cup Z^+$ veya $Z = \{..., -3, -2, -1, 0, 1, 2, 3, ...\}$

▼ 0 sayısı, ne negatif ne de pozitif tam sayıdır. 0 sayısı işaretsizdir.

Definition: Each element of the set {..., –3, –2, –1, 0, 1, 2, 3, ...} is an integer. The set of integers denoted by the symbol Z.

$Z = Z^- \cup \{0\} \cup Z^+$ or $Z = \{... -3, -2, -1, 0, 1, 2, 3, ...\}$

▼ The integer zero is neither positive nor negative. Thus, the number zero is unsigned integer.


Orijin / Origin
–3
–2
–1
0
1
2
3
Negatif Tam sayılar / Negative Integers
Pozitif Tam sayılar / Positive Integers

Önemli / Important

- 0 sayısı ne pozitif ne de negatiftir. Bu yüzden, pozitif tam sayılardan ya da 0 dan bahsederken, negatif olmayan tam sayılar ifadesi kullanılabilir.
- The integer zero is neither positive nor negative. Thus, when you talk about integers that are positive or zero, you can use the term non-negative integers.

Önemli / Important

Bir a sayısı için, aşağıdakilerden biri doğru olmalıdır.

- a negatiftir.
- $a = 0$.
- a pozitiftir.

For any number a, exactly one of the following is true:

- a is negative.
- $a = 0$.
- a is positive.

▼ **Sayı doğrusu /** The number line:

Sayıları göstermek için kullanılan şekle **sayı doğrusu** denir. Sayı doğrusu, orijini 0 olarak işaretlenen yatay bir doğrudan oluşur. Sayı doğrusu, herhangi iki tamsayıyı karşılaştırma olanağı sağlar. Bu nedenle, sayı doğrusu üzerinde iki nokta seçilirse, bir nokta diğerinin solunda kalmak zorundadır. Bu durumda, soldaki sayı, sağdakinden küçük veya sağdaki sayı soldakinden daha büyük olmalıdır.

The picture that is used to represent the numbers is called the **number line**. It consists of a horizontal line with a point (the origin) labeled as 0. The number line provides you with a way of comparing any two integers. For instance, if you choose any two (different) numbers on the number line, one of the numbers must be to the left of the other number. The number to the left is **less than** the number to the right, and the number to the right is **greater than** the number to the left.

POZİTİF TAM SAYILAR / POSITIVE INTEGERS

- {1, 2, 3, ...} kümesine **pozitif tam sayılar kümesi** denir ve Z^+ ile gösterilir. Pozitif tam sayılar kümesinin her elemanına **pozitif tam sayı** denir.

$$Z^+ = \{1, 2, 3, ...\}$$

- {1, 2, 3, ...} is called **positive integers set** and denoted by the symbol Z^+. Each element of a positive integer set is called a **positive integer**.

$$Z^+ = \{1, 2, 3, ...\}$$

NEGATİF TAM SAYILAR / NEGATIVE INTEGERS

- {..., –3, –2, –1} kümesine **negatif tam sayılar kümesi** denir ve Z^- ile gösterilir. Negatif tam sayılar kümesinin her elemanına **negatif tam sayı** denir.

$$Z^- = \{..., -3, -2, -1\}$$

- {..., –3, –2, –1} is called **negative integers set** and denoted by the symbol Z^-. Each element of a negative integer set is called a **negative integer**.

$$Z^- = \{..., -3, -2, -1\}$$

TEK ve ÇİFT TAM SAYILAR / ODD and EVEN INTEGERS

Tanım: {…, –5, –3, –1, 1, 3, 5, …} kümesinin her bir elemanına **tek tam sayı** denir. n bir tam sayı iken, tek sayılar **2n – 1** ile gösterilir.

Definition: Each element of the set {…, –5, –3, –1, 1, 3, 5…} is an odd number. While n is integer, the odd numbers are represented by **2n – 1**.

Tanım: {…, –4, –2, 0, 2, 4,…} kümesinin her bir elemanına **çift tam sayı** denir. n bir tam sayı iken, çift sayılar **2n** ile gösterilir.

Definition: Each element of the set {…, –4, –2, 0, 2, 4,…} is an even number. While n is integer, the even numbers are represented by **2n**.

Önemli / Important

- Çift sayılar 2'nin tam katlarıdır. Tek sayılar 2'nin tam katı değildir.
- Even integers are the multiples of 2. Odd integers are the non-multiples of 2.

TEK ve ÇİFT TAM SAYILARDA İŞLEMLER
OPERATIONS on ODD AND EVEN INTEGER

Tek tam sayılar / Odd integers: T/O

Çift tam sayılar / Even integers: Ç/E

- $T \pm T = Ç$
- $T \pm Ç = T$
- $Ç \pm Ç = Ç$
- $T \cdot T = T$
- $T \cdot Ç = Ç$
- $Ç \cdot Ç = Ç$

Önemli / Important

- Pozitif bir çift sayının pozitif tam sayı kuvveti çift, tek bir sayının pozitif tam sayı kuvveti tektir.

$$n \in Z^+ \Rightarrow T^n = T\ ,\ Ç^n = Ç$$

- The positive integer power of a positive even number is even, the positive integer power of a positive odd number is odd.

$$n \in Z^+ \Rightarrow T^n = T\ ;\ Ç^n = Ç$$

ARDIŞIK SAYILAR / CONSECUTIVE NUMBERS

Tanım: Belirli bir kurala göre sıralanmış sayılara **ardışık sayılar** denir.

Definition: It is a group of terms going forward following a certain rule.

Örneğin / For example:

1, 2, 3, 4, ..., n, ...	[ardışık sayılar (consecutive numbers)]
2, 4, 6, 8, ..., 2n, ...	[ardışık çift sayılar (consecutive even numbers)]
1, 3, 5, 7, ..., 2n – 1, ...	[ardışık tek sayılar (consecutive odd numbers)]
3, 8, 13, 18, ..., 78, ...	[ardışık sayı dizisi (consecutive number sequence)]

ARDIŞIK SAYILARIN TOPLANMASI / SUM of CONSECUTIVE NUMBERS

▼ Genel Toplam Formülü / General Summation:

Dizide kaç tane eleman olduğunu ve bu elemanların toplamını bulmak için aşağıda verilen formüller kullanılır.

To find the number of elements and the sum of these elements in a sequence, these formulas are used:

a. Terim sayısı : $T = \dfrac{\text{Son terim} - \text{İlk terim}}{\text{Ortak fark}} + 1$

a. Number of terms : $T = \dfrac{\text{Last term} - \text{First term}}{\text{Common diference}} + 1$

b. Terimlerin toplamı : $S = \dfrac{\text{Son terim} + \text{İlk terim}}{2} \cdot T$

b. Sum of terms : $S = \dfrac{\text{Last term} + \text{First term}}{2} \cdot T$

I. $1 + 2 + 3 + \ldots + n = \dfrac{n.(n+1)}{2}$ [Ardışık tam sayıların toplamı]

[Sum of consecutive integers]

II. $2 + 4 + 6 + \ldots + 2n = n \cdot (n+1)$ [Ardışık çift tam sayıların toplamı]

[Sum of consecutive even integers]

III. $1 + 3 + 5 + \ldots + (2n - 1) = n^2$ [Ardışık tek tam sayıların toplamı]

[Sum of consecutive odd integers]

ASAL SAYILAR / PRIME NUMBERS

Tanım : Kendisinden ve 1 sayısından başka pozitif tam sayı böleni olmayan, 1'den büyük pozitif tam sayılardır.

{2, 3, 5, 7, 11, 13, 17, 19, 23, ...}

Definition: Any positive integer bigger than 1 which is divisible only by 1 and the number itself is called a prime number.

{2, 3, 5, 7, 11, 13, 17, 19, 23, ...}

ARALARINDA ASAL SAYILAR / RELATIVELY PRIME NUMBERS

Tanım: 1'den başka ortak pozitif tam sayı böleni olmayan pozitif tam sayılardır.

Örneğin; 25 ve 12 , 32 ve 15…

Definition: Two integers of which only common factor is 1 are called relatively prime numbers.

For example ; 25 and 12 , 32 and 15…

Önemli / Important

- 1 asal sayı değildir.
- En küçük asal sayı 2'dir.
- Hem çift hem de asal olan tek sayı 2'dir.
- Ardışık iki tam sayı, aralarında asaldır.
- 1 is not prime
- The smallest prime number is 2.
- The only even prime number is 2.
- Two successive integer numbers are relatively prime numbers.

ASAL ÇARPANLARA AYIRMA / PRIME FACTORIZATION

Tanım: Bir doğal sayının hangi asal sayıların çarpımı olduğunun bulunmasıdır.

Definition: If we consider a natural number as the product of prime numbers, then finding those prime numbers is called prime factorization.

Özellik / Property

★ **Bir tam sayının pozitif tam sayı bölenlerinin sayısı:**

$x, y, z \in Z^+$, a, b, c birbirinden farklı asal sayılar olmak üzere

$A = a^x \cdot b^y \cdot c^z$ ise A sayının pozitif tamsayı bölenlerinin sayısı $\mathbf{(x + 1)\cdot(y + 1)\cdot(z + 1)}$ şeklinde bulunur.

★ **The number of positive integer divisors of an integer:**

$x, y, z \in Z^+$, a, b, c are different prime numbers;

if $A = a^x \cdot b^y \cdot c^z$, The number of positive integer divisors of A will be calculated by

$(x + 1)\cdot(y + 1)\cdot(z + 1)$

Önemli / Important

- Bir sayının pozitif tam sayı bölenlerinin sayısı n ise tamsayı bölenlerinin sayısı 2n'dir.
- If the number of positive integer divisors of a certain number is n, the number of its integer divisors is 2n

Özellik / Property

★ **Bir tam sayının pozitif tam sayı bölenlerinin toplamı:**

$m, n \in Z^+$, a ve b birbirinden farklı asal sayılar olmak üzere;

$A = a^m \cdot b^n$ ise A sayısının pozitif tam sayı bölenlerinin toplamı $\left(\frac{a^{m+1}-1}{a-1}\cdot\frac{b^{n+1}-1}{b-1}\right)$ dir.

★ **Sum of positive integer divisors of an iteger:**

$m, n \in Z^+$, a and b are two different prime numbers; if $A = a^m \cdot b^n$, sum of positive integer divisors of A is $\left(\frac{a^{m+1}-1}{a-1}\cdot\frac{b^{n+1}-1}{b-1}\right)$

Önemli / Important

- Bir sayının tam sayı bölenlerinin toplamı 0 dır.
- Sum of integer divisors of a number is 0.

FAKTÖRİYEL / FACTORIAL

Tanım: 1 den n ye kadar olan ardışık doğal sayıların çarpımına **n faktöriyel** denir ve **n!** şeklinde gösterilir.

Definition: For any positive integer n, n factorial represents the product of the first n positive integers and is shown by **n!**

$n! = n \cdot (n-1) \cdot (n-2) \cdot \ldots \cdot 3 \cdot 2 \cdot 1$

$1! = 1$

$2! = 1 \cdot 2 = 2$

$3! = 1 \cdot 2 \cdot 3 = 6$

$4! = 1 \cdot 2 \cdot 3 \cdot 4 = 24$

$5! = 1 \cdot 2 \cdot 3 \cdot 4 . 5 = 120$

$6! = 1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 \cdot 6 = 720$

$\vdots$

Önemli / Important

- 0! sayısı 1 e eşittir.
- 0! is equal to 1.

0! = 1

DOĞAL SAYILARDA BÖLME / DIVISION in the SET of NATURAL NUMBERS

Tanım / Definition: A, B, C, D $\in N^+$

$$\begin{array}{r|l} A & B \\ \cline{2-2} - & C \\ \hline D & \end{array}$$

A → Bölünen (Dividend)
B → Bölen (Divisor)
C → Bölüm (Quotient)
D → Kalan (Remainder)

▼ **A = B·C + D**

▼ D < B

Örneğin / For example:

$$\begin{array}{r|l} 421 & 12 \\ \cline{2-2} -\;36 & 35 \\ \hline 61 & \\ -\;60 & \\ \hline 1 & \end{array}$$

TAM BÖLÜNEBİLME KURALLARI / DIVISIBILITY RULES

2 ile TAM BÖLÜNEBİLME / DIVISIBILITY by 2

▼ Bir doğal sayının 2 ile tam bölünebilmesi için, sayı çift olmalıdır. Birler basamağındaki rakam çift ise kalan 0, tek ise kalan 1'dir.

▼ The number must be even to be divisible by 2. If the ones-place of the numbers is even, the remainder is 0, and if the ones-place of the number is odd, the remainder is 1.

3 ile TAM BÖLÜNEBİLME / DIVISIBILITY by 3

▼ Bir doğal sayının, 3 ile tam bölünebilmesi için, bu sayının rakamları toplamı 3'ün katları olmalıdır.

▼ The sum of the digits of a number must be multiple of 3 in order to the number itself be divisible by 3.

Önemli / Important

- Bir sayının rakamları toplamının 3 ile bölümünden kalan, o sayının 3'ün ile bölümünden kalana eşittir.
- The remainder of the sum of digits of a number when it is divided by 3 is equal to the remainder of that number, when it is divided by 3.

4 ile TAM BÖLÜNEBİLME / DIVISIBILITY by 4

▼ Bir doğal sayının 4 ile tam bölünebilmesi için, sayının son iki basamağının 4 ile tam bölünebilmesi gerekir.

▼ The last two digits of the number must be divisible by 4 in order to the number itself be divisible by 4.

Önemli / Important

- Bir sayının 4 ile bölümünden kalan, sayının son iki basamağının 4 ile bölümünden kalana eşittir.
- The remainder of the number when it is divided by 4 is equal to the number when the last two digits are divided by 4.

5 ile TAM BÖLÜNEBİLME / DIVISIBILITY by 5

▼ Bir doğal sayının 5 ile tam bölünebilmesi için sayının birler basamağı, 0 ya da 5 olmalıdır.

▼ The units digit of the number must be 0 or 5 to be divisible by 5.

Önemli / Important

- Bir doğal sayının 5 ile bölümünden kalan, sayının birler basamağındaki rakamın 5 ile bölümünden kalana eşittir.
- The remainder of the number when it is divided by 5 is equal to the remainder, when the units digit of the number is divided by 5.

9 ile TAM BÖLÜNEBİLME / DIVISIBILITY by 9

▼ Bir doğal sayının 9 ile tam bölünebilmesi için, bu sayının rakamlarının toplamı 9'un katları olmalıdır.

▼ The sum of the digits of the number must be multiple of 9 in order to the number itself be divisible by 9.

Önemli / Important

- Bir sayının 9 ile bölümünden kalan, o sayının rakamları toplamının 9 ile bölümünden kalana eşittir.
- The remainder of the number when it is divided by 9 is equal to the remainder, when the sum of the digits of the number is divided by 9.

11 ile TAM BÖLÜNEBİLME / DIVISIBILITY by 11

▼ Bir doğal sayının 11 ile tam bölünebilmesi için sayının birler basamağından başlayarak her bir rakam sağdan sola sırasıyla "+ – + – + – ..." işaretleriyle işaretlenir. Daha sonra (+) işaretliler ile (–) işaretliler toplanarak farkları alınır ve fark 11'in katı ise bu sayı 11 ile tam bölünür.

▼ In order to a number be divisible by 11, the digits of the number, starting from the units digit, from left to right get signs "+ – + – + – ...." respectively; afterwars, the positive and negative numbers sum up to find their difference, if the difference is a multiple of 11, then this number is divisible by 11.

Önemli / Important

Verilen bir doğal sayının;

- 6 ile bölünebilmesi için 2 ve 3'e
- 15 ile bölünebilmesi için 3 ve 5'e
- 36 ile bölünebilmesi için 4 ve 9'a
- 45 ile bölünebilmesi için 5 ve 9'a tam bölünebilmesi gerekir.

A natural number;

- must be divisible by 2 and 3 to be divisible by 6
- must be divisible by 3 and 5 to be divisible by 15
- must be divisible by 4 and 9 to be divisible by 36
- must be divisible by 5 and 9 to be divisible by 45

OBEB (ORTAK BÖLENLERİN EN BÜYÜĞÜ) / GCF (GREATEST COMMON FACTOR)

Tanım: İki ya da daha fazla doğal sayıyı ortak olarak tam bölebilen, en büyük doğal sayıya, bu sayıların **ortak bölenlerin en büyüğü (OBEB)** denir.

Definition: The greatest number, which divides two or more than two numbers commonly, is called GCF(Greatest Common Factor).

Önemli / Important

- a ve b aralarında asal ise, **OBEB(a, b)** = 1'dir.
- $x, a, b, c, d \in Z$

  $x \cdot a = c \, , \, x \cdot b = d$

  $\Rightarrow \max(x) = \text{OBEB}(c, d)$
- If a and b are relatively prime, then GCF(a,b) = 1.
- $x, a, b, c, d \in Z$

  $x \cdot a = c \, , \, x \cdot b = d$

  $\Rightarrow \max(x) = \text{GCF}(c, d)$

OKEK (ORTAK KATLARIN EN KÜÇÜĞÜ) / LCM (LEAST COMMON MULTIPLE)

Tanım: İki ya da daha fazla doğal sayının ortak katlarının en küçüğüne o sayıların **ortak katlarının en küçüğü (OKEK)** denir.

Definition: The least common multiple of two or more than two numbers is called **LCM(Least Common Multiple)** of those numbers.

Önemli / Important

- a ve b aralarında asal ise, **OKEK(a, b) = a · b**'dir.
- $a, b \in N^+$

  $\mathbf{OKEK(a, b) \cdot OBEB(a, b) = a \cdot b}$
- If a and b are relatively prime, then LCM(a, b) = a · b
- $a, b \in N^+$

  $\text{LCM}(a, b) \cdot \text{GCF}(a, b) = a \cdot b$

1. $a, b, c, d, e, f \in Z^+$

$$\left.\begin{array}{l} a-b=4\\ c-d=7\\ e-f=11\end{array}\right\} \Rightarrow \dfrac{\mathbf{-(a+c)+(b+d)}}{\mathbf{f-e}}=\mathbf{?}$$

A) −2 B) −1 C) 0 D) 1 E) 2

2. $$\left.\begin{array}{l} x\cdot y\cdot z>0\\ x\cdot y<0\\ y\cdot z<0\end{array}\right\} \Rightarrow \mathbf{x, y, z = ?}$$

A)+, +, − B) −, +, − C) +, +, +
D) −, +, + E) −, −, −

3. $a, b, c, d \in R$,
$\dfrac{2a-c}{a}=0,\ \ a\cdot d=0,\ \ 3a-5d=6 \ \Rightarrow\ \mathbf{c = ?}$

A) − 4 B) −2 C) 0 D) 2 E) 4

4. $a \cdot b = 4,\ a \cdot b^2 = 16 \ \Rightarrow\ \dfrac{\mathbf{a+b}}{\mathbf{a}} = \mathbf{?}$

A) 2 B) 3 C) 4 D) 5 E) 6

5. $a, b, c \in Z,\quad a \neq b \neq c$
$a\cdot b = 12,\ \ \dfrac{a}{c} = b \ \Rightarrow\ \mathbf{a + b + c = \ ?}$

A) 10 B) 11 C) 12 D) 15 E) 25

6. $x, y, z \in Z^+$
$x \cdot y = 5\ ,\ \ x \cdot z = 3 \ \Rightarrow\ \mathbf{x - y - z = ?}$

A) −9 B) −8 C) −7 D) −6 E) − 5

7. $a, b, c \in N^+$
$$\left.\begin{array}{l} a\cdot b=13\\ b\cdot c=17\end{array}\right\} \Rightarrow \mathbf{a+b+c=?}$$

A) 20 B) 21 C) 30 D) 31 E) 40

8. $a, b, c \in Z^+$
$$\left.\begin{array}{l} a\cdot b=21\\ b\cdot c=7\\ b>c\end{array}\right\} \Rightarrow \mathbf{a+b+c=?}$$

A) 11 B) 10 C) 9 D) 8 E) 7

9. $a, b, c \in \mathbb{N}, \quad 3 < c < 6,$

$\frac{b}{a} = c, \quad a + b = 10 \quad \Rightarrow \quad \mathbf{a \cdot b = ?}$

A) 10 B) 12 C) 14 D) 16 E) 18

10. $a, b, c \in \mathbb{R},$

$3a + b = 20, \ 2a = c \quad \Rightarrow \quad \mathbf{a + b + c = ?}$

A) 30 B) 25 C) 20 D) 15 E) 10

11. $x, y, z \in \mathbb{Z}^+, x \neq y \neq z$

$2x + y + 3z = 34 \quad \Rightarrow \quad \mathbf{max(x) = ?}$

A) 10 B) 11 C) 12 D) 13 E) 14

12. $x, y, z \in \mathbb{Z},$

$4x + 2y + 3z = 37 \quad \Rightarrow \quad \mathbf{max(x) = ?}$

A) 5 B) 6 C) 7 D) 8 E) 9

13. $x, y \in \mathbb{N}$

$x + y = 24$

$\Rightarrow \quad \mathbf{min(x \cdot y) + max(x \cdot y) = ?}$

A) 144 B) 148 C) 156 D) 164 E) 168

14. $x, y \in \mathbb{N}^+, x \neq y$

$x \cdot y = 256$

$\Rightarrow \quad \mathbf{min(x + y) = ?}$

A) 32 B) 34 C) 36 D) 38 E) 40

15. $x, \frac{1}{y} \in \mathbb{N}$

$x + \frac{1}{y} = 24 \quad \Rightarrow \quad \mathbf{max\left(\frac{x}{y}\right) = ?}$

A) 132 B) 136 C) 140 D) 144 E) 148

16. $x, y, z \in \mathbb{Z},$

$$\left.\begin{aligned} x \cdot y &= 28 \\ y \cdot z &= 16 \end{aligned}\right\} \Rightarrow \mathbf{min(x + y + z) = ?}$$

A) 15 B) 5 C) – 15 D) – 36 E) – 45

YANITLAR / ANSWERS

1	2	3	4	5	6	7	8
D	B	E	D	B	C	D	A
9	**10**	**11**	**12**	**13**	**14**	**15**	**16**
D	C	E	D	A	E	D	E

1. $[8 : 2 - 3 \cdot 2 + 16] - 10 : 2 = ?$

A) 5 B) 7 C) 9 D) 13 E) 16

2. $[2 + 3 - 4 : 2 - 10 + 2 - 5] : 2 = ?$

A) –5 B) –4 C) 2 D) 3 E) 10

3. $[-5 + 2 \cdot 10 - 3 - 2] + (10 : 2 - 5) = ?$

A) 5 B) 6 C) 7 D) 8 E) 10

4. $[(-2 + 1 - 3 \cdot (4 : 2 - 5 + 10))] = ?$

A) –22 B) 21 C) 19 D) 17 E) 12

5. $[(6 : 3 - (4 : 2) + 18 \cdot 2)] + (20 : 2) \cdot 2 = ?$

A) 2 B) 18 C) 36 D) 41 E) 56

6. a ve b birer çift sayıdır. / a and b are even numbers.

Buna göre aşağıdakilerden hangisi kesinlikle tek sayıdır?

Accordingly, which of the following is definitely an odd number?

A) $5a + 7b$ B) $\frac{a-b}{2}$ C) $a + b$

D) $\frac{a}{2} + b$ E) $(a+1) \cdot (b-1)$

7. **$x \in Z^+$ olmak üzere aşağıdakilerden hangisi kesinlikle tek sayıdır?**

Which of the following is definitely an odd number if $x \in Z^+$?

A) $x^2 + 5$ B) $x \cdot (x - 1) + 4$ C) $x \cdot (x + 1) + 3$

D) $5^x + x^5$ E) $x! + 1$

8. **x tek, y çift sayı ise aşağıdakilerden hangisi kesinlikle tek sayıdır?**

Which of the following is definitely an odd number if x is an odd and y is an even number?

A) $2x + y$ B) $2\left(\frac{x}{y}\right)$ C) $x + y - 2$

D) $x \cdot y^2$ E) $4x + y$

9. $x, y, z \in Z^+$

$\frac{3x+y}{6} = z$ ise aşağıdakilerden hangisi kesinlikle doğrudur?

If $\frac{3x+y}{6} = z$, which of the following is definitely true?

A) $x \cdot y + z$ çifttir. / is even.

B) $x + y + z$ tektir. / is odd.

C) x çift ise y tektir. / If x is even, y is an odd number.

D) y çift ise x tektir. / If y is even, x is an odd number.

E) $x + 3y + 2z$ çifttir. / is even.

10. $x, y, z \in Z$

$\frac{2x-3y}{z} = 3$ ise aşağıdakilerden hangisi kesinlikle doğrudur?

If $\frac{2x-3y}{z} = 3$, which of the following is definitely true?

A) x çift ise y çifttir. / If x is even, y is an even number.

B) x tek ise z tektir. / If x is odd, z is an odd number.

C) y tek ise z çifttir. / If y is odd, z is an even number.

D) y çift ise z çifttir. / If y is even, z is an even number.

E) y çift ise z tektir. / If y is even, z is an odd number.

11. a, b ve c sırasıyla ardışık üç çift sayıdır.

a, b and c are three consecutive even numbers.

Buna göre / Accordingly, **$(a-b)^2 + (a-c)^2 - (b-c)^2 = ?$**

A) 2 B) 4 C) 8 D) 16 E) 32

12. $x, y, z \in Z^+$

$\frac{x^2+y+z}{4} = z$ ise aşağıdakilerden hangisi kesinlikle doğrudur?

If $\frac{x^2+y+z}{4} = z$, which of the following is definitely true?

A) $x \cdot z$ çifttir. / is even.

B) $y \cdot z$ çifttir. / is even.

C) $x \cdot y$ tektir. / is odd.

D) $x - y$ çifttir. / is even.

E) $x + y - z$ çifttir. / is even.

13. $A, B \in Z$

$A = \frac{2a-1}{a+4}$, $B = \frac{a+4}{2a-1}$

$\Rightarrow \sum a = ?$

A) – 1 B) 1 C) 2 D) 4 E) 5

14. $A = \frac{8x+15}{x}$, $A \in Z \Rightarrow \sum x = ?$

A) 24 B) 9 C) 0 D) – 9 E) – 24

15. $A = \frac{3x+13}{x+1}$, $A \in Z^+ \Rightarrow \sum x = ?$

A) – 3 B) 0 C) 1 D) 5 E) 11

YANITLAR / ANSWERS

1	2	3	4	5	6	7	8
C	A	E	A	E	E	C	C
9	10	11	12	13	14	15	
E	D	D	D	D	C	A	

1. $ab, ba, bb \in N$

$\frac{ab+ba}{bb} = 7 \Rightarrow \mathbf{\frac{a+b}{a-5b} = ?}$

A) 4 B) 5 C) 6 D) 7 E) 8

2. $ab \in N$

$ab = 6 \cdot (a+b) + 8 \Rightarrow \mathbf{a - b = ?}$

A) 2 B) 3 C) 4 D) 5 E) 6

3. $abc, bac \in N$

$\left.\begin{array}{l} abc - bac = 630 \\ 4 \cdot a + c - 4 \cdot b = 35 \end{array}\right\} \Rightarrow \mathbf{c = ?}$

A) 5 B) 6 C) 7 D) 8 E) 9

4. $ABC, ABC5 \in N$

$ABC = x \Rightarrow \mathbf{ABC5 = ?}$

A) 10x B) x + 5 C) 10x – 5 D) 2x + 5 E) 10x + 5

5. $baa, abb \in N$

$baa - abb = 356 \Rightarrow \mathbf{max(a+b) = ?}$

A) 9 B) 10 C) 11 D) 14 E) 15

6. $aa, bb, cc \in N$

$\left.\begin{array}{l} aa - bb = cc \\ aa + bb + cc = 176 \end{array}\right\} \Rightarrow \mathbf{a = ?}$

A) 4 B) 5 C) 6 D) 7 E) 8

7. $ab, ba \in N$

$ab + ba = 7 \cdot (a+b) + 48 \Rightarrow \mathbf{a + b = ?}$

A) 9 B) 10 C) 11 D) 12 E) 13

8. $ab3, ab \in N$

$ab3 = 3 \cdot (ab) + 171 \Rightarrow \mathbf{a \cdot b = ?}$

A) 6 B) 8 C) 10 D) 12 E) 15

9. $ab,\ ba \in N$

$ab + ba = 88 \Rightarrow$ **$a + b = ?$**

A) 6 B) 7 C) 8 D) 9 E) 10

10. $ab,\ ba \in N$

$ab - ba = 54 \Rightarrow$ **$a - b = ?$**

A) 5 B) 4 C) 5 D) 6 E) 7

11. $ab,\ ba \in N$

$ab + ba = 132$

$\Rightarrow$ **$\min(a) = ?$**

A) 1 B) 2 C) 3 D) 4 E) 5

12. $abc,\ ab,\ ca,\ bc \in N$

$ab + ca + bc = 88$

$\Rightarrow$ **$\max(abc) = ?$**

A) 521 B) 611 C) 620 D) 710 E) 800

13. $abc,\ cba \in N,\ a \neq b \neq c$

$abc - cba = 495$

$\Rightarrow$ **$\min(abc) = ?$**

A) 691 B) 500 C) 611 D) 601 E) 600

14. $abc,\ cba \in N$

$abc - cba = 792$

$\Rightarrow$ **$\max(abc) = ?$**

A) 982 B) 990 C) 991 D) 992 E) 998

15. $ab \in N$

$ab = 8\cdot(a+b) \Rightarrow$ **$a\cdot b = ?$**

A) 10 B) 11 C) 12 D) 13 E) 14

16. $\left.\begin{array}{l} x = A5B4 \\ y = A3B6 \end{array}\right\} \Rightarrow$ **$x - y = ?$**

A) 298 B) 208 C) 198 D) 108 E) 98

YANITLAR / ANSWERS

1	2	3	4	5	6	7	8
D	B	C	E	D	E	D	B
9	**10**	**11**	**12**	**13**	**14**	**15**	**16**
C	D	C	A	D	C	E	C

1. $(21ab)_5 = 294 \Rightarrow \mathbf{a = ?}$

A) 0 B) 1 C) 2 D) 3 E) 4

2. $\dfrac{\mathbf{(145)_6}}{\mathbf{(12)_3}} = \mathbf{(?)_2}$

A) 1011 B) 1101 C) 101 D) 110 E) 1111

3. $(333)_x - (33)_x = 108 \Rightarrow \mathbf{x = ?}$

A) 4 B) 5 C) 6 D) 7 E) 8

4. $\mathbf{(442)_5 + (224)_5 = ?}$

A) $(123)_5$ B) $(1221)_5$ C) $(2102)_5$ D) $(2002)_5$ E) $(3022)_5$

5. $\mathbf{(443)_5 \cdot (34)_5 = ?}$

A) $(22302)_5$ B) $(32322)_5$ C) $(33322)_5$ D) $(30522)_5$ E) $(32202)_5$

6. $(1aa1)_4 = (a25)_8 \Rightarrow \mathbf{a = ?}$

A) 0 B) 1 C) 2 D) 3 E) 4

7. $(xxy)_6 = (324)_5 \Rightarrow \mathbf{x + y = ?}$

A) 12 B) 10 C) 9 D) 7 E) 5

8. $(102)_a = 51 \Rightarrow \mathbf{a = ?}$

A) 3 B) 4 C) 5 D) 6 E) 7

9. $(3x2)_5 + (214)_5 = (1111)_5 \Rightarrow$ **$x = ?$**

A) 0 B) 1 C) 2 D) 3 E) 4

10. $(3x21)_5 = (4031)_5 - (410)_5 \Rightarrow$ **$x = ?$**

A) 4 B) 3 C) 2 D) 1 E) 0

11. $(2x2)_4 + (1xx)_4 = (10x1)_4 \Rightarrow$ **$x = ?$**

A) 0 B) 1 C) 2 D) 3 E) 4

12. $(xy44)_5 = (43)_5 \cdot (23)_5 \Rightarrow$ **$x + y = ?$**

A) 2 B) 3 C) 4 D) 5 E) 6

13. $(41)_x = 37 \Rightarrow$ **$x = ?$**

A) 6 B) 7 C) 8 D) 9 E) 10

14. $(14)_x + (35)_x = (51)_x \Rightarrow$ **$x = ?$**

A) 4 B) 5 C) 6 D) 7 E) 8

15. $(1201)_x = 46 \Rightarrow$ **$x = ?$**

A) 3 B) 4 C) 5 D) 6 E) 7

16. $A = 3 \cdot 7^5 + 4 \cdot 7^3 + 2 \cdot 7^2 + 25$

$\Rightarrow$ **$(A)_7 = ?$**

A) 34225 B) 304234 C) 304225 D) 304226 E) 304224

17. $(111n)_4 < (2n2)_6 \Rightarrow$ **$n = ?$**

A) 0 B) 1 C) 2 D) 3 E) 4

18. $\left.\begin{array}{l} x = (1a4)_5 \\ y = (233)_a \end{array}\right\} \Rightarrow$ **$x + y = ?$**

A) 90 B) 94 C) 96 D) 98 E) 100

YANITLAR / ANSWERS

1	2	3	4	5	6	7	8	9
D	B	C	B	C	B	D	E	E
10	11	12	13	14	15	16	17	18
D	D	B	D	E	A	B	D	C

1. **$1 + 2 + 3 + \ldots + 20 = ?$**

A) 200 B) 205 C) 210 D) 220 E) 240

2. **$12 + 16 + 20 + \ldots + 116 = ?$**

A) 1619 B) 1728 C) 1816
D) 1921 E) 2110

3. **$17 + 21 + 25 + 29 + \ldots + 81 + 85 = ?$**

A) 902 B) 918 C) 924
D) 936 E) 940

4. **$70 + 75 + 80 + \ldots + 145 + 150 = ?$**

A) 1825 B) 1850 C) 1870
D) 1885 E) 2000

5. **$10 + 12 + 14 + 16 + \ldots + 98 + 100 = ?$**

A) 2530 B) 2540 C) 2548
D) 2550 E) 2556

6. **$40 + 42 + 44 + \ldots + 94 + 96 = ?$**

A) 1958 B)1960 C) 1964
D) 1970 E) 1972

7. **$3 + 6 + 9 + \ldots + 24 + 27 + 28 + 26 + 24 + \ldots + 12 = ?$**

A) 315 B) 350 C) 380
D) 385 E) 400

8. **$1 - 2 + 3 - 4 + 5 - 6 + \ldots + 59 - 60 = ?$**

A) –92 B) –81 C) –30 D) 72 E) +68

9. $1 - 2 + 3 - 4 + 5 - 6 + ... + 71 - 72 + 73 = ?$

A) 36 B) 37 C) 73 D) 145 E) 189

10. $\frac{1}{2} + \frac{2}{3} + \frac{3}{2} + \frac{4}{3} + \frac{5}{2} + ... + \frac{23}{2} + \frac{24}{3} = ?$

A) 198 B) 156 C) 140 D) 126 E) 124

11. $1 + 2 + 3 + ... + 2n = 465 \Rightarrow n = ?$

A) 15 B) 18 C) 20 D) 25 E) 30

12. $11 + 13 + 15 + ... + (2n - 1) = 375 \Rightarrow n = ?$

A) 10 B) 15 C) 20 D) 22 E) 25

13. $21 + 23 + 25 + ... + (2n - 1) = 800 \Rightarrow n = ?$

A) 24 B) 26 C) 28 D) 30 E) 40

14. $1 + 2 + 3 + ... + 7n = x - 2,$

$8 + 10 + 12 + ... + 14n = 202 \Rightarrow x = ?$

A) 104 B) 105 C) 106 D) 107 E) 109

15. $A = 5 + 8 + 11 + ... + 98$

$B = 7 + 10 + 13 + ... + 100$

$\Rightarrow B - A = ?$

A) 60 B) 62 C) 64 D) 66 E) 68

16. $A = 13 + 14 + 15 + ... + n$

$B = 16 + 17 + 18 + ... + n$

$A + B = 222 \Rightarrow A = ?$

A) 130 B) 131 C) 132 D) 134 E) 136

17. $a + (a + 1) + (a + 2) + ... + (a + 14) = 165$

$\Rightarrow a = ?$

A) 2 B) 3 C) 4 D) 5 E) 6

18. $A = 12 + 13 + ... + n$

$B = 11 + 12 + ... + n + (n + 1)$

$\Rightarrow B - A = ?$

A) $n - 10$ B) $n + 10$ C) $2n - 1$ D) $n + 11$ E) $n + 12$

YANITLAR / ANSWERS

1	2	3	4	5	6	7	8	9
C	B	B	C	A	E	A	C	B
10	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**
E	A	C	D	E	C	C	C	E

1. $a, b, c \in \mathbb{Z}$

$360 = 2^a \cdot 3^b \cdot 5^c \Rightarrow$ **$a + b + c = ?$**

A) 4 B) 5 C) 6 D) 7 E) 8

2. $a, b \in \mathbb{N}^+$

$756 \cdot a = b^3 \Rightarrow$ **$\min(a) = ?$**

A) 42 B) 49 C) 70 D) 98 E) 147

3. $a, b \in \mathbb{N}$

$720 \cdot a = b^3 \Rightarrow$ **$\min(a + b) = ?$**

A) 150 B) 135 C) 210 D) 300 E) 360

4. **750 sayısının asal bölenlerinin toplamı kaçtır?**

What is sum of the prime number divisors of the number 750?

A) 2 B) 3 C) 5 D) 8 E) 10

5. **840 sayısının kaç tane asal böleni vardır?**

How many prime number divisors does the number 840 have?

A) 2 B) 3 C) 4 D) 5 E) 6

6. **480 sayısının kaç tane pozitif tam sayı böleni vardır?**

How many integer divisors does the number 480 have?

A) 24 B) 18 C) 16 D) 12 E) 6

7. **1200 sayısının kaç tane tam sayı böleni vardır?**

How many integer divisors does the number 1200 have?

A) 75 B) 60 C) 45 D) 30 E) 15

8. **540 sayısının asal olmayan kaç tane pozitif tam sayı böleni vardır?**

How many non-prime positive integer divisors does the number 540 have?

A) 24 B) 23 C) 22 D) 21 E) 20

9. **700 sayısının asal olmayan kaç tane tam sayı böleni vardır?**

How many non-prime integer divisors does the number 700 have?

A) 36 B) 34 C) 33 D) 32 E) 30

10. **400 sayısının pozitif tam sayı bölenlerinin toplamı kaçtır?**

What is sum of the positive integer divisors of the number 400?

A) 900 B) 961 C) 980 D) 1000 E) 1020

11. **300 sayısının asal olmayan pozitif bölenlerinin toplamı kaçtır?**

What is sum of the non-prime positive divisors of the number 300?

A) 858 B) 860 C) 862 D) 864 E) 868

12. **1240 sayısının tam sayı bölenlerinin toplamı kaçtır?**

What is sum of the integer divisors of the number 1240?

A) 4450 B) 3200 C) 960 D) 400 E) 0

13. **320 sayısının asal olmayan tam sayı bölenlerinin toplamı kaçtır?**

What is sum of the non-prime integer divisors of the number 320?

A) 7 B) 4 C) 0 D) 4 E) – 7

14. **420 sayısının pozitif tam sayı bölenlerinin kaç tanesi 3 ün katı değildir?**

How many of positive integer divisors of the number 420 are not multiples of 3?

A) 24 B) 16 C) 12 D) 9 E) 6

15. **600 sayısının pozitif tam sayı bölenlerinin kaç tanesi 5 ile tam bölünmez?**

How many of positive integer divisors of the number 600 can not be divided by 5?

A) 10 B) 8 C) 6 D) 4 E) 2

16. **120 sayısının pozitif tam sayı bölenlerinin çarpımı kaçtır?**

What is the product of the positive integer divisors of the number 120?

A) 120^2 B) 120^4 C) 120^6 D) 120^8 E) 120^{16}

YANITLAR / ANSWERS

1	2	3	4	5	6	7	8
C	D	E	E	C	A	B	D
9	**10**	**11**	**12**	**13**	**14**	**15**	**16**
C	B	A	E	E	C	B	D

1. $\dfrac{8!+7!}{8!-7!}=?$

A) $\frac{7}{8}$ B) $\frac{8}{7}$ C) $\frac{9}{7}$ D) $\frac{8}{9}$ E) $\frac{9}{8}$

2. $\dfrac{5\cdot6!+12\cdot5!}{7!}=?$

A) 1 B) 3 C) 5 D) 12 E) 15

3. $\dfrac{8!-5!}{5\cdot5!}=?$

A) 35 B) 48 C) 54 D) 67 E) 72

4. $\dfrac{12!-6\cdot10!}{63\cdot10!}=?$

A) 1 B) 2 C) 3 D) 4 E) 5

5. $\dfrac{6\cdot(6!+5!)}{7!-6!}=?$

A) 5 B) 6 C) $\frac{7}{6}$ D) 8 E) $\frac{9}{10}$

6. $\dfrac{12!+3\cdot10!}{4\cdot9!-9\cdot8!}=?$

A) 380 B) 400 C) 420 D) 435 E) 450

7. $\dfrac{8!-7!}{2\cdot6!+9\cdot5!}=?$

A) 15 B) 14 C) 13 D) 12 E) 11

8. $6!\cdot x+7!=9! \Rightarrow x=?$

A) 482 B) 490 C) 497 D) 501 E) 504

9. $\dfrac{14!-a\cdot 12!}{2}=12! \Rightarrow$ **a = ?**

A) 100 B) 120 C) 180 D) 182 E) 198

10. $24\cdot(21!)=x \Rightarrow$ **22! + 23! =?**

A) $24\cdot x$ B) $23\cdot x$ C) $22\cdot x$
D) $21\cdot x$ E) $20\cdot x$

11. $(n+1)!=3!\cdot n! \Rightarrow$ **n! = ?**

A) 6 B) 24 C) 30 D) 120 E) 720

12. $\dfrac{(n+2)!-n!}{n!}=29 \Rightarrow$ **n = ?**

A) 7 B) 6 C) 5 D) 4 E) 2

13. $\dfrac{n!}{(n-2)!}=12 \Rightarrow$ **n = ?**

A) 2 B) 3 C) 4 D) 5 E) 6

14. $\dfrac{(3n)!\cdot(n-3)!}{(3n-1)!\cdot(n-2)!}=?$

A) n B) 3n – 3 C) $\dfrac{3n}{n-2}$
D) $\dfrac{n-3}{3n-1}$ E) $\dfrac{n-3}{n-2}$

15. $\dfrac{(n-1)!+(n-2)!}{(n-2)!\cdot(n-1)!}=?$

A) $\dfrac{n-1}{n!}$ B) $\dfrac{n^2}{n!}$ C) $\dfrac{n-2}{(n-1)!}$
D) $\dfrac{n}{(n-1)!}$ E) $\dfrac{n^2}{n-1}$

16. $\dfrac{(n-6)!}{2\cdot(n-5)!}=\dfrac{1}{14} \Rightarrow$ **n = ?**

A) 14 B) 13 C) 12 D) 10 E) 1

17. $\dfrac{8!+9!+10!}{100\cdot(6!+7!)}=?$

A) 7 B) 8 C) 10 D) 20 E) 50

18. $\dfrac{a}{12}=10! \Rightarrow$ **12! + 13! = ?**

A) $130\cdot a$ B) $144\cdot a$ C) $148\cdot a$
D) $150\cdot a$ E) $154\cdot a$

YANITLAR / ANSWERS

1	2	3	4	5	6	7	8	9
C	A	D	B	C	E	B	C	C
10	11	12	13	14	15	16	17	18
C	D	D	C	C	D	C	A	E

1. $\frac{(n+1)!-n!}{3n\cdot(n-1)!}=2 \Rightarrow \mathbf{n=?}$

A) 2 B) 4 C) 5 D) 6 E) 7

2. $\frac{(n+1)!+(n-1)!}{2\cdot(n!)+n-1)!}=1 \Rightarrow \mathbf{n=?}$

A) 1 B) 2 C) 3 D) 4 E) 5

3. $\frac{(n-1)!}{n!}+\frac{n!}{(n+1)!}=\frac{9}{20} \Rightarrow \mathbf{n=?}$

A) 3 B) 4 C) 5 D) 6 E) 8

4. $\frac{(n+1)!+2(n!)}{(n+2)!+(n+1)!}=\frac{1}{5} \Rightarrow \mathbf{n=?}$

A) 8 B) 7 C) 6 D) 5 E) 4

5. $\frac{a!\cdot(2a+1)!}{(a-1)!\cdot(2a+2)!}=\frac{4}{9} \Rightarrow \mathbf{a=?}$

A) 8 B) 6 C) 4 D) 3 E) 2

6. $\mathbf{\frac{n!+2\cdot(n-1)!}{(n+1)!+n!}=?}$

A) $\frac{n-1}{n}$ B) $\frac{1}{n}$ C) $\frac{1}{n^2}$ D) $\frac{n-1}{n}$ E) $\frac{n+2}{n^2-1}$

7. $\frac{(2n-5)!}{(3n)!}:\frac{(2n-4)!}{(3n+1)!}=\frac{11}{5} \Rightarrow \mathbf{n=?}$

A) 11 B) 10 C) 9 D) 8 E) 7

8. $\frac{(n+1)!+n!}{(n+2)!}=\frac{1}{10} \Rightarrow \mathbf{n=?}$

A) 1 B) 4 C) 6 D) 8 E) 9

9. $n > 1$

$$\frac{(n+1)!}{(n-1)!} = 7n-5 \Rightarrow \mathbf{n = ?}$$

A) 2 B) 3 C) 4 D) 5 E) 6

10. $n, A \in N$

$$84! = 5^n \cdot A \Rightarrow \mathbf{max(n) = ?}$$

A) 17 B) 18 C) 19 D) 20 E) 21

11. $n, m, A \in N$

$$73! = 3^n \cdot 4^m \cdot A \Rightarrow \mathbf{max(n+m) = ?}$$

A) 68 B) 69 C) 70 D) 72 E) 73

12. $n, A \in N$

$$27! + 28! = A \cdot 10^n \Rightarrow \mathbf{max(n) = ?}$$

A) 4 B) 5 C) 6 D) 7 E) 8

13. $n, A \in N$

$$78! + 79! = A \cdot 10^n \Rightarrow \mathbf{max(n) = ?}$$

A) 15 B) 16 C) 17 D) 18 E) 19

14. $$\frac{(n-2)!}{a!} = 120 \Rightarrow \mathbf{min(n+a) = ?}$$

A) 5 B) 6 C) 7 D) 8 E) 9

15. $$\frac{(n-3)!}{72} = 7! \Rightarrow \mathbf{n = ?}$$

A) 9 B) 10 C) 11 D) 12 E) 13

16. $$\frac{(n+1)!}{1+2+3+\ldots+n} = 2 \cdot 7! \Rightarrow \mathbf{n = ?}$$

A) 6 B) 7 C) 8 D) 9 E) 10

17. $A = 0! + 1! + 2! + \ldots + 43!$

$$\Rightarrow \begin{array}{r|l} A & 5 \\ \hline \underline{=\quad\quad} & \\ \mathbf{?} & \end{array}$$

A) 0 B) 1 C) 2 D) 3 E) 4

18. $a, b \in N$

$$\Rightarrow \frac{\mathbf{[(a-b)\cdot(a+b)]!}}{\mathbf{(a^2-b^2-1)!\cdot(a+b)}} = \mathbf{?}$$

A) $a+b-1$ B) 1 C) $a-b$ D) $a+b$ E) $\frac{a-b}{a+b}$

YANITLAR / ANSWERS

1	2	3	4	5	6	7	8	9
D	A	B	E	A	B	E	E	D
10	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**
C	B	C	E	C	D	C	E	C

1. $\begin{array}{c|c} A & B \\ \cline{2-2} - & 4 \\ \cline{1-1} 1 & \end{array}$ ve / and $A + B = 56$

$\Rightarrow$ **A = ?**

A) 40 B) 41 C) 42 D) 43 E) 45

2. $\begin{array}{c|c} A & B \\ \cline{2-2} - & 3 \\ \cline{1-1} 4 & \end{array}$ ve / and $\begin{array}{c|c} B & 5 \\ \cline{2-2} - & C \\ \cline{1-1} 2 & \end{array}$

$\Rightarrow$ **C = ?**

A) $\frac{A-10}{15}$ B) $\frac{A+10}{5}$ C) $\frac{A+10}{15}$ D) $\frac{A-6}{5}$ E) $\frac{A-4}{5}$

3. $\begin{array}{c|c} A & B^2 \\ \cline{2-2} - & 5 \\ \cline{1-1} 4 & \end{array}$ $\Rightarrow$ **min(A) = ?**

A) 9 B) 24 C) 29 D) 129 E) 134

4. $\begin{array}{c|c} A & 25 \\ \cline{2-2} - & 4 \\ \cline{1-1} B^3 & \end{array}$ $\Rightarrow$ **max(A) = ?**

A) 100 B) 101 C) 124 D) 127 E) 225

5. $\begin{array}{c|c} A & B \\ \cline{2-2} - & 9 \\ \cline{1-1} 3 & \end{array}$ ve / and $\begin{array}{c|c} A+2 & B+1 \\ \cline{2-2} - & 8 \\ \cline{1-1} 2 & \end{array}$

$\Rightarrow$ **A = ?**

A) 45 B) 48 C) 50 D) 54 E) 64

6. $\begin{array}{c|c} AB0AB3 & AB \\ \cline{2-2} - & x \\ \cline{1-1} y & \end{array}$ $\Rightarrow$ **x + y = ?**

A) 104 B) 1003 C) 1013 D) 1113 E) 10013

7. $\begin{array}{r|l} A & 4 \\ \cline{2-2} - \quad & B \\ \cline{1-1} 2 & \end{array}$ ve / and $\begin{array}{r|l} B & 3 \\ \cline{2-2} - \quad & C \\ \cline{1-1} 1 & \end{array}$

$\Rightarrow \begin{array}{r|l} A & 12 \\ \cline{2-2} - \quad & \\ \cline{1-1} ? & \end{array}$

A) 3 B) 5 C) 6 D) 7 E) 9

8. $\begin{array}{r|l} A & 7 \\ \cline{2-2} - \quad & \\ \cline{1-1} 4 & \end{array}$ ve / and $\begin{array}{r|l} B & 7 \\ \cline{2-2} - \quad & \\ \cline{1-1} 5 & \end{array}$

$\Rightarrow \begin{array}{r|l} 2A + 3B & 7 \\ \cline{2-2} - \quad & \\ \cline{1-1} ? & \end{array}$

A) 1 B) 2 C) 3 D) 5 E) 6

9. $\begin{array}{r|l} A & B \\ \cline{2-2} - \quad & 3 \\ \cline{1-1} 4 & \end{array}$ $\begin{array}{r|l} C & 6 \\ \cline{2-2} - \quad & D \\ \cline{1-1} B & \end{array}$

$\Rightarrow$ **A = ?**

A) 16 B) 19 C) 22 D) 25 E) 28

10. $\begin{array}{r|l} 4325471 & 9 \\ \cline{2-2} - \quad & \\ \cline{1-1} ? & \end{array}$

A) 2 B) 4 C) 5 D) 7 E) 8

11. $\begin{array}{r|l} 3472643 & 5 \\ \cline{2-2} - \quad & \\ \cline{1-1} ? & \end{array}$

A) 0 B) 1 C) 2 D) 3 E) 4

12. A = 28!

$\Rightarrow \begin{array}{r|l} A & 5 \\ \cline{2-2} - \quad & \\ \cline{1-1} ? & \end{array}$

A) 0 B) 1 C) 2 D) 3 E) 4

YANITLAR / ANSWERS

1	2	3	4	5	6	7	8	9	10	11	12
E	A	C	C	B	E	C	B	B	E	D	A

1. **OBEB / GCF (72,96) + OKEK / LCM (72,96) = ?**

A) 288 B) 292 C) 296 D) 308 E) 312

2. **OKEK / LCM (48,64,72) = ?**

A) 144 B) 288 C) 576 D) 840 E) 1200

3. x ve y aralarında asal sayılar olmak üzere / relatively prime numbers

$x \cdot y = 240 \Rightarrow$ **OKEK / LCM (x,y) = ?**

A) 240 B) 196 C) 188 D) 160 E) 120

4. $AB, CD \in N$, $AB \neq CD$

OBEB / GCF (AB,CD) = 12 $\Rightarrow$ **max(AB + CD) = ?**

A) 176 B) 180 C) 184 D) 186 E) 192

5. x, y, z, t asal sayılar / prime numbers

$A = x^4 \cdot y^3 \cdot z$

$B = x^3 \cdot y \cdot z^2 \cdot t^2$

$\Rightarrow$ **OBEB / GCF (A,B) = ?**

A) $x^4 \cdot y^3 \cdot z^2 \cdot t^2$ B) $x^3 \cdot y \cdot z^2$ C) $x^3 \cdot y \cdot z$

D) $x^3 \cdot y \cdot z$ E) $x^4 \cdot y^3 \cdot z^2$

6. $A = 2^4 \cdot 3^2 \cdot 5$

$B = 2 \cdot 3^3 \cdot 5 \cdot 7$

$\Rightarrow$ **OKEK (A, B) = ?**

A) $2 \cdot 3^2 \cdot 5 \cdot 7$ B) $2 \cdot 3^2 \cdot 5$ C) $2^5 \cdot 3^5 \cdot 5^2 \cdot 7$

D) $2^4 \cdot 3^3 \cdot 5^2 \cdot 7$ E) $2^4 \cdot 3^3 \cdot 5 \cdot 7$

7.

$$\begin{array}{ccc|c} X & Y & Z & 2 \\ K & Y & M & 2 \\ P & Y & M & 3 \\ 1 & L & 1 & 5 \\ & 1 & & \end{array} \Rightarrow \mathbf{X+Y+Z=?}$$

A) 33 B) 32 C) 31 D) 30 E) 29

8. $A = 3k + 1 = 5m + 1 = 7n + 1$

$\Rightarrow$ **min(A) = ?**

A) 104 B) 105 C) 106 D) 107 E) 108

9. $A = 8k + 6 = 10n + 8 = 15m + 13$

$\Rightarrow$ **min(A) = ?**

A) 116 B) 118 C) 120 D) 122 E) 124

10. $A = 6k + 3 = 7m + 4 = 8n + 5$

$300 < A < 350 \Rightarrow$ **A = ?**

A) 330 B) 331 C) 332 D) 333 E) 334

11.

1	2	3	4	5	6	7	...
G	Ü	L	G	Ü	L	G	...
S	A	R	I	S	A	R	...

2 satır ve 50 sütunu bulunan yukarıdaki tablonun birinci satırına G, Ü, L harfleri ikinci satırına S, A, R, I harfleri yazılmıştır.

Buna göre, $\boxed{\begin{matrix}G\\S\end{matrix}}$ **harfleri ikinci kez hangi sütunda alt alta gelir?**

In the first row of the table above consisting of two rows and 50 columns, the letters G, Ü, L and in the second row, the letters S, A, R, I are spelled.

Accordingly, in which column the letters $\boxed{\begin{matrix}G\\S\end{matrix}}$ will come across each other?

A) 16 B) 17 C) 18 D) 19 E) 20

12. Boyutları 125 cm ve 200 cm olan bir odanın tabanı kare fayanslar ile kaplanacaktır.

Buna göre en az kaç fayans gereklidir?

The floor of a room with dimensions of 125 cm and 200 cm is going to be furnished with tiles.

Accordingly, at least how many tiles are needed?

A) 30 B) 32 C) 34 D) 40 E) 44

13. 24 litre su ile 42 litre kola birbirine karıştırılmadan eşit hacimli şişelere doldurulacaktır.

Bunun için en az kaç şişeye ihtiyaç vardır?

24 litres of water and 42 litres of cola are to be bottles without them mixing.

Accordingly, at least how many bottles are needed?

A) 10 B) 11 C) 12 D) 13 E) 14

14. İki farklı ip, eşit uzunlukta küçük parçalara ayrılacaktır. İplerden biri 24 m diğeri 16 m dir.

Bu ipleri en az sayıda parçaya ayırmak için toplam kaç defa kesme işlemi yapılmalıdır?

Two different rods are to be cut to equal length parts. The length of one of them is 24 m and the length of the other is 16 m.

In order for these rods to be cut into the least number of small pieces, how many operations of cutting should be performed?

A) 2 B) 3 C) 4 D) 5 E) 6

15. İki doktordan biri 6 günde diğeri 10 günde bir nöbet tutmaktadır.

Birlikte tuttukları ilk nöbetten kaç gün sonra ikinci kez birlikte nöbet tutarlar?

One of two doctors takes a shift in every 6 days, and the other takes a shift in every 10 days.

How many days after their first shift together, the take a shift in same day for the second time?

A) 24 B) 26 C) 28 D) 30 E) 32

16. Boyutları 12 cm, 15 cm ve 18 cm olan prizma şeklindeki kutularla içi dolu bir küp yapılacaktır.

Buna göre, en az kaç kutu gereklidir?

We are going to form a filled cube using rectangular prism shaped boxes with dimensions of 12 cm, 15 cm and 18 cm.

Accordingly, how many of these boxes are needed?

A) 900 B) 1200 C) 1500 D) 1800 E) 2100

YANITLAR / ANSWERS

1	2	3	4	5	6	7	8
E	C	A	C	D	E	A	C
9	**10**	**11**	**12**	**13**	**14**	**15**	**16**
B	D	D	D	B	B	D	D

BÖLÜM
CHAPTER
2

TARİHSEL NOT / HISTORICAL NOTE

Ali Kuşçu [1403 – 1474]

Ali Kuşçu asıl adı Ali Bin Muhammed Türk gök bilimci, matematikçi ve dilbilimcidir. Küçük yaştan itibaren matematik ve astronomiye ilgi duyan Ali Kuşçu, Uluğ Bey'in oğlu ve Bursalı Kadızade Rumi'nin öğrencisidir. Felsefeden bağımsız olarak astronomi ile de ilgilenmiştir.

Ali Bin Muhammed known as Ali Qushji was a persian astronomer, mathematician and physicist.He was the son of an official of Ulugh Beg, and also a student of Qadi Zadeh. He is best known for his development of an astronomical physics independent from natural philosophy.

RASYONEL ve ONDALIK SAYILAR
RATIONAL and DECIMAL NUMBERS

Bu bölüm, 150 test sorusu içermektedir.

This chapter includes 150 test questions.

BÖLÜM / CHAPTER 2

RASYONEL ve ONDALIK SAYILAR / RATIONAL and DECIMAL NUMBERS

- Rasyonel ve Ondalık Sayılar / Rational and Decimal Numbers .... 35 – 62

BÖLÜM

02

CHAPTER

RASYONEL ve ONDALIK SAYILAR
RATIONAL and DECIMAL NUMBERS

RASYONEL SAYILAR / RATIONAL NUMBERS

KESİR KAVRAMI / FRACTION CONCEPT

Tanım: a ve b tam sayı ve $b \neq 0$ olmak üzere $\frac{a}{b}$ ifadesi bir kesirdir.

Definition: Provided that a and b are integers and $b \neq 0$, the expression $\frac{a}{b}$ is called a ***fraction.***

Önemli / Important

- $\frac{a}{b}$ kesrinde, a sayısına pay, b sayısına payda, aradaki çizgiye kesir çizgisi denir.

kesir çizgisi ← $\frac{a}{b}$, a → pay, b → payda

- In the fraction $\frac{a}{b}$, a is called numerator, b is called denominator and the line in between is called fraction bar.

Fraction bar ← $\frac{a}{b}$, a → Numerator, b → Denominator

BASİT KESİR / PROPER FRACTION

Tanım: $\frac{a}{b}$ kesrinde $|a| < |b|$ ise $\frac{a}{b}$ kesrine **basit kesir** denir.

Definition: In the fraction $\frac{a}{b}$, if $|a| < |b|$, the fraction $\frac{a}{b}$ is called a **proper fraction.**

BİLEŞİK KESİR / IMPROPER FRACTION

Tanım: $\frac{a}{b}$ kesrinde $|a| \geq |b|$ ise $\frac{a}{b}$ kesrine **bileşik kesir** denir.

Definition: In the fraction $\frac{a}{b}$, if $|a| \geq |b|$, the fraction $\frac{a}{b}$ is called a **improper fraction**.

TAM SAYILI KESİR / MIXED FRACTION

Tanım: $a\frac{b}{c}$ kesrine **tam sayılı kesir** denir. a ya kesrin **tam kısmı**, $\frac{b}{c}$ ye **kesir kısmı** denir.

Definition: $a\frac{b}{c}$ is called a **mixed fraction**. The number a is called the **whole part**, and $\frac{b}{c}$ is called the **fraction part**.

Önemli / Important

- $a\frac{b}{c}$ tamsayılı kesrinde, $\frac{b}{c}$ kesri basit kesir olmak zorundadır.

- In the mixed fraction $a\frac{b}{c}$, the fraction $\frac{b}{c}$ must be a proper fraction.

Önemli / Important

- $a\frac{b}{c} = a + \frac{b}{c} = \frac{a \cdot c + b}{c}$

- $-a\frac{b}{c} = -\left(a + \frac{b}{c}\right) = -\frac{a \cdot c + b}{c}$

KESİRLERDE SADELEŞTİRME ve GENİŞLETME / SIMPLIFICATION and EXPANSION OF FRACTIONS

▼ **Sadeleştirme:** Bir kesrin pay ve paydasının aynı sayıya bölünerek küçültülmesine sadeleştirme denir.

▼ **Simplification (or reduction):** Dividing both the numerator and denominator of a fraction to the same number to form a new fraction with the same value of the original one is called reducing or simplifying the fraction.

▼ **Genişletme:** Bir kesrin pay ve paydasının aynı sayı ile çarpılarak büyütülmesine genişletme denir.

▼ **Expansion;** Multiplying both the numerator and denominator of a fraction by the same number to form a new fraction with the same value of the original one is called expanding the fraction.

DENK KESİRLER / EQUIVALENT FRACTIONS

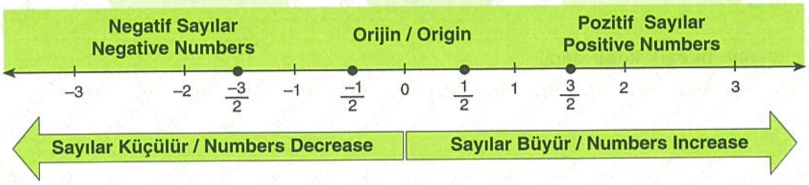
Tanım: İki veya daha fazla kesrin pay ve paydaları sadeleştirme veya genişletme işlemleri yaparak birbirlerine eşit hale getirilebiliyor ise bu kesirlere denk kesir denir

Definition: Two or more than two fractions that look different but can be equal to each other by reducing or expanding are called equivalent fractions.

RASYONEL SAYILAR / RATIONAL NUMBERS

Tanım: Birbirine denk kesirlerin oluşturduğu her küme bir sayı belirtir bu sayıya rasyonel sayı denir. Rasyonel sayıların oluşturduğu kümeye **rasyonel sayılar kümesi** denir ve **Q** ile gösterilir.

Definition: The representative element of a set of equivalent fractions is called a rational number. The rational numbers form a set that is called **the set of rational numbers** and is shown by the letter **Q**.



▼ 0'ın sağındaki rasyonel sayılara **pozitif rasyonel sayılar** denir ve **Q^+** ile gösterilir.

The rational numbers to the right side of zero are called **positive rational numbers** and shown by **Q^+.**

$$Q^+ = \left\{\ldots, \frac{1}{4}, \frac{1}{2}, \ldots, 1, \ldots, \frac{3}{2}, \ldots, 2, \ldots, 3, \ldots\right\}$$

▼ 0'ın solundaki rasyonel sayılara **negatif rasyonel sayılar** denir ve **Q^-** ile gösterilir.

The rational numbers to the left side of zero are called negative rational numbers and shown by Q^-.

$$Q^- = \left\{\ldots, -4, -3, -2, -\frac{3}{2}, -1, -\frac{1}{2}, \ldots\right\}$$

- Her tam sayı bir **rasyonel sayı**dır.

  Every integer is a **rational number**.

  $3 = \frac{3}{1}, 4 = \frac{4}{1}, \ldots$

  O halde / Therefore, $N \subset Z \subset Q$

RASYONEL SAYILAR KÜMESİNDE İŞLEMLER / OPERATIONS in the SET of RATIONAL NUMBERS

TOPLAMA ve ÇIKARMA İŞLEMİ / ADDITION and SUBTRACTION OPERATIONS

- Paydaları eşit olan iki kesir toplanırken veya çıkarılırken payların toplamı paya, ortak payda da paydaya yazılır. Bunun için aşağıdaki genel kural yazılabilir.
- To add or subtract rational numbers with the same denominators, add or subtract the numerators and write the sum or difference as the numerator. Then write the common denominator as the denominator. The following general rule can be written:

$$a, b, c, d, \in Z \text{ ve/and } b, d \neq 0 \Rightarrow \frac{a}{b} \mp \frac{c}{d} = \frac{a \cdot d \mp b \cdot c}{b \cdot d}$$

- Paydaları farklı rasyonel sayılar toplanırken veya çıkarılırken, toplama veya çıkarma işleminden önce kesirler ortak paydaya sahip kesirler olarak yazılır.
- To add or subtract rational numbers with the different denominators, we write the fractions as fractions with common denominators before adding or subtracting.

ÇARPMA ve BÖLME İŞLEMİ / MULTIPLICATION and DIVISION OPERATION

- Çarpma işleminde iki rasyonel sayının çarpımı bulunurken, payların çarpımı paya, paydaların çarpımı ise paydaya yazılır. Bölmede ise birinci kesrin aynısı yazılıp, ikinci kesir ters çevrilir ve işlem çarpmaya dönüştürülür.
- To multiply two fractions multiply their numerators and write the result to the numerator and multiply their denominators and write the result to the denominator.

  To divide any fraction by another fraction, multiply the first fraction by the reciprocal of the second fraction.

- $\frac{a}{b} \cdot \frac{c}{d} = \frac{a \cdot c}{b \cdot d}$
- $\frac{a}{b} : \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c} = \frac{a \cdot d}{b \cdot c}$
- $\frac{\frac{a}{b}}{\frac{c}{d}} = \frac{a \cdot d}{b \cdot c}$
- $\frac{1}{\frac{a}{b}} = \frac{b}{a}$

Önemli / Important

- Tam sayılı kesirleri çarparken tam sayılı kesirler bileşik kesirlere dönüştürülür.
- İki sayının çarpımı 1 ise, bu sayılar çarpma işlemine göre birbirinin tersidir.
- Sıfırın başka bir rasyonel sayıya bölümü sıfırdır.
- Bir rasyonel sayının sıfıra bölümü tanımsızdır.
- To multiply mixed fractions we first convert each mixed fraction to an improper one
- Two numbers whose product is 1 are called multiplicative inverses or reciprocals.
- The division of zero by any other rational number is zero.
- The division of a rational number by zero is undefined.

Önemli / Important

- $\dfrac{a}{\frac{b}{c}} = \dfrac{a}{1} \cdot \dfrac{c}{b} = \dfrac{a \cdot c}{b}$
- $\dfrac{\frac{a}{b}}{c} = \dfrac{a}{b} \cdot \dfrac{1}{c} = \dfrac{a}{b \cdot c}$
- $a^{-1} = \dfrac{1}{a}$
- $\left(\dfrac{a}{b}\right)^{-1} = \dfrac{b}{a}$

ZİNCİRLEME KESİRLER / CONTINUED FRACTIONS

▼ $a \neq 0$, a bir tam sayı olmak üzere,

▼ a is an integer and $a \neq 0$. The fractions defined as:

$$x = a + \cfrac{1}{b + \cfrac{1}{c + \cfrac{1}{d + \cfrac{1}{e}}}}$$

biçiminde tanımlanan kesirlere **sonlu zincirleme kesirler** denir.

are called *finite continued fractions.*

Önemli / Important

- $a, b \in Z$, $a \neq 0$ olmak üzere

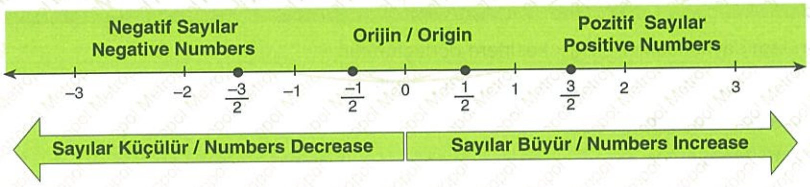
  $$x = a + \cfrac{b}{a + \cfrac{b}{\vdots}}$$

  biçiminde tanımlanan kesirlere **sonsuza giden zincirleme kesirler** denir.

- $a, b, \in z$, $a \neq 0$

  Fractions defined as $x = a + \cfrac{b}{a + \cfrac{b}{\vdots}}$ are called **infinite continued fractions.**

RASYONEL SAYILARIN SIRALANMASI / THE ARRANGEMENT of RATIONAL NUMBERS


Negatif Sayılar
Negative Numbers
Orijin / Origin
Pozitif Sayılar
Positive Numbers
–3
–2
$\frac{-3}{2}$
–1
$\frac{-1}{2}$
0
$\frac{1}{2}$
1
$\frac{3}{2}$
2
3
Sayılar Küçülür / Numbers Decrease
Sayılar Büyür / Numbers Increase

▼ Sayı doğrusu üzerinde, 0 dan sağa doğru uzaklaştıkça sayıların değeri büyür ve 0 dan sola doğru uzaklaştıkça sayıların değeri küçülür.

▼ In the number line the values of numbers increase by going to the right from zero and decrease by going to the left from zero.

Özellik / Property

★ Negatif bir rasyonel sayı, pozitif bir rasyonel sayıdan daha küçüktür.

A negative rational number is smaller than a positive one.

$-\frac{2}{5} < \frac{2}{5},\ -\frac{4}{7} < \frac{4}{7},\ldots$

★ Paydaları eşit olan iki pozitif rasyonel sayıdan, payı küçük olan daha küçüktür.

If the fractions have the same positive denominator, the fraction with the smaller numerator is smaller.

$\frac{3}{8} < \frac{5}{8},\ \frac{1}{7} < \frac{2}{7},\ldots$

★ Payları eşit olan pozitif iki rasyonel sayıdan, paydası küçük olan sayı daha büyüktür.

If the fractions have the same numerator and the denominators are positive, the fraction with the smaller denominator is bigger.

$\frac{6}{5} < \frac{6}{4},\ \frac{7}{8} < \frac{7}{4},\ldots$

★ Pozitif her rasyonel sayı sıfırdan büyük, negatif her rasyonel sayı sıfırdan küçüktür.

Any positive rational number is bigger than zero and any negative rational number is smaller than zero.

$\frac{1}{4} > 0,\ \frac{-1}{4} < 0,\ldots$

★ $\frac{a}{b}, \frac{c}{d} \in Q$

$\frac{a}{b} < \frac{c}{d} \Rightarrow \frac{a}{b} < \frac{1}{2}\left(\frac{a}{b} + \frac{c}{d}\right) < \frac{c}{d}$

★ Payı ile paydası arasındaki fark aynı olan pozitif basit kesirlerden payı küçük olan daha küçüktür.

In positive proper fractions whose differences between numerators and denominators are equal, the fraction with smaller numerator is the smaller fraction.

$\frac{23}{27} < \frac{25}{29} < \frac{37}{41},\ \frac{1}{2} < \frac{15}{16} < \frac{51}{52},\ldots$

★ Payı ile paydası arasındaki fark aynı olan, pozitif bileşik kesirlerden payı büyük olan daha küçüktür.

In positive improper fractions whose differences between numerators and denominators are equal, the fraction with bigger numerator is the smaller fraction.

$\frac{36}{31} < \frac{24}{19} < \frac{12}{7},\ \frac{50}{49} < \frac{25}{24} < \frac{3}{2},\ldots$

★ Pozitif basit kesirlerin kuvvetleri büyüdükçe kesrin değeri küçülür.

Increasing the powers of the positive proper fractions, the values of fractions decrease.

$\left(\frac{1}{3}\right)^2 > \left(\frac{1}{3}\right)^3 > \left(\frac{1}{3}\right)^4 > \ldots$

★ İki rasyonel sayının ortasındaki sayıyı bulmak için bu sayılar, toplanıp ikiye bölünür.

$\frac{3}{4}$ ile $\frac{11}{6}$ sayılarının ortasındaki sayı,

To find the number in the middle of two rational numbers, the sum of them is divided by 2.

The number in the middle of $\frac{3}{4}$ and $\frac{11}{6}$ is

$\frac{1}{2}\cdot\left(\frac{3}{4} + \frac{11}{6}\right) = \frac{1}{2}\cdot\frac{31}{12} = \frac{31}{24}$

Önemli / Important

- Negatif rasyonel sayılar kendi aralarında sıralanırken, pozitif rasyonel sayılardaki gibi sıralama yapılır sonra sıralamanın tam tersi alınır.
- When arranging negative numbers, the arrangement will be done as in positive numbers, and then the order of numbers will be reversed.

Önemli / Important

$\frac{a}{b}, \frac{c}{d}, \frac{e}{f} \in Q,$

- $\frac{a}{b} > \frac{c}{d}$ ve / and $\frac{c}{d} > \frac{e}{f} \Rightarrow \frac{a}{b} > \frac{e}{f}$
- $\frac{a}{b} < \frac{c}{d}$ ve / and $\frac{c}{d} < \frac{e}{f} \Rightarrow \frac{a}{b} < \frac{e}{f}$

ONDALIK SAYILAR / DECIMAL NUMBERS

Tanım: Paydası 10, 100, 1000 gibi 10 un kuvveti olan kesirlere ***ondalık kesir,*** bu kesrin belirttiği sayıya da **ondalık sayı** denir.

Definition: The fractions whose denominators are powers of 10 like 10, 100, 1000 are called *decimal fractions* and the values of these fractions are called *decimal numbers.*

BİR RASYONEL SAYIYI ONDALIK SAYIYA ÇEVİRME / CONVERTING a RATIONAL NUMBER into a DECIMAL NUMBER

Bir rasyonel sayıyı, ondalık sayıya çevirirken, aşağıdaki iki kuraldan biri uygulanır:

▼ Paydaki sayı, paydadaki sayıya bölünür.

▼ Paydadaki sayı, 10 un kuvveti olarak yazıldıktan sonra ondalık sayıya çevrilir.

To convert a rational number into a decimal number we use either of the two following rules:

▼ Divide the numerator into the denominator.

▼ After writing denominator as a power of 10 convert it into decimal number.

ONDALIK SAYILARDA İŞLEMLER / OPERATIONS in DECIMAL NUMBERS

TOPLAMA / ADDITION

▼ Ondalık sayılar alt alta yazılır.

▼ Ondalık virgülünün sağındaki her bir sütunda yer alan her sayı için ondalık virgülünün sağına sıfır eklenir.

▼ Tam sayılardaki gibi toplama yapılır.

▼ Sayılar toplandıktan sonra ondalık virgülü yerleştirilir.

▼ Decimals are written in a column vertically.

▼ Add zeros to the right of the decimal comma for all of the numbers having an entry in each column on the right of the decimal comma.

▼ Add the numbers in the same way with the whole numbers.

▼ After adding the numbers a decimal comma is placed.

ÇIKARMA / SUBTRACTION

- Virgüller ve aynı adlı basamaklar, alt alta gelecek şekilde yazılır.
- Eksilen sayının, çıkan ondalıklı kesre göre eksik basamağı varsa, yerine sıfır yazılır.
- Sayılar tam sayılardaki gibi çıkarılır.
- Bulunan fark, virgüllerin hizasından yine virgülle ayrılır.
- Commas and places with the same name are written in a column vertically.
- If the minuend number has deficient places according to the subtrahend decimal fraction, add zeros instead.
- Subtract the numbers as you would with the whole numbers.
- The difference is seperated with a decimal comma.

ÇARPMA / MULTIPLICATION

- Çarpma işlemi virgül yokmuş gibi yapılır. İşlemin sonunda, çarpılan sayıların virgülden sonraki basamak sayılarının toplamı kadar, sağdan sola doğru virgül ile ayrılır.
- In the multiplication of decimal numbers, do the multiplication like there is no comma. At the end of the operation, the result is seperated with a comma from right to left as much as sum of the numbers of the digits after comma in multiplicand numbers.

BÖLME / DIVISION

- Bölen sayı ondalık sayıdan kurtuluncaya kadar, bölendeki ondalık virgülü sağa taşınır.
- Adım 1'de taşınan ondalık virgül kadar bölünen sayıda da sağa taşınır.
- Bölünen ve bölen tam sayı ise 2. adımdaki sonuç 1.ye bölünür.
- Bölümün ondalık kısmının sayısı, 2. adımdaki sonucun ondalık kısmının sayısına eşittir.
- Bölünen sayının sağına sıfır ekleyerek elde edilebilir ve 3. adım tekrar edilir. Bölünene eklenen herbir sıfır için bölümde birden fazla ondalık kısım gerekir.
- Move the decimal comma in the divisor to the right until there is no decimal part in the divisor (this is the same as multiplying the divisor by a power of 10.)
- Move the decimal comma in the dividend the same number of places to the right as you moved the decimal comma in step 1.
- Divide the result of step (2) by the result of step (1) as if they were whole numbers.
- The number of decimal places in the result (quotient) is equal to the number of decimal places in the result of step (2).
- You may obtain as many decimal places as you wish in the quotient by adding zeros to the right of the dividend and then repeating step (3). For each zero you add to the dividend, you need one more decimal place in the quotient.

ONDALIK BİR SAYIYI RASYONEL BİR SAYIYA ÇEVİRME / CONVERTING a DECIMAL NUMBER into a RATIONAL NUMBER

Ondalık bir sayıyı rasyonel sayıya çevirirken;

- Tam kısmı varsa yazılır.
- Virgülden sonra kaç tane sıfır varsa, o sayı 10 un kuvveti olarak paydaya yazılır.
- Virgülden sonraki rakamlar olduğu gibi paya yazılır.
- Varsa sadeleştirme yapılır.

To convert a decimal number into a rational number;

- If there is a whole part present, write it down.
- The number of zeros after comma is written as the power of 10 to denominator.
- The digits after comma are written to the numerator as they are.
- If simplification is possible, it's done.

DEVİRLİ ONDALIK SAYILAR / RECURRING DECIMALS

Tanım: Bir rasyonel sayı ondalık sayı biçiminde yazıldığında virgülden sonra gelen kısımdaki sayılar belli bir rakamdan sonra tekrar ediyorsa bu sayılara **devirli ondalık sayı** denir.

Definition: When a rational number is written in the decimal form, if the numbers coming after the comma repeats infinitely afterwards a certain number, then these numbers are called *recurring (repeating) or periodic decimals.*

Önemli / Important

- Devirli ondalık sayılarda toplama işlemi yapılırken aynı basamakta devreden sayıların toplamı 10 dan küçükse toplama işlemi tam sayılarda olduğu gibi yapılır.
- In reccuring decimals, when doing the addition, if the reccuring digits in same place add up to a number less than 10, the addition is done as in integers.

DEVİRLİ ONDALIK SAYININ RASYONEL SAYIYA DÖNÜŞTÜRÜLMESİ
CONVERTING a RECCURING DECIMAL NUMBER into a RATIONAL NUMBER

- $\text{ab, cd}\overline{\text{efg}} = \dfrac{\text{abcdefg} - \text{abcd}}{99900}$

Önemli / Important

- Devirli ondalık sayılarda devreden kısım 9 ise sayı bir üst basamağa yuvarlanır.

  $4,1\overline{9} = 4,2$

  $3,\overline{9} = 4$
- In reccuring numbers, if the repeating part is 9, the number will round-up to one bigger place

  $4,1\overline{9} = 4,2$

  $3,\overline{9} = 4$

1. $\frac{1}{3}-2:\frac{3}{4}+\frac{1}{3}=?$

A) -2 B) $-\frac{5}{3}$ C) $-\frac{4}{3}$ D) -1 E) $-\frac{1}{3}$

2. $\frac{2}{5}-4\cdot\left(1-\frac{1}{3}\right)=?$

A) -6 B) $-\frac{34}{15}$ C) $-\frac{27}{5}$ D) -4 E) $-\frac{22}{5}$

3. $2-\frac{3}{4}:\left(1-\frac{1}{4}\right)^{-1}=?$

A) 1 B) $\frac{13}{16}$ C) $\frac{19}{16}$ D) 2 E) $\frac{23}{16}$

4. $(3^{-1}+2^{-1}):\frac{1}{6}-3=?$

A) 3 B) 2 C) 1 D) 0 E) -1

5. $\left(1-\frac{1}{1-\frac{2}{3}}\right)^{-1}=?$

A) -2 B) $-\frac{3}{2}$ C) $-\frac{1}{2}$ D) $\frac{1}{2}$ E) 2

6. $\dfrac{\left(3-\frac{3}{2}\cdot\frac{7}{3}\right):\left(\frac{5}{4}-2\right)}{2+\frac{1}{1-\frac{1}{3}}}=?$

A) $\frac{4}{25}$ B) $\frac{4}{21}$ C) $\frac{1}{3}$ D) $\frac{1}{4}$ E) $\frac{4}{27}$

7. $\frac{3}{4}-\frac{3}{4}\cdot\left(\frac{2}{5}:\frac{6}{5}\right)+\frac{7}{2}=?$

A) 1 B) 2 C) 3 D) 4 E) 5

8. $\dfrac{1-\frac{1-\frac{1}{3}}{3}}{1-\frac{1}{1-\frac{1}{3}}}=?$

A) $\frac{16}{3}$ B) $-\frac{14}{9}$ C) $\frac{23}{9}$ D) $\frac{31}{9}$ E) $-\frac{41}{3}$

9. $\frac{1}{\frac{7}{3}}:\left[\frac{3}{\frac{7}{3}+\frac{7}{3}+\frac{7}{3}}\right]=?$

A) $\frac{1}{3}$ B) $\frac{2}{3}$ C) 1 D) 2 E) 3

10. $\left(1-\frac{1}{4}\right)\cdot\left(1-\frac{1}{5}\right)\cdot\ldots\cdot\left(1-\frac{1}{n}\right)=?$

A) $\frac{3}{n}$ B) $\frac{n-1}{n}$ C) $\frac{4}{n}$ D) $\frac{2}{n}$ E $\frac{3}{2n}$

11. $1+\frac{2}{1-\frac{1}{2-\frac{2}{3}}}=?$

A) 4 B) $\frac{8}{3}$ C) $\frac{11}{6}$ D) 7 E) 9

12. $\left[5-\frac{1-\frac{1}{3}}{1+\frac{1}{3}}\right]\cdot\left(2+\frac{2}{3}\right)=?$

A) 12 B) 8 C) $\frac{7}{9}$ D) $\frac{5}{3}$ E) $\frac{5}{6}$

13. $\left[2-\frac{1}{1-\frac{1}{1-\frac{1}{3}}}\right]\cdot\left[1+\frac{1}{1+\frac{1}{3}}\right]=?$

A) – 3 B) 7 C) 6 D) $\frac{1}{3}$ E) 9

14. $\frac{1}{3}-\frac{1}{2}\left[2-\frac{1}{2}\cdot\left(1+\frac{1}{3}\right)\right]+1=?$

A) $\frac{1}{3}$ B) $-\frac{4}{3}$ C) $\frac{1}{6}$ D) $\frac{5}{6}$ E) $\frac{2}{3}$

15. $\left[12:\frac{2}{1-\frac{2}{3}}-1\right]:\frac{1}{5}=?$

A) 5 B) 7 C) $\frac{4}{7}$ D) 1 E) $\frac{11}{8}$

16. $\left(1-\frac{1}{2}\right):\left(\frac{\frac{1}{2}-\frac{1}{4}}{1+\frac{1}{2}}\right)=?$

A) $\frac{1}{4}$ B) $\frac{2}{3}$ C) 2 D) 3 E) $\frac{1}{6}$

17. $\frac{1+\frac{1}{2}}{1-\frac{1}{2}}:\left(1-\frac{1}{1-\frac{1}{2}}\right)=?$

A) – 3 B) $\frac{1}{2}$ C) 0 D) $\frac{3}{2}$ E) 1

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1	2	3	4	5	6	7	8	9
A	B	E	B	C	B	D	B	C
10	11	12	13	14	15	16	17	
A	E	A	B	E	A	D	A	

1. $\dfrac{1}{1-\dfrac{1}{2}}-\dfrac{3-\dfrac{1}{3}}{3}=?$

A) $\frac{1}{2}$ B) $-\frac{2}{3}$ C) $-\frac{5}{3}$ D) $\frac{10}{9}$ E) $\frac{2}{3}$

2. $1-\dfrac{1}{3-\dfrac{1}{\dfrac{4}{5}-1}}=?$

A) $\frac{3}{2}$ B) $-\frac{3}{2}$ C) 1 D) $-\frac{7}{8}$ E) $\frac{7}{8}$

3. $1-\dfrac{1}{1-\dfrac{1}{1-\dfrac{1}{1-\dfrac{1}{2}}}}=?$

A) 2 B) $\frac{3}{2}$ C) 1 D) $\frac{1}{2}$ E) 0

4. $\left[\dfrac{\dfrac{3}{4}}{\dfrac{1}{4}+1}\right]\cdot\left[\dfrac{3+\dfrac{1}{3}}{1-\dfrac{2}{3}}\right]=?$

A) $\frac{2}{3}$ B 1 C) $\frac{3}{5}$ D) – 5 E)6

5. $\frac{1}{5}-\left(\frac{1}{5}-\frac{1}{6}-\frac{1}{2}\right)-\left(\frac{1}{3}+\frac{1}{5}\right)=?$

A) $\frac{2}{15}$ B) $\frac{3}{5}$ C) $-\frac{7}{30}$ D) $\frac{7}{2}$ E) $\frac{-12}{3}$

6. $\dfrac{\dfrac{1}{3}+\dfrac{1}{\dfrac{1}{3}+\dfrac{1}{\dfrac{1}{3}}}}{\dfrac{1}{3}+\dfrac{1}{\dfrac{1}{3}}}=?$

A) 19 B) $\frac{19}{10}$ C) $\frac{19}{100}$ D) $\frac{9}{10}$ E) $\frac{9}{100}$

7. $\left(\dfrac{1-\dfrac{2}{5}}{1+\dfrac{2}{5}}\right):\left(1-\dfrac{1}{-\dfrac{3}{4}}\right)=?$

A) 1 B) $\frac{3}{4}$ C) $\frac{3}{7}$ D) $\frac{9}{28}$ E) $\frac{9}{49}$

8. $\dfrac{\left(2+\frac{1}{2}\right)+\left(4+\frac{1}{4}\right)}{\left(5+\frac{1}{5}\right)-\left(3+\frac{1}{3}\right)}\cdot\left(\frac{4}{9}\right)^2=?$

A) $\frac{7}{5}$ B) 1 C) $\frac{5}{7}$ D) $-\frac{5}{7}$ E) 1

9. $3-\dfrac{2-\frac{1}{2}}{3-\dfrac{2}{1-\frac{3}{2}}}=?$

A) $\frac{19}{7}$ B) $\frac{9}{2}$ C) $\frac{37}{9}$ D) $\frac{39}{14}$ E) $-\frac{3}{2}$

10. $\frac{2}{3}\cdot\left(\dfrac{4+\frac{1}{3}}{1-\frac{1}{3}}\right)-\frac{1}{6}:\left[\frac{3}{2}\cdot\left(1-\frac{1}{2}\right)\right]=?$

A) $\frac{13}{9}$ B) $\frac{37}{9}$ C) $-\frac{5}{6}$ D) $\frac{11}{3}$ E) $\frac{26}{27}$

11. $a=\frac{3}{4},\ b=\frac{5}{6},\ c=\frac{2}{3}$

$\Rightarrow$ **? < ? < ?**

A) $a<b<c$

B) $a<c<b$

C) $b<a<c$

D) $b<c<a$

E) $c<a<b$

12. $a=-\frac{19}{21},\ b=-\frac{73}{75},\ c=-\frac{47}{49}$

$\Rightarrow$ **? < ? < ?**

A) $a<b<c$

B) $c<a<b$

C) $b<c<a$

D) $a<c<b$

E) $b<a<c$

13. $a=\frac{2}{171},\ b=\frac{3}{191},\ c=\frac{4}{137}$

$\Rightarrow$ **? < ? < ?**

A) $a<b<c$

B) $a<c<b$

C) $c<b<a$

D) $c<a<b$

E) $b<a<c$

14. $\left(\frac{a+b}{ab}+\frac{a+c}{ac}-\frac{b+c}{bc}\right):\frac{1}{ab}=?$

A) 2(a+b) B) 2b C) 2a

D) 2(a + b + c) E) 2c

15. $a\neq 1,\ a\neq 0,\ ab+b+1=0\ \Rightarrow\ b-\dfrac{1}{1+\frac{1}{a}}=?$

A) 2 B) 1 C) 0 D) –1 E) –2

16. $a>0,$

$$\left.\begin{aligned}x&=\frac{1}{a}\\ y&=\frac{2}{a}\\ z&=\frac{3}{a}\end{aligned}\right\}\Rightarrow\ ?>?>?$$

A) $x<y<z$ B) $x<z<y$ C) $y<x<z$

D) $y<z<x$ E) $z<x<y$

17. $x=\left(1+\frac{1}{4}\right)\left(1+\frac{1}{5}\right)\left(1+\frac{1}{6}\right)\ldots\left(1+\frac{1}{47}\right)$

$y=\left(1-\frac{1}{3}\right)\left(1-\frac{1}{4}\right)\left(1-\frac{1}{5}\right)\ldots\left(1-\frac{1}{24}\right)$

$\Rightarrow$ **$x\cdot y=?$**

A) $\frac{2}{3}$ B) $\frac{3}{4}$ C) 1 D) $\frac{11}{12}$ E) $\frac{3}{2}$

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1	2	3	4	5	6	7	8	9
D	E	D	E	A	C	E	C	D
10	11	12	13	14	15	16	17	
B	E	C	A	B	D	A	C	

1. $\left(\frac{1}{2}+\frac{2}{3}-\frac{1}{7}\right):\left(\frac{2}{7}-\frac{4}{3}-1\right)=?$

A) $-\frac{3}{2}$ B) $-\frac{1}{2}$ C) 0 D) $\frac{1}{2}$ E) $\frac{3}{2}$

2. $a=\frac{5}{2},\ b=\frac{2}{3},\ c=\frac{3}{7}$

$\Rightarrow a+\frac{1}{b\cdot c}=?$

A) 4 B) 5 C) 6 D) 7 E) 8

3. $3-\dfrac{1+\frac{1}{2}}{1-\dfrac{3}{3+\frac{1}{3}}}:\dfrac{1-\frac{1}{2}}{1+\frac{1}{2}}=x-27 \Rightarrow x=?$

A) −18 B) −15 C) −13 D) 13 E) 15

4. $2-\dfrac{1-\frac{1}{2}}{2+\dfrac{1}{1-\frac{1}{3}}}=?$

A) $\frac{10}{9}$ B) $\frac{11}{9}$ C) $\frac{13}{9}$ D) $\frac{13}{7}$ E) $\frac{15}{7}$

5. $\dfrac{13}{3-\dfrac{1}{3-\frac{1}{x}}}-2=3 \Rightarrow x=?$

A) 1 B) 2 C) 3 D) 4 E) 5

6. $\dfrac{\dfrac{5-\frac{1}{4}}{4-\frac{1}{5}}+\frac{2}{5}}{\frac{1}{4}+\frac{1}{5}\cdot 2}=?$

A) $\frac{33}{13}$ B) $\frac{23}{15}$ C) $\frac{13}{11}$ D) $\frac{1}{13}$ E) $\frac{1}{11}$

7. $\left(3+\frac{5}{4}\right)+\left(3+\frac{6}{4}\right)+\left(3+\frac{7}{4}\right)+\ldots+\left(3+\frac{20}{4}\right)=?$

A) 92 B) 96 C) 98 D) 100 E) 108

8. $x=\left(1-\frac{1}{2}\right)\cdot\left(1-\frac{1}{4}\right)\cdot\left(1-\frac{1}{6}\right)\cdot\ldots\cdot\left(1-\frac{1}{18}\right)$

$y=\left(1+\frac{1}{3}\right)\cdot\left(1+\frac{1}{5}\right)\cdot\left(1+\frac{1}{7}\right)\cdot\ldots\cdot\left(1+\frac{1}{17}\right)$

$\Rightarrow x\cdot y=?$

A) $\frac{2}{17}$ B) $\frac{1}{9}$ C) $\frac{18}{17}$ D) $\frac{1}{2}$ E) 1

9. $A=\frac{7}{3}+\frac{9}{4}+\frac{11}{5}+\ldots+\frac{23}{11}$

$B=\frac{5}{3}+\frac{7}{4}+\frac{9}{5}+\ldots+\frac{21}{11}$

$\Rightarrow$ **A + B = ?**

A) 24 B) 26 C) 32 D) 36 E) 48

10. $A=\left(1-\frac{5}{2}\right)\cdot\left(1-\frac{5}{3}\right)\cdot\left(1-\frac{5}{4}\right)\cdot\ldots\cdot\left(1-\frac{5}{97}\right)$

$\Rightarrow$ **A = ?**

A) $-\frac{3}{97}$ B) $-\frac{1}{97}$ C) 0 D) $\frac{1}{97}$ E) $\frac{3}{97}$

11. $A=\frac{4}{17}+\frac{5}{18}+\frac{6}{19}$

$B=\frac{38}{17}-\frac{13}{18}+\frac{63}{19}$ $\Rightarrow$ **B = ?**

A) A + 4 B) A + 2 C) A – 1

D) A – 3 E) A – 4

12. $A=\frac{x}{2}+\frac{2y}{3}+\frac{z}{7}$

$B=\frac{2x+4}{4}+\frac{2y+9}{3}+\frac{z+49}{7}$ $\Rightarrow$ **B = ?**

A) A + 7 B) A + 9 C) A + 11

D) A + 13 E) A + 15

13. $x, y, z \in Z^+$,

$$1+\cfrac{1+\cfrac{1+\frac{1}{z}}{y}}{x}=\frac{41}{24} \Rightarrow \mathbf{\min(x\cdot y\cdot z)=?}$$

A) 12 B) 16 C) 18 D) 20 E) 24

14. $$1-\cfrac{1-\cfrac{1-\cfrac{1-\cfrac{\vdots}{2}}{2}}{2}}{2}=x \Rightarrow \mathbf{x=?}$$

A) $\frac{1}{3}$ B) $\frac{2}{3}$ C) 1 D) 2 E) 3

15. $$\frac{3+5\frac{2011}{2012}}{6+2\frac{2011}{2012}}+2011=?$$

A) $\frac{3}{2012}$ B) $\frac{2011}{2012}$ C) $\frac{2013}{2012}$

D) 2012 E) 2013

16. $a=\frac{75}{68}$, $b=\frac{64}{57}$, $c=\frac{48}{55}$

$\Rightarrow$ **? < ? < ?**

A) a < b < c B) a < c < b C) c < b < a

D) c < a < b E) b < a < c

YANITLAR / ANSWERS

1	2	3	4	5	6	7	8
B	C	B	D	B	A	C	D
9	**10**	**11**	**12**	**13**	**14**	**15**	**16**
D	C	A	C	E	B	D	D

1. $$\frac{\left(\frac{1}{3}-\frac{1}{4}+\frac{1}{5}\right)-\left(\frac{1}{5}+\frac{1}{4}+\frac{1}{3}\right)}{\left(\frac{1}{5}-\frac{1}{4}\right)}=?$$

A) 1 B) 5 C) 9 D) 10 E) 12

2. $$\frac{1+\cfrac{1}{2+\cfrac{1}{3}}}{2-\cfrac{1}{2-\cfrac{1}{3}}}:\frac{25}{49}+1=?$$

A) 1 B) 2 C) 3 D) 4 E) 5

3. $$1+\cfrac{1}{2-\cfrac{1}{2+\cfrac{1}{x}}}=\frac{14}{9}\Rightarrow x=?$$

A) $\frac{1}{3}$ B) $\frac{1}{2}$ C) 1 D) 2 E) 3

4. $$3+\cfrac{4}{3+\cfrac{4}{3+\cfrac{4}{3+\cfrac{4}{\vdots}}}}=?$$

A) 3 B) 4 C) 5 D) 6 E) 7

5. $$2+\cfrac{2+\cfrac{2+\cfrac{\vdots}{6}}{6}}{6}=?$$

A) $\frac{4}{5}$ B) 1 C) $\frac{6}{5}$ D) $\frac{12}{5}$ E) $\frac{14}{5}$

6. $$1-\cfrac{1}{1+\cfrac{1}{1-\cfrac{1}{x}}}=3\Rightarrow x=?$$

A) $-\frac{3}{5}$ B) $-\frac{1}{5}$ C) 1 D) $\frac{1}{5}$ E) $\frac{3}{5}$

7. $$\left[\frac{\frac{a}{b}}{c}:\frac{a}{\frac{b}{c}}\right]:\left(1-\frac{1}{c}\right)=\frac{1}{20}$$

$c\in Z^{+}\Rightarrow$ **c = ?**

A) 2 B) 3 C) 4 D) 5 E) 6

8. $$2-\cfrac{1}{2-\cfrac{1}{2-\cfrac{1}{3}}}:\frac{5}{7}=?$$

A) $\frac{2}{17}$ B) $\frac{1}{9}$ C) $\frac{18}{17}$ D) $\frac{1}{2}$ E) 1

9. $$\frac{\left(2011-\frac{1}{3}\right)-\left(2012-\frac{1}{3}\right)}{\left(\frac{2}{3}-2012\right)+\left(2011-\frac{1}{6}\right)}=?$$

A) –2　B) –1　C) $\frac{1}{2}$　D) 1　E) 2

10. $$\frac{\frac{7}{11}-\frac{6}{13}+\frac{4}{7}}{\frac{7}{22}-\frac{3}{13}+\frac{2}{7}}=?$$

A) –2　B) –1　C) $\frac{1}{2}$　D) 1　E) 2

11. $$\frac{13}{5}=a+\cfrac{1}{b+\cfrac{1}{c+\cfrac{1}{d}}}$$

$\Rightarrow$ **a + b + c + d = ?**

A) 4　B) 5　C) 6　D) 7　E) 8

12. $$\cfrac{4}{3+\cfrac{4}{3+\cfrac{4}{\vdots}}}+\cfrac{5}{4+\cfrac{5}{4+\cfrac{5}{\vdots}}}+3(x+1)=23$$

$\Rightarrow$ **x = ?**

A) 4　B) 5　C) 6　D) 7　E) 8

13. $$A=\frac{5}{8}+\frac{9}{10}+\frac{11}{12}$$

$$B=\frac{31}{24}+\frac{7}{30}+\frac{1}{4}\quad\Rightarrow\quad \mathbf{B=?}$$

A) 3A – 2　B) 3A + 2　C) 3A + 3　D) $\frac{3A-2}{3}$　E) $\frac{3A+2}{3}$

14. $$\left(2-\frac{2}{3}\right)\cdot\left(2-\frac{3}{4}\right)\cdot\left(2-\frac{4}{5}\right)\cdot\ldots\cdot\left(2-\frac{x+1}{x+2}\right)=7$$

$\Rightarrow$ **x = ?**

A) 12　B) 14　C) 16　D) 18　E) 24

15. $$\cfrac{3}{3+\cfrac{4-x}{3+\cfrac{1}{4+\cfrac{4}{3}}}}=1\quad\Rightarrow\quad \mathbf{x=?}$$

A) 2　B) 3　C) 4　D) 5　E) 6

16. $$\left(\frac{1}{49^2+49}+\frac{1}{50^2+50}+\frac{1}{49^2+98}\right)\cdot(49\cdot50\cdot51)=?$$

A) 147　B) 148　C) 149　D) 150　E) 151

YANITLAR / ANSWERS

1	2	3	4	5	6	7	8
D	C	A	B	D	E	D	E
9	**10**	**11**	**12**	**13**	**14**	**15**	**16**
E	E	C	C	D	D	C	D

1. $\frac{3,2}{0,16}+\frac{1,3}{0,1}=?$

A) 11 B) 12 C) 13 D) 14 E) 33

2. $\frac{0,02}{1,4}+\frac{0,21}{0,7}-\frac{0,36}{2,1}=?$

A) $\frac{1}{6}$ B) $\frac{1}{7}$ C) $\frac{1}{9}$ D) $\frac{1}{15}$ E) $\frac{1}{21}$

3. $\frac{0,00018}{0,001}:\frac{0,009}{0,0005}=?$

A) $\frac{1}{10}$ B) $\frac{5}{10}$ C) $\frac{1}{100}$ D $\frac{5}{100}$ E) $\frac{1}{1000}$

4. $\dfrac{\frac{0,5}{0,03}+\frac{0,4}{0,2}}{1,2}=?$

A) $\frac{28}{9}$ B) $\frac{14}{3}$ C) $\frac{13}{4}$ D) $\frac{5}{3}$ E) $\frac{140}{9}$

5. $\frac{0,4+0,04+0,004}{0,000111}=?$

A) 4 B) 40 C) 400 D) 4000 E) 40000

6. $2,5+\dfrac{25}{2+\frac{12}{0,25}}=?$

A) 2,5 B) 3 C) 3,5 D) 7,5 E) 8

7. $\frac{5\cdot(0,002)^3}{(0,01)^4}=?$

A) 8 B) 7 C) 6 D) 5 E) 4

8. $\dfrac{3}{\frac{1}{2}+1}-\dfrac{0,1\overline{5}}{1,\overline{5}}=\dfrac{6}{x}\ \Rightarrow\ x=?$

A) – 10 B) – 1 C) $\frac{1}{10}$ D) $-\frac{1}{10}$ E) 10

9. $\dfrac{1,\overline{3}-2,4\overline{1}}{10+0,\overline{7}}=?$

A) –10 B) –1 C) $\frac{1}{10}$ D) $-\frac{1}{10}$ E) 10

10. $0,0\overline{12}-0,0\overline{6}=?$

A) $-\frac{4}{27}$ B) $-\frac{9}{22}$ C) $-\frac{3}{55}$ D) $-\frac{5}{8}$ E) $\frac{7}{99}$

11. $(0,06)^2-(0,04)^2=2\cdot10^x \Rightarrow$ **$x=?$**

A) –6 B) –5 C) –4 D) –3 E) –2

12. $\left(\frac{1}{8}\right)^{0,\overline{3}}=?$

A) $\frac{1}{2}$ B) $\frac{1}{4}$ C) $\frac{1}{16}$ D) $\frac{1}{24}$ E) $\frac{1}{36}$

13. $\left.\begin{array}{l} x=1,\overline{2} \\ y=0,\overline{4} \end{array}\right\} \Rightarrow \frac{x}{y}+\frac{y}{x}=?$

A) $\frac{135}{44}$ B) $\frac{137}{44}$ C) $\frac{69}{22}$ D) $\frac{67}{22}$ E) $\frac{53}{22}$

14. $\frac{0,xy+0,yx}{0,\overline{xy}+0,\overline{yx}}=?$

A) 0,9 B) 0,99 C) 1 D) $\frac{10}{9}$ E) $\frac{100}{99}$

15. $a=0,\overline{3},\ b=0,\overline{6} \Rightarrow \left(\frac{1}{a}+\frac{1}{b}\right)\cdot\frac{4}{9}=?$

A) 1 B) 2 C) 3 D) $\frac{2}{3}$ E) $\frac{3}{2}$

16 $a, b \in Z^+$

$1,\overline{36}\cdot a=b \Rightarrow$ **min(b) = ?**

A) 6 B) 9 C) 11 D) 15 E) 26

17. $\frac{2002}{2003}+\frac{2003}{2004}=a \Rightarrow \frac{1}{2003}+\frac{1}{2004}=?$

A) a+1 B) 1–a C) a–2 D) 2–a E) a+2

18. $\frac{0,abc}{0,bc}=\frac{10\cdot a}{bc}+m \Rightarrow$ **m = ?**

A) 0,5 B) 0,4 C) 0,3 D) 0,2 E) 0,1

YANITLAR / ANSWERS

1	2	3	4	5	6	7	8	9
E	B	C	E	D	B	E	D	D
10	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**
C	D	A	B	B	B	D	D	E

1. $\frac{0,8}{0,04}+\frac{0,34}{0,14}:\frac{0,85}{2,1}=?$

A) 23 B) 24 C) 25 D) 26 E) 27

2. $\dfrac{\dfrac{0,55}{0,011}-\dfrac{9,1}{0,07}}{\dfrac{0,06}{0,03}+\dfrac{0,04}{0,005}}:8=?$

A) –2 B) –1 C) 1 D) 2 E) 3

3. $(1-0,4:0,2+0,5):\frac{1}{8}+\frac{1}{0,2}=?$

A) 1 B) 2 C) 3 D) 4 E) 5

4. $(4,\overline{9}+0,2\overline{9}-1,\overline{6})\cdot\left(\frac{3}{109}\right)=?$

A) $\frac{1}{30}$ B) $\frac{1}{10}$ C) $\frac{1}{5}$ D) 1 E) 10

5. $A=2\cdot10^{0}+10^{-1}+6\cdot10^{-2}+6\cdot10^{-3}+6\cdot10^{-4}+\ldots$

$\Rightarrow \frac{A}{13}=?$

A) $\frac{1}{9}$ B) $\frac{1}{8}$ C) $\frac{1}{6}$ D) $\frac{1}{3}$ E) $\frac{1}{2}$

6. $\frac{1}{9\cdot10}+\frac{1}{10\cdot11}+\frac{1}{11\cdot12}+\ldots+\frac{1}{17\cdot18}=?$

A) $\frac{1}{18}$ B) $\frac{1}{15}$ C) $\frac{1}{12}$ D) $\frac{1}{10}$ E) $\frac{1}{9}$

7. $\dfrac{\left(1-\dfrac{0,\overline{3}}{0,3}\right):\left(1-\dfrac{0,3}{0,\overline{3}}\right)+1}{0,\overline{2}+0,\overline{3}-0,\overline{4}}=?$

A) $-\frac{1}{3}$ B) $-\frac{1}{2}$ C) -1 D) $\frac{1}{2}$ E) 1

8. $x=1,3333\ldots$

$y=0,272727\ldots \Rightarrow 2\frac{3}{4}\cdot x\cdot y=?$

A) $\frac{2}{3}$ B) $\frac{1}{3}$ C) $\frac{1}{2}$ D) 1 E) $\frac{3}{2}$

9. $$\frac{1+\frac{1}{0,\overline{3}}}{2-\frac{1+0,\overline{2}}{0,1\overline{2}}}+4=?$$

A) $\frac{9}{2}$ B) $\frac{9}{4}$ C) $\frac{7}{2}$ D) $\frac{7}{4}$ E) $\frac{5}{2}$

10. $$\frac{0,001}{0,01}+\frac{16,9}{13}-\frac{0,288}{0,24}=?$$

A) $\frac{4}{5}$ B) $\frac{3}{5}$ C) $\frac{2}{5}$ D) $\frac{1}{5}$ E) 1

11. $$\left(\frac{0,\overline{21}}{0,\overline{24}}+\frac{1}{2}\right)^{-1}\cdot\frac{1}{8}=?$$

A) $-\frac{1}{8}$ B) $-\frac{1}{11}$ C) $\frac{1}{11}$ D) $\frac{1}{8}$ E) $\frac{1}{4}$

12. $x=1,\overline{745}$

$x=1,7\overline{45}$

$x=1,74\overline{5}$ ⇒ **? < ? < ?**

A) $y<z<x$ B) $y<x<z$ C) $z<y<x$

D) $x<y<z$ E) $x<z<y$

13. $$\frac{a,0\overline{b}-b,0\overline{a}}{0,0\overline{a}+0,0\overline{b}}=17,8 \Rightarrow \mathbf{max(a+b)=?}$$

A) 10 B) 12 C) 14 D) 15 E) 18

14. $$\left[\left(\frac{1}{0,\overline{3}}\right)^{-2}+\left(\frac{1}{0,\overline{6}}\right)^{-2}\right]:0,\overline{8}=?$$

A) $\frac{3}{8}$ B) $\frac{5}{8}$ C) 1 D) 2 E) $\frac{27}{8}$

15. $$\frac{0,\overline{3}+0,\overline{5}}{8,8\overline{8}}+3,\overline{9}:5=?$$

A) $\frac{3}{10}$ B) $\frac{7}{10}$ C) $\frac{9}{10}$ D) $\frac{11}{10}$ E) $\frac{13}{10}$

16. $x=5,\overline{72}$

$y=6,363636\ldots$

$A\cdot\frac{y}{x}\in Z \Rightarrow$ **min(A) = ?**

A) 9 B) 18 C) 24 D) 25 E) 27

YANITLAR / ANSWERS

1	2	3	4	5	6	7	8
D	B	A	B	C	A	C	D
9	**10**	**11**	**12**	**13**	**14**	**15**	**16**
C	D	C	A	D	B	C	A

1. $\dfrac{0{,}5}{0{,}000005} + \dfrac{0{,}4}{0{,}000004} = ?$

A) 10^5
B) $2 \cdot 10^5$
C) 10^6
D) $2 \cdot 10^6$
E) $3 \cdot 10^6$

2. $x = 0{,}3 + 0{,}03 + 0{,}003 + \ldots$

$y = 10^0 - 5 \cdot 10^{-1} - 5 \cdot 10^{-2} - 5 \cdot 10^{-3} - \ldots$

$\Rightarrow x + y = ?$

A) $\frac{4}{9}$
B) $\frac{5}{9}$
$\frac{2}{3}$
D) $\frac{7}{9}$
E) $\frac{8}{9}$

3. $\dfrac{x,\overline{x} - y,\overline{y}}{x,\overline{0x} - y,\overline{0y}} = ?$

A) $\frac{11}{10}$
B) $\frac{111}{100}$
C) $\frac{101}{100}$
D) $\frac{111}{101}$
E) $\frac{100}{101}$

4. $x = 0{,}0\overline{3},\ y = 1{,}\overline{2}$

$\Rightarrow \dfrac{1}{x} + \dfrac{22}{y} = ?$

A) 36
B) 42
C) 44
D) 48
E) 52

5. $x = \dfrac{23}{22},\quad y = \dfrac{223}{222},\quad z = \dfrac{2223}{2222}$

$\Rightarrow ? < ? < ?$

A) $z < y < x$
B) $y < z < x$
C) $x < y < z$
D) $y < x < z$
E) $z < x < y$

6. $\dfrac{0,\overline{x}}{0{,}0\overline{x}} - \dfrac{1,\overline{3}}{\frac{1}{2} - 0{,}1\overline{6}} = ?$

A) 2
B) 3
C) 4
D) 5
E) 6

7. $\dfrac{0{,}0\overline{xy}}{0,\overline{xy}} + \dfrac{1}{9} \cdot \dfrac{0,x}{0{,}0\overline{x}} = ?$

A) 1,1
B) 1,2
C) 1,3
D) 1,4
E) 1,5

8. $\left(0{,}5 \cdot 2\frac{2}{5}\right)^{-1} : \left(1{,}4\overline{9} - 2^{-1}\right) = ?$

A) $0,\overline{83}$
B) $0{,}8\overline{3}$
C) $1,\overline{83}$
D) $1{,}8\overline{3}$
E) $1,\overline{3}$

9. $\frac{0,002}{0,0002}+\frac{0,49}{0,7}-\frac{0,27}{0,3}+\frac{0,5}{0,05}=?$

A) 19,9 B) 19,8 C) 19,6 D) $19,\overline{8}$ E) $19,\overline{6}$

10. $x, y \in Z$,

$23 < x + y < 42$,

$\frac{x-y}{y} = 4 \Rightarrow \min(x \cdot y) = ?$

A) 64 B) 72 C) 80 D) 84 E) 96

11. $\frac{1}{4} < x < y < \frac{1}{2} \Rightarrow \{x, y\} = ?$

A) $\left\{\frac{5}{12}, \frac{1}{2}\right\}$ B) $\left\{\frac{5}{16}, \frac{5}{8}\right\}$ C) $\left\{\frac{3}{8}, \frac{7}{16}\right\}$

D) $\left\{\frac{3}{4}, \frac{5}{4}\right\}$ E) $\left\{\frac{1}{4}, \frac{7}{16}\right\}$

12. $100 \cdot x + x + 3 \cdot 10^{-1} \cdot x + \frac{x}{100} = A \Rightarrow A = ?$

A) xx,3x B) x3,0x C) x0x,1x

D) x0x,3x E) xx,30x

13. $x = -\frac{5}{13},\ y = -\frac{3}{8},\ z = -\frac{25}{66}$

$\Rightarrow ? < ? < ?$

A) x < z < y B) x < y < z C) y < x < z

D) y < z < x E) z < x < y

14. $A = 0,2 - 0,02 + 0,002 - 0,0002\ldots \Rightarrow A = ?$

A) $\frac{1}{18}$ B) $\frac{1}{9}$ C) $\frac{4}{99}$ D) $\frac{2}{33}$ E) $\frac{2}{11}$

15. $\left(\frac{0,08\overline{3}}{0,2\overline{7}+0,7\overline{2}}\right) \cdot 144 = ?$

A) 8 B) 9 C) 10 D) 12 E) 16

16. $\left(1-\frac{1}{9}\right) \cdot \left(1-\frac{1}{16}\right) \cdot \left(1-\frac{1}{25}\right) \cdot \ldots \cdot \left(1-\frac{1}{196}\right) = ?$

A) $\frac{5}{7}$ B) $\frac{1}{3}$ C) $\frac{6}{7}$ D) $\frac{2}{3}$ E) $\frac{16}{21}$

YANITLAR / ANSWERS

1	2	3	4	5	6	7	8
B	D	A	D	A	E	A	B
9	**10**	**11**	**12**	**13**	**14**	**15**	**16**
B	C	C	D	A	E	D	A

1. $\dfrac{1+\frac{1}{3}}{1-\frac{1}{5}} : \dfrac{1-\frac{1}{3}}{1+\frac{1}{5}} = ?$

A) 1 B) 2 C) –3 D) 3 E) 5

2. $\left(\dfrac{\frac{1}{3}}{4} - \dfrac{2}{\frac{3}{4}}\right) : (12)^{-1} = ?$

A) 0 B) 2 C) –21 D) –31 E) 26

3. $\left[\frac{1}{2}\left(2-\frac{1}{3}\right)-\frac{1}{5}\left(1+\frac{1}{4}\right)\right] : \frac{1}{3} = ?$

A) 2 B) $\frac{5}{2}$ C) $\frac{6}{7}$ D) $\frac{7}{4}$ E) $\frac{8}{4}$

4. $\left(\left(1-\frac{1}{7}\right) : \frac{6}{7}\right) : \left(1-\frac{3}{10}\right) = ?$

A) $\frac{-7}{6}$ B) $\frac{-5}{6}$ C) $\frac{10}{7}$ D) $\frac{3}{7}$ E) $\frac{-5}{4}$

5. $\dfrac{a-b}{2a+b} = \dfrac{1}{4} \Rightarrow \dfrac{a+b}{3 \cdot a} = ?$

A) $\frac{1}{3}$ B) $\frac{5}{4}$ C) $\frac{7}{4}$ D) $\frac{-15}{2}$ E) $\frac{7}{15}$

6. $\dfrac{-2\frac{1}{3}}{4-\frac{2}{3}} : \dfrac{5+\frac{1}{2}}{7-\frac{1}{3}} = ?$

A) $-\frac{19}{3}$ B) $\frac{-28}{33}$ C) $\frac{5}{8}$ D) $\frac{29}{18}$ E) $\frac{7}{2}$

7. $\dfrac{0,2}{0,02} + \dfrac{0,3}{0,03} - \dfrac{4}{0,4} = ?$

A)20 B) 30 C) –10 D) 10 E) 40

8. $2,4 \cdot a - 0,6 = 0,8 \cdot a + 0,04 \Rightarrow$ **a = ?**

A) 1,2 B) 0,16 C) 0,4 D) 3,2 E) 4,1

9. $\dfrac{0,\overline{4}\cdot a}{0,\overline{3}} - \dfrac{0,6\cdot a}{\dfrac{1}{0,\overline{2}}} = 1 \Rightarrow \mathbf{a = ?}$

A) $\frac{2}{3}$ B) $\frac{4}{9}$ C) $\frac{5}{6}$ D) $\frac{7}{2}$ E) $\frac{8}{3}$

10. $1 + \dfrac{12}{1 + \dfrac{12}{1 + \dfrac{12}{\vdots}}} = ?$

A) 0 B) 2 C) 4 D) 6 E) 12

11. $2 - \dfrac{1}{1 + \dfrac{2}{1 + \dfrac{1}{a}}} = \dfrac{3}{2} \Rightarrow \mathbf{a = ?}$

A) 0 B) 1 C) 2 D) 3 E) 4

12. $\dfrac{0,003}{0,002} - \dfrac{0,05}{0,10} = ?$

A) 1 B) 2 C) –3 D) 4 E) 8

13. $\dfrac{1 + \dfrac{1}{a}}{1 + \dfrac{1}{1 - \dfrac{2}{a+2}}} = ?$

A) 2a B) –2 C) $\frac{1}{2}$ D) $\frac{4}{a}$ E) $\frac{a}{4}$

14. $\dfrac{\dfrac{2}{3}}{4} - \dfrac{2}{\dfrac{3}{4}} = ?$

A) $\frac{-12}{7}$ B) $\frac{-5}{2}$ C) $\frac{8}{3}$ D) $\frac{9}{2}$ E) $\frac{19}{2}$

15. $\dfrac{\left(1-\frac{1}{3}\right)+\left(1+\frac{1}{2}\right)}{\left(2-\frac{1}{3}\right)+\left(\frac{4}{3}-1\right)} = ?$

A) $\frac{13}{24}$ B) $\frac{1}{9}$ C) $\frac{4}{99}$ D) $\frac{2}{33}$ E) $\frac{2}{11}$

16. $\dfrac{13-x}{23-x} = \dfrac{1}{2} \Rightarrow \mathbf{x = ?}$

A) 1 B) 2 C) 3 D) 4 E) 5

17. $\dfrac{\left(1-\frac{1}{2}\right)\cdot\left(3-\frac{2}{3}\right)}{\left(2+\frac{1}{2}\right)\cdot\left(1-\frac{1}{5}\right)} = ?$

A) 4 B) $\frac{3}{2}$ C) $\frac{7}{12}$ D) $\frac{3}{5}$ E) 8

YANITLAR / ANSWERS

1	2	3	4	5	6	7	8	9
D	D	D	C	E	B	D	C	C
10	11	12	13	14	15	16	17	
C	B	A	C	B	B	C	C	

1. $\dfrac{\mathbf{m:(n\cdot t)}}{\mathbf{(m:n)\cdot t}}=\mathbf{?}$

A) $\frac{1}{m^2}$ B) $\frac{1}{n^2}$ C) $\frac{1}{t^2}$ D) $\frac{t}{mn}$ E) $\frac{m}{nt}$

2. $\dfrac{\frac{x}{y}}{z}:\dfrac{x}{\frac{y}{z}}=9 \Rightarrow \mathbf{z=?}$

A) 3 B) $\frac{1}{3}$ C) $\frac{1}{2}$ D) 2 E) $\frac{1}{9}$

3. $1+\dfrac{2}{3-\dfrac{1}{1+\dfrac{2}{3-x}}}=4 \Rightarrow \mathbf{x=?}$

A) $-\frac{12}{7}$ B) $\frac{24}{4}$ C) $\frac{31}{5}$ D) $\frac{13}{2}$ E) $\frac{2}{3}$

4. $\left[\left(\dfrac{\mathbf{2a-3}}{\mathbf{b}}\right):\left(2-\dfrac{3}{a}\right)\right]:\dfrac{\mathbf{4a}}{\mathbf{b}}=\mathbf{?}$

A) a·b B) 2a – 3 C) 2a +1

D) $\frac{1}{4}$ E) 4

5. $\dfrac{\mathbf{3-0,\overline{6}}}{\mathbf{1-0,\overline{7}}}=\mathbf{?}$

A) $\frac{7}{2}$ B) $\frac{7}{6}$ C) 7 D) $\frac{21}{2}$ E) 21

6. $1+\dfrac{2}{1+\dfrac{2}{1+\dfrac{2}{\vdots}}}=\mathbf{?}$

A) $\frac{1}{2}$ B) 1 C) $\frac{3}{2}$ D) 2 E) $\frac{4}{3}$

7. $1-\dfrac{2}{1+\dfrac{2}{1+\dfrac{2}{1+\dfrac{2}{\vdots}}}}=\mathbf{?}$

A) –2 B) –1 C) 0 D) 1 E) 2

8. $\left(\frac{1}{4}-1\right)\cdot\left(\frac{1}{5}-1\right)\cdot\left(\frac{1}{6}-1\right)\cdot\ldots\cdot\left(\frac{1}{24}-1\right)=\mathbf{?}$

A) 1 B) $\frac{1}{8}$ C) 0 D) $-\frac{1}{8}$ E) –1

9. $\frac{3}{5}+\frac{4}{7}+\frac{5}{9}=x \Rightarrow \frac{2}{5}+\frac{3}{7}+\frac{4}{9}=?$

A) 2 + x B) 2 − x C) x − 4 D) 3 − x E) x + 4

10. $\dfrac{2-\left(\frac{1}{2}-2:\frac{1}{3}\right)^{-1}}{2-\cfrac{1}{2-\cfrac{1}{2-\cfrac{1}{\vdots}}}}=?$

A) $\frac{1}{2}$ B) $\frac{11}{2}$ C) $-\frac{8}{11}$ D) $\frac{3}{8}$ E) $\frac{24}{11}$

11. $\frac{4}{x}+\frac{5}{y}+\frac{3}{z}=12$

$\Rightarrow \frac{4+x}{x}+\frac{5+2y}{y}+\frac{3z+3}{z}=?$

A) 9 B) 12 C) 15 D) 18 E) 24

12. $\dfrac{\frac{3}{5}+\frac{3}{6}+\frac{3}{7}+\ldots+\frac{3}{19}}{\frac{1}{5}+\frac{1}{6}+\frac{1}{7}+\ldots+\frac{1}{19}}=?$

A) $\frac{1}{3}$ B) $\frac{2}{3}$ C) $\frac{3}{2}$ D) 3 E) 4

13. $\frac{1}{13}+\frac{1}{15}-\frac{1}{17}=x$

$\Rightarrow \frac{11}{13}+\frac{13}{15}+\frac{2}{17}=?$

A) x + 3 B) x − 3 C) 2 − x D) 2 + x E) 2 − 2x

14. $\frac{2}{3}+\frac{22}{33}+\frac{222}{333}+\ldots+\frac{222222}{333333}=?$

A) $\frac{2}{3}$ B) 2 C) $\frac{4}{3}$ D) 4 E) $\frac{8}{3}$

15. $\frac{43}{13}=a+\cfrac{1}{b+\cfrac{1}{c}} \Rightarrow a+b+c=?$

A) 9 B) 10 C) 11 D) 12 E) 13

16. $x = 0,4 + 0,04 + 0,004 + \ldots\ldots$

$\Rightarrow x=?$

A) $\frac{2}{9}$ B) $\frac{1}{3}$ C) $\frac{4}{9}$ D) $\frac{5}{9}$ E) 1

17. $\left(1-\frac{1}{4}\right)\cdot\left(1-\frac{1}{9}\right)\cdot\left(1-\frac{1}{16}\right)\cdot\ldots\cdot\left(1-\frac{1}{169}\right)=?$

A) $\frac{1}{13}$ B) $\frac{7}{13}$ C) $\frac{14}{13}$ D) 7 E) 13

YANITLAR / ANSWERS

1	2	3	4	5	6	7	8	9
C	B	D	D	D	C	D	D	D
10	**11**	**12**	**13**	**14**	**15**	**16**	**17**	
E	D	D	E	D	B	C	B	

BÖLÜM CHAPTER 3

TARİHSEL NOT / HISTORICAL NOTE

Omar Khayyam [1048 – 1122]

Ömer Hayyam, anlaşılması güç matematiksel konuları, şiirsel bir dille açıklayan, derin felsefi bilgisi olan bir düşünür ve şairdir. O astronomik tablolar oluşturarak, yeni bir takvimin ortaya çıkmasına katkıda bulunmuştur. Ayrıca bir çemberin, bir parabol ile kesişiminden oluşan ve üçüncü dereceden denklemleri içeren problemleri çözerek yeni geometrik yöntemler de geliştirmiştir.

Omar Khayyam was a scholar who was a poet as well as a mathematician. He compiled astronomical tables and contributed to calendar reform and discovered a geometrical method of solving cubic equations by intersecting a parabola with a circle.

ÜSLÜ SAYILAR
EXPONENTIALS

Bu bölüm, 119 test sorusu içermektedir.
This chapter includes 119 test questions.

BÖLÜM / CHAPTER 3

ÜSLÜ SAYILAR / EXPONENTIALS

● Üslü Sayılar / Exponentials .... 63 – 84

Metropol

BÖLÜM

03

CHAPTER

ÜSLÜ SAYILAR
EXPONENTIALS

ÜSLÜ SAYILAR / EXPONENTIALS

Tanım: a sıfırdan farklı bir reel (gerçel) sayı ve n bir doğal sayı olmak üzere,

$\underbrace{a\cdot a\cdot a\cdot \ldots\cdot a}_{n\ \text{çarpan}} = a^n$ ifadesine, **üslü ifade** denir.

n : Üs (kuvvet) , **a :** Taban

a^n ifadesi, a nın n. kuvveti olarak okunur.

Definition: Provided that n is a natural number and a is a real number different than zero, the expression

$\underbrace{a\cdot a\cdot a\cdot \ldots\cdot a}_{n\ \text{factors}} = a^n$ is called ***exponential expression.***

n : power , **a :** base.

The expression a^n is read as " a to the n^{th} power".

Özellik / Property

★ Sıfırdan farklı bir sayının sıfırıncı kuvveti 1 dir.

$a \neq 0 \Rightarrow a^0 = 1$

The zeroth power of a number different than zero is 1.

$a \neq 0 \Rightarrow a^0 = 1$

Önemli / Important

- Eğer kuvvet 0 ise, ifadenin değeri her zaman 1 dir.
- Eğer kuvvet 1 ise, ifadenin değeri aynı tabandaki sayıya eşittir.
- If the exponent is 0, the expression always has a value of 1.
- If the exponent is 1, the value of the expression is equal to the base number.

Özellik / Property

★ $a \in R^+$ ve / and $n \in Z$

I. $(-a)^{2n} = a^{2n}$

II. $(-a)^{2n-1} = -a^{2n-1}$

Önemli / Important

- $-a^{2n}$, a^{2n} in negatifi anlamındadır, –a'nın 2n. kuvveti anlamında değildir. –a'nın 2n. kuvveti $(-a)^{2n}$ ile gösterilir.
- $-a^{2n}$ means the negative of a^{2n}, not the $(2n)^{th}$ power of –a which would be represented by $(-a)^{2n}$

$-a^{2n} \neq a^{2n}$

$-4^4 = -4 \cdot 4 \cdot 4 \cdot 4 = -256$

$4^4 = 4 \cdot 4 \cdot 4 \cdot 4 = 256 \Rightarrow -4^4 \neq 4^4$

Özellik / Property

★ **I.** $x^{2n} \geq 0, \ n \in Z$

II. $(f(x))^{2n} + (g(x))^{2n} = 0$ gibi denklemlerin tek çözümü, $f(x) = 0$ ve $g(x) = 0$ dır.

★ **I.** $x^{2n} \geq 0, \ n \in Z$

II. The only solution of the equations like $(f(x))^{2n} + (g(x))^{2n} = 0$ is $f(x) = 0$ and $g(x) = 0$

ÜSLÜ İFADELERDE İŞLEMLER / OPERATIONS in EXPONENTIALS

TOPLAMA ve ÇIKARMA / ADDITION and SUBSTRACTION

▼ Ortak çarpan parantezine alma özelliği kullanılarak üslü sayıların toplam veya farkı tek bir eşitlik ile ifade edilebilir.

▼ Using the common factor brackets property, you can express the sum and difference of exponents as a single form.

$$a \cdot x^n + b \cdot x^n - c \cdot x^n = (a + b - c) \cdot x^n$$

ÇARPMA / MULTIPLICATION

▼ Tabanları aynı olan üslü sayıların çarpımında üsler toplanır.

In multiplication of exponentials with same base, add exponents.

$$a^n \cdot a^m = a^{n+m}$$

▼ Üsleri aynı olan üslü sayıların çarpımında, tabanlar çarpılarak ortak üs, üs olarak yazılır.

In multiplication of exponentials with same power, multiply the base and write the power.

$$a^n \cdot b^n = (a \cdot b)^n$$

BÖLME / DIVISION

▼ Tabanları aynı olan iki üslü sayının bölümünde, ortak taban üzerinde payın üssünden paydanın üssü çıkarılır.

▼ In the division of exponentials with same base, subtract the power of denominator from that of numerator.

$$\frac{a^n}{a^m} = a^{n-m}$$

▼ Üsleri aynı olan iki üslü sayının bölümünde, tabanların bölümüne ortak üs, üs olarak yazılır.

▼ In the division of exponentials with same power, divide bases and write common power as the power.

$$\frac{a^n}{b^n} = \left(\frac{a}{b}\right)^n, \ b \neq 0$$

Önemli / Important

- Çarpma ve bölme için verilen kurallar, toplama ve çıkarma işlemi için uygulanamaz.
- The rules of multiplying and dividing exponents do not apply to addition or subtraction.

BİR ÜSLÜ İFADENİN KUVVETİ / POWER of an EXPONENTIAL EXPRESSION

▼ Bir üslü ifadenin kuvveti alınırken, üsler çarpılıp aynı taban üzerine üs olarak yazılır.

▼ To raise an exponential expression to another power, multiply the powers and write the same base.

$$(a^m)^n = (a^n)^m = a^{m\cdot n}$$

Önemli / Important

- $(a^n)^m \neq (a)^{n^m}$
- a^{n^m} ifadesi tanımsızdır.
- Expression a^{n^m} is undehined.

ÜSLÜ SAYILARDA NEGATİF KUVVET / NEGATIVE POWERS of EXPONENTIALS

▼ $a^{-1} = \frac{1}{a}$, $a^{-n} = \frac{1}{a^n}$

ONDALIK SAYILARIN ÜSLÜ GÖSTERİMİ / WRITING EXPONENTIAL FORM of DECIMAL NUMBERS

▼ $0,\underbrace{000\ \ldots\ 000a}_{\text{n rakam / n digits}} = a\cdot 10^{-n}$ [virgülden sonra n rakam]

[n digits after comma]

ÜSLÜ DENKLEMLER / EXPONENTIAL EQUATIONS

Özellik / Property

★ $a \neq \{-1, 0, 1\}$

$a^n = a^m \Rightarrow n = m$

Özellik / Property

★ $\left.\begin{matrix} a^m = b^x \\ a^n = b^y \end{matrix}\right\} \Rightarrow \frac{m}{n} = \frac{x}{y}$

Özellik / Property

★ $x^n = y^n \Rightarrow \begin{cases} x = y, & \text{n tek tam sayı ise / if n is an odd integer.} \\ x = y \text{ veya / or } x = -y, & \text{n çift tam sayı ise / if n is an even integer.} \end{cases}$

Önemli / Important

- $x^y = 1 \Rightarrow \begin{cases} x = 1 & \\ y = 0, & x \neq 0 \\ x = -1, & \text{y çift / even} \end{cases}$

Özellik / Property

★ x ve y aralarında asal sayılardır.

x and y are relatively prime numbers.

$x^a = y^b \Rightarrow a = b = 0$

ÜSLÜ SAYILARDA EŞİTSİZLİKLER / INEQUALITIES in EXPONENTIALS

▼ $0 < x < y$ ve $m \in Z^+$ olmak üzere, $x^m < y^m$ dir.

▼ $1 < x$ ve $m < n$ olmak üzere, $x^m < x^n$ dir.

▼ $0 < x < 1$ ve $m < n$ olmak üzere, $x^m > x^n$ dir.

▼ If $0 < x < y$ and $m \in Z^+$ then, $x^m < y^m$

▼ If $1 < x$ and $m < n$ then, $x^m < x^n$

▼ If $0 < x < 1$ and $m < n$ then, $x^m > x^n$

1. $4^x + 4^x + 4^x + 4^x = ?$

A) 4^{4x} B) 4^{x+4} C) 4^{x+5} D) 4^{x+1} E) 4^{2x+1}

2. $x = 3^a \Rightarrow 3^{a-3} = ?$

A) x B) $\frac{x}{3}$ C) $\frac{x}{9}$ D) $\frac{x}{27}$ E) $\frac{x}{81}$

3. $\frac{3 \cdot 2^x - 4 \cdot 2^{x-2}}{2^x - 2^{x-1}} = ?$

A) 0 B) 1 C) 2 D) 4 E) 8

4. $4^{2a-1} = 32 \Rightarrow a = ?$

A) $\frac{7}{4}$ B) $\frac{4}{7}$ C) 2 D) $\frac{1}{2}$ E) 3

5. $16^x = 36 \Rightarrow 4^{3x} = ?$

A) 1 B) 3 C) 6 D) 36 E) 216

6. $\frac{9^{10}}{3} = \frac{2^{20}}{2} \cdot x^{19} \Rightarrow x = ?$

A) $\frac{3}{2}$ B) 2 C) $\frac{5}{2}$ D) 3 E) $\frac{7}{2}$

7. $\frac{1}{3^{x-1}} + \frac{1}{3^{x-2}} + \frac{1}{3^{x-3}} = 13 \Rightarrow x = ?$

A) 1 B) 2 C) 3 D) 4 E) 5

8. $\frac{3^x + 3^x + 3^x}{9^x + 9^x + 9^x} = \frac{3}{27} \Rightarrow x = ?$

A) 1 B) 2 C) 3 D) 4 E) 5

9. $\dfrac{(0,001)^{a-2}}{(0,1)^{5-a}} = 10^{2a+4} \Rightarrow$ **a = ?**

A) $\frac{1}{7}$ B) $\frac{6}{7}$ C) $\frac{7}{6}$ D) $\frac{1}{6}$ E) 1

10. $27^{a+2b} = 243^{a-b} \Rightarrow \mathbf{\dfrac{2 \cdot a + 11 \cdot b}{22 \cdot b} = ?}$

A) 1 B) 2 C) 3 D) 4 E) 5

11. $\dfrac{6^x + 6^x + 6^x + 6^x}{12^x + 12^x} = \dfrac{1}{32} \Rightarrow$ **x = ?**

A) 3 B) 4 C) 5 D) 6 E) 7

12. $\mathbf{\dfrac{1}{1+2^{a-b}} + \dfrac{1}{1+2^{b-a}} = ?}$

A) 2^a B) 2^b C) 0 D) –1 E) 1

13. $2^x = a \Rightarrow \mathbf{\dfrac{4^{x+1}-2^4}{2^{x+1}-2^2} = ?}$

A) 2a + 1 B) 2a – 4 C) 4 – 2a
D) 4 + 2a E) 4a + 4

14. $\left.\begin{aligned} 2^a \cdot 3^b &= 0,1\overline{6} \\ 2^b \cdot 3^a &= 36 \end{aligned}\right\} \Rightarrow$ **a + b = ?**

A) 0 B) 1 C) 2 D) 3 E) 4

15. $\left(\dfrac{x}{y}\right)^{a+b} = \left(\dfrac{y}{x}\right)^{2a+5b} \Rightarrow \mathbf{(5)^{\frac{a}{b}} = ?}$

A) $\frac{1}{125}$ B) $\frac{1}{50}$ C) $\frac{1}{25}$ D) $\frac{1}{5}$ E) 1

16. $\left.\begin{aligned} 9^{a+b} &= 243 \\ 4^{2a-b} &= 8 \\ 5^{ma+b} &= 25 \end{aligned}\right\} \Rightarrow$ **m = ?**

A) $\frac{3}{5}$ B) $\frac{5}{8}$ C) $\frac{5}{9}$ D) $\frac{2}{3}$ E) $\frac{7}{9}$

17. $\left.\begin{aligned} 3^x &= 45 \\ 3^y &= 25 \end{aligned}\right\} \Rightarrow$ **2x–y + 1 = ?**

A) 1 B) 2 C) 3 D) 4 E) 5

18. $x^a = 16x^b$, $a + b = 4$
$x^3 = 2 \Rightarrow$ **a = ?**

A) 5 B) 6 C) 7 D) 8 E) 9

YANITLAR / ANSWERS

1	2	3	4	5	6	7	8	9
D	D	D	A	E	A	A	B	C
10	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**
A	D	E	D	B	C	B	E	D

1. $3^{x-1} = 27 \Rightarrow \mathbf{4^x = ?}$

A) 16 B) 64 C) 128 D) 256 E) 512

2. $x^a + x^b = 4 \Rightarrow \mathbf{x^{x+a} + x^{x+b} = ?}$

A) 4 B) 4^x C) $4x^x$ D) $4x^{2x}$ E) 4x+2

3. $4^a = 7 \Rightarrow \mathbf{2^{2a+4} = ?}$

A) 102 B) 112 C) 120 D) 128 E) 256

4. $\left.\begin{array}{l} 2^{x+1} = 7 \\ 2^y = 28 \end{array}\right\} \Rightarrow \mathbf{y - x = ?}$

A) 2 B) 3 C) 6 D) 7 E) 8

5. $\left.\begin{array}{l} 12^a = 16 \\ 12^b = 9 \end{array}\right\} \Rightarrow \mathbf{a + b = ?}$

A) 1 B) 2 C) 3 D) 4 E) 5

6. $3^{a+b-c+4} = 5^{a+b+8} = 6^{c+4}$

$\Rightarrow \mathbf{a + b + c = ?}$

A) 3 B) 0 C) –3 D) –8 E) –12

7. $\dfrac{9^{1-a}}{3^{a-1}} = 81 \Rightarrow \mathbf{a = ?}$

A) $-\frac{1}{3}$ B) 0 C) $\frac{1}{3}$ D) 1 E) 3

8. $\left.\begin{array}{l} x = (2^3)^5 \\ y = 2^{(3^5)} \\ z = 2^{(5^3)} \end{array}\right\} \Rightarrow \mathbf{? > ? > ?}$

A) z > y > x B) y > z > x C) x > y > z D) x > z > y E) z > x > y

9. $\left(\frac{3^x}{3^y}\right)^{y-x} \cdot \left(\frac{3^x}{3^y}\right)^{x-y} = ?$

A) 3^{x-y} B) 3^{y-x} C) 3^y D) 3^x E) 1

10. $(x-4)^{x+1} = 1 \Rightarrow \sum x = ?$

A) 4 B) 5 C) 6 D) 7 E) 8

11. $\left.\begin{matrix} 8^a = 125 \\ 16^b = 36 \end{matrix}\right\} \Rightarrow 2^{a+2b-1} = ?$

A) 2 B) 7 C) 15 D) 27 E) 30

12. $3^{x+1} - 6 \cdot 3^{x-1} + 7 \cdot 3^x = 72 \Rightarrow x = ?$

A) 1 B) 2 C) 3 D) 4 E) 5

13. $\frac{5^x + 5^{-x}}{5^x - 5^{-x}} = \frac{13}{12} \Rightarrow x = ?$

A) –1 B) 0 C) 1 D) 2 E) 3

14. $\frac{27^x - 81^x}{9^x - 27^x} = ?$

A) 1 B) 3 C) 3^x D) 3^{2x} E) 3^{3x}

15. $3^{x-1} = a$

$\Rightarrow 3^x + 3^{x+1} + 3^{x-2} = ?$

A) $\frac{37 \cdot a}{3}$ B) 12a C) $\frac{96 \cdot a}{9}$ D) 9a E) $\frac{78 \cdot a}{9}$

16. $3^a = 12$,

$5^b = 24$,

$4^c = 3$

$\Rightarrow ? > ? > ?$

A) a > b > c B) b > a > c C) a > c > b D) c > b > a E) b > c > a

17. $\frac{0,8.10^{-2} + 0,02.10^{-1}}{10^{-3}} = ?$

A) 10^{-2} B) 10^{-1} C) 1 D) 10 E) 10^2

YANITLAR / ANSWERS

1	2	3	4	5	6	7	8	9
D	C	B	B	B	E	A	B	E
10	11	12	13	14	15	16	17	
D	C	B	C	C	A	A	D	

1. $2^{x+1} = 6^{x+2} \Rightarrow$ **3^{x+2} = ?**

A) 9 B) 3 C) 2 D) $\frac{1}{2}$ E) $\frac{1}{3}$

2. $2^{n-1} = a,$

$3^n = b \Rightarrow$ **24^n = ?**

A) $\frac{a^3 \cdot b^2}{8}$ B) $\frac{a \cdot b^2}{8}$ C) $\frac{a^3 \cdot b}{8}$ D) $\frac{8 \cdot a^3}{b}$ E) $8 \cdot a^3 \cdot b$

3. $3^{n-1} = 0,\overline{6} \Rightarrow \left(\frac{1}{0,\overline{3}}\right)^n$

A) $\frac{1}{3}$ B) $\frac{1}{2}$ C) 1 D) 2 E) 3

4. $2^a \cdot 3^b = 16$ ve / and

$2^a \cdot 3^{1-b} = 16 \Rightarrow$ **b = ?**

A) $-\frac{1}{9}$ B) $\frac{1}{2}$ C) 1 D) $\frac{3}{2}$ E) 2

5. $\frac{36^{x+1} - 4.36^x + 36^x}{6^{x+1} + 5.6^x} = 108 \Rightarrow$ **x = ?**

A) 2 B) 3 C) 5 D) 6 E) 7

6. $\left(\frac{3}{2}\right)^{a-b} \cdot \left(\frac{10}{3}\right)^{a-b} = 25$, $a^2 - b^2 = 16 \Rightarrow$ **2a + 3b = ?**

A) 10 B) 17 C) 18 D) 19 E) 20

7. $\frac{4^a + 4^a + 4^a + 4^a}{2^b + 2^b + 2^b} = \frac{8}{3} \Rightarrow$ **b = ?**

A) 4 – a B) a + 2 C) 2a + 1 D) 2a – 1 E) 2a

8. $3^x = 2^{x+1} \Rightarrow \left(\frac{4}{9}\right)^{x-1}$ **= ?**

A) $\frac{9}{4}$ B) $\frac{4}{9}$ C) $\frac{9}{16}$ D) $\frac{16}{9}$ E) $\frac{9}{2}$

9. $3^{-0,01} = x \Rightarrow \mathbf{3^{3,99} = ?}$

A) $9 \cdot x$ B) $27 \cdot x$ C) $48 \cdot x$ D) $81 \cdot x$ E) $243 \cdot x$

10. $\mathbf{(0,125)^{\frac{2}{3}} + (0,04)^{-\frac{1}{2}} = ?}$

A) 5 B) $\frac{9}{4}$ C) $\frac{21}{4}$ D) $\frac{23}{5}$ E) $\frac{21}{5}$

11. $8^x = 9$ ve / and $3^y = 2 \Rightarrow \mathbf{x \cdot y = ?}$

A) $\frac{1}{6}$ B) $\frac{3}{2}$ C) 3 D) $\frac{2}{3}$ E) 6

12. $12^n = a$,

$108^n = b \Rightarrow \mathbf{3^n = ?}$

A) $\frac{a^2}{b}$ B) $\left(\frac{b}{a}\right)^{\frac{1}{2}}$ C) $\frac{b^2}{4}$ D) $\frac{a}{b}$ E) $\left(\frac{a}{b}\right)^{\frac{1}{2}}$

13. $\left.\begin{matrix} 5^a = (0,25)^{-1} \\ 25^b = 8 \end{matrix}\right\} \Rightarrow \mathbf{\frac{a^2 + b^2}{(a-b)^2} = ?}$

A) 15 B) 16 C) 25 D) 29 E) 33

14. $\left.\begin{matrix} x^{\frac{a}{2}} = y^b \\ (y)^{b^2} = x^6 \end{matrix}\right\} \Rightarrow \mathbf{a.b = ?}$

A) 6 B) 10 C) 12 D) 14 E) 16

15. $x = 2y = 3z$ ve / and

$z^y = y^x \Rightarrow \mathbf{9\,(x + y + z) = ?}$

A) 18 B) 20 C) 22 D) 24 E) 28

16. $(0,16)^a + (0,4)^{2a} = 5^{1-2a} \Rightarrow \mathbf{4^a = ?}$

A) 5 B) 3 C) $\frac{5}{2}$ D) 2 E) 1

17. $(0,008)^a = 2$ ve / and

$16^b = 25 \Rightarrow \mathbf{2^{6a \cdot b+1} = ?}$

A) 0 B) 1 C) $\frac{1}{2}$ D) 18 E) $\frac{1}{16}$

YANITLAR / ANSWERS

1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
D	E	D	B	A	D	D	C	D
10	**11**	**12**	**13**	**14**	**15**	**16**	**17**	
C	D	B	C	C	C	C	B	

1. $2^{x-y} + 2^{y-x} = 9 \Rightarrow \dfrac{4^x + 4^y}{2^{x+y-2}} = ?$

A) 36 B) 18 C) 9 D) $\frac{9}{2}$ E) $\frac{9}{4}$

2. $\left[(-2)^{-3} : \left(-\frac{1}{2}\right)^4\right]^3 : \left(-\frac{1}{3}\right)^{-1} = ?$

A) $\frac{8}{3}$ B) $\frac{1}{24}$ C) $-\frac{1}{24}$ D) – 8 E) – 24

3. $\left(x + \frac{3}{y}\right)^{-a} . \left(y + \frac{3}{x}\right)^a = ?$

A) x^a B) y^a C) $\left(\frac{y}{x}\right)^a$ D) $\left(\frac{x}{y}\right)^a$ E) x^{-a}

4. $\dfrac{9^a}{27^{a+b}} = \dfrac{1}{243}$, $\dfrac{5^{a+b}}{25^b} = 5 \Rightarrow 11^{\frac{a}{b}} = ?$

A) 1 B) 11 C) 22 D) 33 E) 121

5. $\left.\begin{array}{l} 5^{m+2} - 5^{n+1} = 354 \\ 5^{m+1} + 5^{n+1} = 396 \end{array}\right\} \Rightarrow m = ?$

A) 1 B) 2 C) 3 D) 4 E) 5

6. $\left.\begin{array}{l} m^{x-2} = 0,03 \\ m^{x+3} = 0,48 \end{array}\right\} \Rightarrow m^5 = ?$

A) 2 B) 4 C) 8 D)16 E) 32

7. $\dfrac{(1-a^{-1})^8 \cdot (1+a^{-1})^6}{(a-1)^8 \cdot (a+1)^6} = ?$

A) 1 B) $\frac{1}{a}$ C) $\frac{1}{a^2}$ D) $\frac{1}{a^{10}}$ E) $\frac{1}{a^{14}}$

8. $\dfrac{2^x + 1}{2^{x+y} - 2^x + 2^y - 1} = \dfrac{1}{127} \Rightarrow y = ?$

A) 4 B) 5 C) 6 D) 7 E) 8

9. $\dfrac{5\cdot m^{-3}\cdot n^{3}\cdot t^{-4}}{10\cdot m^{-1}\cdot n^{-2}\cdot t}:\dfrac{t^{-5}}{2\cdot m^{-1}\cdot n^{-4}}=?$

A) $m\cdot n^2$ B) $m\cdot n\cdot t^2$ C) $\dfrac{m\cdot n^2}{t}$

D) $\dfrac{m\cdot n}{6}$ E) $\dfrac{n}{m^3}$

10. $x\in Z^{+}$

$4\cdot 4^2\cdot 4^3 \ldots 4^x = 16^{x+1} \Rightarrow$ **x = ?**

A) 2 B) 4 C) 6 D) 8 E) 10

11. $(2x+y)\in R^{+}$

$\dfrac{a^{2x+y}}{a^3}=b^4,\ \dfrac{b^{2x+y}}{b^3}=a^4 \Rightarrow$ **2x + y = ?**

A) 3 B) 4 C) 5 D) 6 E) 7

12. $\left.\begin{array}{l} x=1+3^{t} \\ y=5+3^{1-t} \end{array}\right\} \Rightarrow$ **x = ?**

A) $\dfrac{2-y}{y+1}$ B) $3y+1$ C) $\dfrac{y-5}{2y-7}$

D) $\dfrac{7-2y}{5-y}$ E) $\dfrac{y-2}{y-5}$

13. $\dfrac{9^{2x}+9^{2x}+9^{2x}-81^{x}}{27^{x}+27^{x}}=\dfrac{1}{243} \Rightarrow$ **x = ?**

A) $-\dfrac{5}{2}$ B) -4 C) -5 D) -6 E) -7

14. $(-x^{-1})^4\cdot(-x^{-2})^3\cdot(-(-x)^{-1})^{-5}=?$

A) $-x^5$ B) $-\dfrac{1}{x^5}$ C) 1 D) x^5 E) $\dfrac{1}{x^5}$

15. $2\cdot 2^2\cdot 2^3\cdot\ldots\cdot 2^{23}=(0{,}25)^{3x} \Rightarrow$ **x = ?**

A) –24 B) –36 C) –40 D) –46 E) –50

16. $\left.\begin{array}{l} a^3b=54 \\ a^2b^3=72 \end{array}\right\} \Rightarrow$ **a^{10} = ?**

A) 6^2 B) 3^5 C) 3^6 D) 2^{10} E) 3^{10}

YANITLAR / ANSWERS

1	2	3	4	5	6	7	8
A	A	C	E	B	D	E	D
9	**10**	**11**	**12**	**13**	**14**	**15**	**16**
E	B	E	E	C	B	D	E

1. $\dfrac{12^3+12^3+12^3}{6^4+6^4+6^4+6^4}=?$

A) 1 B) 2 C) 3 D) 4 E) 5

2. $\dfrac{5^{1,1}}{(125)^{-0,3}}+\dfrac{9^{0,4}}{3^{-0,2}}=?$

A) 3 B) 5 C) 8 D) 25 E) 28

3. $\dfrac{2\cdot 7^{22}-9\cdot 7^{21}}{(49)^{10}}=?$

A) 49 B) 35 C) 7 D) 5 E) 1

4. $(-x^2)^3\cdot(-x^{-3})^{-1}\cdot(x^{-2})^3=?$

A) 1 B) $-x^{-3}$ C) $-x^4$ D) x^4 E) x^3

5. $3^{x-1}=m \Rightarrow 3^{x+3}-3^{x+2}-3^{x+1}=?$

A) $7\cdot m$ B) $15\cdot m$ C) $34\cdot m$ D) $45\cdot m$ E) $68\cdot m$

6. $(0,5)^{2x}=3 \Rightarrow (0,25)^{2x}=?$

A) 27 B) 9 C) 3 D) 3^{-1} E) $3^{\frac{1}{2}}$

7. $4^x=5 \Rightarrow 16^{x-1}=?$

A) -2 B) $-\dfrac{1}{8}$ C) $\dfrac{1}{5}$ D) $\dfrac{25}{16}$ E) 4

8. $\dfrac{10^x-5^x}{6^x-3^x}=\dfrac{9}{25} \Rightarrow x=?$

A) -1 B) -2 C) -3 D) -4 E) -5

9. $(a-1)^3 \cdot (1-a)^{-2} = ?$

A) $a-1$ B) $1-a$ C) $(a-1)^2$ D) a E) -1

10. $27^{-4} \cdot \dfrac{3^x}{81^{-1}} = 1 \Rightarrow x = ?$

A) 5 B) 6 C) 7 D) 8 E) 9

11. $2^x \cdot 5^2 = 800$ ve / and $5^z \cdot 5^x = 125 \Rightarrow z = ?$

A) –1 B) –2 C) –3 D) –4 E) –5

12. $\left(\dfrac{81}{16}\right)^{-2x+5} = \left(\dfrac{2}{3}\right)^{3x+10} \Rightarrow x = ?$

A) 3 B) 4 C) 5 D) 6 E) 7

13. $10^{x+1} = 2^{x+2} \Rightarrow 5^{-2x} = ?$

A) $\dfrac{5}{4}$ B) $\dfrac{25}{4}$ C) $\dfrac{125}{4}$ D) $\dfrac{625}{4}$ E) 1

14. $2^x = \dfrac{1}{y} \Rightarrow 16^{1-\frac{x}{4}} = ?$

A) $\dfrac{16}{y}$ B) $16y$ C) $4y$ D) $\dfrac{4}{y}$ E) $8y$

15. $\dfrac{a^{x+y+3} - a^{x+3}}{a^{x+y} - a^x} = ?$

A) a B) a^2 C) a^3 D) a^x E) a^{x+y}

16. $\left(\dfrac{1}{9^x} - 1\right) : \left(\dfrac{1}{3^x} + 1\right) = 2 \Rightarrow x = ?$

A) 3 B) $\dfrac{1}{3}$ C) $-\dfrac{1}{3}$ D) –1 E) –3

17. $\left.\begin{array}{l}(0,125)^a = (0,25)^b \\ 2a + b = \dfrac{7}{2}\end{array}\right\} \Rightarrow 5a - 4b = ?$

A) –2 B) –1 C) 1 D) 2 E) 3

YANITLAR / ANSWERS

1	2	3	4	5	6	7	8	9
A	E	B	E	D	B	D	B	A
10	**11**	**12**	**13**	**14**	**15**	**16**	**17**	
D	B	D	B	B	C	D	B	

1. $5 \cdot 10^4 + 2 \cdot 9^{-2} + 7 \cdot 9^{-2} + 8 \cdot 9^{-1} = ?$

A) 50099 B) 50001 C) 50000,9

D) 50000,7 E) 50000,3

2. $2^a \cdot 0{,}25 = 2 \cdot 5^b \Rightarrow a + b = ?$

A) –4 B) –3 C) $\frac{1}{2}$ D) 3 E) 4

3. $\left(\frac{8}{2^a}\right)^2 = 128 \Rightarrow a = ?$

A) $-\frac{1}{2}$ B) –1 C) 0 D) 1 E) 2

4. $27^{\frac{1}{2}} \cdot 2^{-\frac{1}{2}} \cdot 6^{\frac{1}{2}} = ?$

A) 9 B) 8 C) $7.2^{\frac{1}{2}}$

D) $6.2^{\frac{1}{2}}$ E) $5.2^{\frac{1}{2}}$

5. $(-2)^{-2} \cdot \left(\frac{1}{2}\right)^{-4} \cdot (-2)^{-4} \cdot (-2^3) = ?$

A) 2 B) –2 C) 2^2 D) -2^2 E) 2^3

6. $\left(\frac{1}{81}\right)^{-0,25} = ?$

A) –3 B) $-\frac{1}{3}$ C) $\frac{1}{3}$ D) 3 E) 9

7. $\frac{(-2)^{x+1} \cdot 3^{x-1}}{(-6)^x} = ?$

A) $\frac{2}{3}$ B) $-\frac{1}{2^x}$ C) $-\frac{3^x}{4}$ D) $-\frac{2}{3}$ E) $-\frac{4}{3}$

8. $\frac{14^6 \cdot 5^{m+7}}{7^6 \cdot 5^{4-2m}} : \frac{16}{5^{-3}} = ?$

A) 4 B) $4 \cdot 5^m$ C) $4 \cdot 3^m$

D) 20 E) $4 \cdot 5^{3m}$

9. $12 \cdot (3^x \cdot 9^{x-2}) = 4 \Rightarrow \mathbf{8^x = ?}$

A) 1 B) 2 C) 3 D) 4 E) 8

10. $4^{x+1} + (2^{x+1})^2 = 8^{x-1} \Rightarrow \mathbf{x = ?}$

A) 2 B) 4 C) 5 D) 6 E) 9

11. $6^x = 2^{x+1} \Rightarrow \mathbf{27^x = ?}$

A) 8 B) 9 C) 12 D) 16 E) 18

12. $3^{x-1} = 2 \Rightarrow \left(\frac{1}{27}\right)^{x-\frac{1}{3}} = \mathbf{?}$

A) $\frac{1}{36}$ B) $\frac{1}{72}$ C) $\frac{1}{108}$ D) $\frac{1}{144}$ E) $\frac{1}{180}$

13. $5^{2a} = 7$ ve ve / and $7^b = 5 \Rightarrow \mathbf{a \cdot b = ?}$

A) $\frac{1}{2}$ B) 1 C) $\frac{4}{3}$ D) $\frac{3}{2}$ E) 2

14. $3^{x+5} = 2^{y-2} \Rightarrow \mathbf{x + y = ?}$

A) 7 B) 5 C) 2 D) 0 E) –3

15. $a^x = 8$ ve / and $a^{3x} = 2^a \Rightarrow \mathbf{a = ?}$

A) 6 B) 7 C) 8 D) 9 E) 10

16. $\dfrac{125^a - 1}{25^a + 5^a + 1} = -0{,}8 \Rightarrow \mathbf{a = ?}$

A) –4 B) –3 C) –2 D) –1 E) 0

17. $9^x - 9^{x-1} = 216 \Rightarrow \mathbf{x = ?}$

A) $\frac{2}{3}$ B) $\frac{3}{4}$ C) $\frac{5}{2}$ D) 2 E) 3

YANITLAR / ANSWERS

1	2	3	4	5	6	7	8	9
B	D	A	A	B	D	D	E	E
10	**11**	**12**	**13**	**14**	**15**	**16**	**17**	
D	A	B	A	E	D	D	C	

1. $\left[(-81)^{-1}:\left(\frac{-3}{5}\right)^{-4}\right]^{-1}:\left(\frac{-2}{5}\right)^{-3}=?$

A) 20 B) 25 C) 30 D) 35 E) 40

2. $A = 76\cdot10^{-2} + 9\cdot10^{-3}$

$B = 2\cdot10^{-1} + 31\cdot10^{-3}$

$\Rightarrow$ **A + B = ?**

A) $\frac{9}{10}$ B) $\frac{99}{100}$ C) 1 D) $\frac{11}{10}$ E) $\frac{19}{10}$

3. $7^{2-a} = 49\cdot3^{a}\cdot b \Rightarrow (21^{3a-1})^{-1} = ?$

A) b^3 B) $3\cdot b^3$ C) $7\cdot b^3$ D) $14\cdot b^3$ E) $21\cdot b^3$

4. $(0{,}25)\cdot100^{2x-3} = (0{,}4)^{-1}\cdot10^{3x-4} \Rightarrow x = ?$

A) 2 B) 3 C) 4 D) 5 E) 6

5. $\frac{3}{4x^{a-b}-3}+\frac{4}{3x^{b-a}-4}=?$

A) –2 B) –1 C) 1 D) 2 E) 3

6. $(12x-4)^{2x^2-\frac{2}{9}}=1 \Rightarrow \sum x = ?$

A) $\frac{1}{3}$ B) $\frac{2}{3}$ C) $\frac{1}{12}$ D) $\frac{3}{4}$ E) $\frac{5}{12}$

7. $x = 4^{y} \Rightarrow (0{,}25)^{y-2} = ?$

A) $\frac{4}{x}$ B) $\frac{x}{16}$ C) $\frac{16}{x}$ D) $4\cdot x$ E) $16\cdot x$

8. $\left(\frac{8}{27}\right)^{-x}\cdot\left(\frac{81}{16}\right)^{1-x}=0,\overline{9} \Rightarrow x = ?$

A) –4 B) –3 C) 2 D) 3 E) 4

9. $n \neq 0,$

$$\underbrace{2^n+2^n+2^n+\ldots+2^n}_{\text{n tane/times}}=\left[n^{\frac{1}{n}}.4^{\frac{n}{3}}\right]^n \Rightarrow \mathbf{n = ?}$$

A) 1 B) $\frac{3}{2}$ C) 2 D) $\frac{5}{2}$ E) 3

10. $A = 5^x - 1$

$B = 5^{-x+1} \Rightarrow \mathbf{A = ?}$

A) $\frac{2+B}{B}$ B) $\frac{5-B}{B}$ C) $\frac{2-B}{B-1}$

D) $\frac{5 \cdot B-1}{B}$ E) $\frac{5 \cdot B+5}{B-1}$

11. $x = 3^{\frac{1}{16}} \Rightarrow \dfrac{\left(3^{\frac{1}{8}}-1\right)\cdot\left(3^{\frac{1}{8}}+1\right)}{\left(3^{\frac{1}{2}}-1\right)} = ?$

A) x^2+1 B) x^4+1 C) $\frac{1}{x^2+1}$

D) $\frac{1}{x^4+1}$ E) $\frac{1}{x^8+1}$

12. $5^{2-x} - \frac{3}{5^{x-1}} = 50 \Rightarrow \mathbf{x = ?}$

A) –3 B) –2 C) –1 D) 1 E) 2

13. $\frac{18^x+18^x}{2^x+2^x+2^x} = 54 \Rightarrow \mathbf{x = ?}$

A) $\frac{1}{2}$ B) 1 C) $\frac{3}{2}$ D) 2 E) $\frac{5}{2}$

14. $(x-2y)^{x^2-4y^2} = 1 \Rightarrow \mathbf{\sum x = ?}$

A) $4y+1$ B) $4y$ C) $2y+1$ D) $2y$ E) 1

15. $x = 2^{-65}$

$y = 3^{-39}$

$z = 5^{-26} \Rightarrow \mathbf{? < ? < ?}$

A) $x < y < z$ B) $x < z < y$ C) $z < y < x$

D) $z < x < y$ E) $y < x < z$

16. $(0,1\overline{6})^{-x-2} = 108 \Rightarrow \mathbf{6^{1-x} + 6^{3-x} = ?}$

A) 72 B) 74 C) 76 D) 80 E) 82

YANITLAR / ANSWERS

1	2	3	4	5	6	7	8
E	C	E	B	B	C	C	E
9	10	11	12	13	14	15	16
B	B	D	C	D	E	A	B

1. $3^{x-1}+3^{x+1}=5 \Rightarrow 3^x+3^{x+2}=?$

A) 5 B) 10 C) 15 D) 20 E) 25

2. $x \in \mathbb{N}$,

$\underbrace{5^x \cdot 5^x \cdot 5^x \ldots 5^x}_{x \text{ tane/times}}=25^{72}$

$\underbrace{5^y+5^y+5^y+\ldots+5^y}_{y \text{ tane/times}}=375 \Rightarrow \frac{x}{y}=?$

A) 1 B) 2 C) 3 D) 4 E) 5

3. $\dfrac{(2^x+5)+(2^x+6)+(2^x+7)+\ldots+(2^x+19)}{2^{x+1}+24}=?$

A) 5 B) 6 C) $\frac{15}{2}$ D) 8 E) $\frac{17}{2}$

4. $5^{\frac{m-n}{m}}=15 \Rightarrow 3^{\frac{m+n}{n}}=?$

A) $\frac{1}{3}$ B) $\frac{1}{2}$ C) 1 D) $\frac{2}{5}$ E) $\frac{3}{5}$

5. $(x-2)^{x^2-5x+6}=1 \Rightarrow \sum x=?$

A) 1 B) 2 C) 3 D) 4 E) 6

6. $\dfrac{3}{2^{x-y}-1}+\dfrac{3}{2^{y-x}-1}=?$

A) –3 B) –1 C) 1 D) 2 E) 3

7. $2^{x-y}=81$

$3^{x^2+xy+y^2}=32 \Rightarrow x^3-y^3=?$

A) 8 B) 16 C) 20 D) 24 E) 36

8. $2^x=3^y \Rightarrow 2^{\frac{3x-y}{y}}\cdot 3^{\frac{4y-3x}{x}}=?$

A) 8 B) 16 C) 27 D) 36 E) 72

9. $5^x = 3,$

$3^y = 4$

$\Rightarrow \mathbf{3^{\frac{xy+1}{x}} + 4^{\frac{y+1}{y}} = ?}$

A) 32 B) 36 C) 48 D) 54 E) 64

10. $54^n = 3^{n-3}.9^{n+\frac{1}{2}} \Rightarrow \mathbf{16^{\frac{-2n+1}{4}} = ?}$

A) 18 B) 54 C) 108 D) 162 E) 216

11. $3^x = m \Rightarrow \mathbf{\dfrac{9^{x+1}-81}{3^{x+1}+9} = ?}$

A) m + 3 B) 2m – 9 C) 3m – 3

D) 3m – 9 E) 3m – 27

12. $2\cdot 3^{n+1} - 3^{n+2} + 5\cdot 3^{n-1} = -36 \Rightarrow \mathbf{n = ?}$

A) –3 B) –2 C) –1 D) 2 E) 3

13. $17^{x^2-4} - \left(\frac{1}{13}\right)^{-x^2+3x+10} = 0 \Rightarrow \mathbf{x = ?}$

A) –5 B) –2 C) 2 D) 3 E) 5

14. $2^a = 6^{\frac{x}{x+y}},\ 3^a = 6^{\frac{y}{x+y}} \Rightarrow \mathbf{8^{\frac{y}{x}+1} = ?}$

A) 72 B) 144 C) 192 D) 216 E) 240

15. $(0,\overline{3})^{x^2+x} < \left(\frac{1}{9}\right)^{x+3},$

$x \in N \Rightarrow \mathbf{min(x) = ?}$

A) 0 B) 1 C) 2 D) 3 E) 4

16. $\left(\frac{5}{3}\right)^{x^2+5x} < \left(\frac{9}{25}\right)^{2x}$

$x \in Z^- \Rightarrow \mathbf{min(x) = ?}$

A) –9 B) –8 C) –5 D) –4 E) –1

YANITLAR / ANSWERS

1	2	3	4	5	6	7	8
C	D	C	E	D	A	C	A
9	**10**	**11**	**12**	**13**	**14**	**15**	**16**
A	D	D	E	B	D	E	B

BÖLÜM
CHAPTER
4

TARİHSEL NOT / HISTORICAL NOTE

Leonhard Euler [1707 – 1783]

Leonhard Euler, matematik, fizik, analitik geometri, trigonometri ve sayı teorisi konusunda katkıları bulunan bir İsviçre matematikçisidir.

Leonhard Euler was a Swiss mathematician who made enormous contibutions to a wide range of mathematics and physics topics including analytic geometry, trigonometry, geometry, calculus and number theory.

KÖKLÜ SAYILAR
RADICALS

Bu bölüm, 133 test sorusu içermektedir.

This chapter includes 133 test questions.

BÖLÜM / CHAPTER 4

KÖKLÜ SAYILAR / RADICALS

- Köklü Sayılar / Radicals.. 85 – 108

BÖLÜM

04

CHAPTER

KÖKLÜ SAYILAR
RADICALS

KÖKLÜ SAYILAR / RADICALS

Tanım: $n \in Z^+$, $n \geq 2$ olmak üzere,

$x^n = a$ eşitliğini sağlayan x sayısına, **a'nın n'inci dereceden kökü** denir ve $x = \sqrt[n]{a}$ ile gösterilir.

$n = 2$ için $\sqrt{a}$ ifadesi **karekök a**,

$n = 3$ için $\sqrt[3]{a}$ ifadesi **küpkök a** diye okunur.

Definition: $n \in Z^+$ $n \geq 2$

For any positive integer n, if $x^n = a$, then x is called the *n^{th} root of the number a*. The symbol $\sqrt[n]{a}$ is used to denote the n^{th} root of a.

For $n = 2$, $\sqrt{a}$ is the square root of a,

For $n = 3$, $\sqrt[3]{a}$ is the cube root of a.

Örneğin; $x^3 = 64$ denkleminde $64 = 4^3$ olduğundan, $x = \sqrt[3]{64} = 4$ tür.

$x^4 = 16$ denkleminde ise $16 = 2^4$ veya $16 = (-2)^4$ olduğundan x hem 2, hem de (–2) değerlerini alabilir. Fakat, 16'nın 4. dereceden kökü 2 dir. ($\sqrt[4]{16} = 2$)

For example; in equation $x^3 = 64$ since $64 = 4^3$

$x = \sqrt[3]{64} = 4$.

In equation $x^4 = 16$, $16 = 2^4$ or $16 = (-2)^4$, x can be both 2 and –2. However, the fourth index root of 16 is 2. ($\sqrt[4]{16} = 2$)

Özellik / Property

★ n tek doğal sayı ise, $\sqrt[n]{a^n} = a$ dır.

★ n çift doğal sayı ise, $\sqrt[n]{a^n} = |a|$ dır.

★ If n is an odd natural number, then $\sqrt[n]{a^n} = a$.

★ If n is an even natural number, then $\sqrt[n]{a^n} = |a|$.

Önemli / Important

- Çift dereceli köklerin sonucu negatif olamayacağından $\sqrt{x} = -4$ şeklindeki denklemler reel sayılarda tanımsızdır.
- The results of even roots can not be negative, so equations like $\sqrt{x} = -4$ are undefined in real numbers.

Önemli / Important

- Reel sayılarda bir negatif sayının çift dereceli kökü yoktur.
- A negative number does not have a square real number root.

Özellik / Property

★ $\sqrt{a} + \sqrt{b} = 0 \Rightarrow \mathbf{a = 0\,,\, b = 0}$

Özellik / Property

★ **I.** p tek sayı ise, $\sqrt[p]{a^n} = a^{\frac{n}{p}}$ dır.

If p is an odd number, $\sqrt[p]{a^n} = a^{\frac{n}{p}}$

II. p ve n çift sayı ise, $\sqrt[p]{a^n} = |a|^{\frac{n}{p}}$ dır.

If p and n are even numbers, $\sqrt[p]{a^n} = |a|^{\frac{n}{p}}$

Özellik / Property

★ **I.** x tek doğal sayı ve a > 0 ise, $\sqrt[n]{a^n} = \sqrt[nx]{a^{nx}}$ dır.

II. a < 0 ve x çift doğal sayı ise, $\sqrt[n]{a^n} \neq \sqrt[nx]{a^{nx}}$ dır.

I. If x is an odd natural number and a > 0, then $\sqrt[n]{a^n} = \sqrt[nx]{a^{nx}}$

II. If a < 0 and x is an even natural number, then $\sqrt[n]{a^n} \neq \sqrt[nx]{a^{nx}}$

Özellik / Property

★ $\sqrt[n]{a^n \cdot b} = \begin{cases} a \cdot \sqrt[n]{b}, & \text{n tek ise / if n is odd} \\ |a| \cdot \sqrt[n]{b}, & \text{n çift ise / if n is even} \end{cases}$

Özellik / Property

★ $a \cdot \sqrt[n]{b} = \sqrt[n]{a^n \cdot b}$

KÖKLÜ SAYILARDA İŞLEMLER / OPERATIONS in RADICALS

TOPLAMA VE ÇIKARMA / ADDITION AND SUBTRACTION

▼ Kök içerisinde yer alan ifade ve kök derecesi aynı olan köklü sayılara **benzer köklü sayılar** denir. Sadece benzer köklü ifadelerde, toplama veya çıkarma yapılabildiğinden benzer kökler toplanacağı veya çıkarılacağı zaman, kök katsayılarının toplamı veya farkı alınır ve sonra benzer olan köklü ifade ile çarpılır.

▼ Radicals are said to be like (or similar) radicals if they are in the same index, and have the same radicand. Only like radicals can be added (or subtracted), so when we add or subtract like radicals, we find the algebraic sum or difference of the coefficients of the radicals and multiply that sum by the like radical.

Önemli / Important

- $\sqrt{a+b} \neq \sqrt{a} + \sqrt{b}$
- $\sqrt{a-b} \neq \sqrt{a} - \sqrt{b}$

KÖKLÜ SAYILARDA ÇARPMA İŞLEMİ / OPERATION of MULTIPLICATION in RADICAL

Özellik / Property

★ İki veya daha fazla köklü ifade aynı kök derecesine sahipse kök içerisinde çarpma işlemi yapılabilir.

If two or more radicals have the same index, they can be multiplied inside radical.

$a, b \in R^+$

I. $\sqrt[n]{a} \cdot \sqrt[n]{b} = \sqrt[n]{ab}$

II. $\sqrt{a} \cdot \sqrt{a} = \sqrt{a^2}$

Özellik / Property

★ Eğer köklü ifadeler aynı kök derecesine sahip değilse, ilk olarak kök dereceleri eşitlenir.

If the radicals do not have the same index, they should first be reduced to radicals with the same index.

$$\sqrt[n]{a} \cdot \sqrt[m]{a} = \sqrt[m \cdot n]{a^m} \cdot \sqrt[m \cdot n]{a^n} = \sqrt[m \cdot n]{a^{m+n}}$$

Özellik / Property

★ $a \in R^+$, $n \in N^+$ ise, $(\sqrt{a})^n = \sqrt{a^n}$ dir.

If $a \in R^+$ and $n \in N^+$, then $(\sqrt{a})^n = \sqrt{a^n}$

KÖKLÜ SAYILARDA BÖLME İŞLEMİ / OPERATION of DIVISION in RADICALS

▼ İki köklü ifade aynı kök derecesine sahip ise ortak kök içerisinde bölme işlemi yapılabilir.

$a, b \in R^+$, $b \neq 0$

$$\frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}}$$

▼ If two radicals have same index, the operation of division can be done inside the common radical.

$a, b \in R^+$, $b \neq 0$

$$\frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}}$$

EŞLENİK İFADELER / CONJUGATE EXPRESSIONS

Eşlenik: Rasyonel olmayan bir ifadeyi rasyonel yapan çarpana **eşlenik** denir.

Conjugate: a multiplier that vationalizes an irrational expresson is called conjugate.

İfade / Expression	Eşlenik / Conjugate
$\sqrt{a}$	$\sqrt{a}$
$\sqrt[n]{a^t}$	$\sqrt[n]{a^{n-t}}$
$\sqrt{a}+\sqrt{b}$	$\sqrt{a}-\sqrt{b}$
$\sqrt[3]{a}-\sqrt[3]{b}$	$\sqrt[3]{a^2}+\sqrt[3]{ab}+\sqrt[3]{b^2}$
$\sqrt{a}+\sqrt{b}+\sqrt{c}$	$\sqrt{a}+\sqrt{b}-\sqrt{c}$, $\sqrt{a}-\sqrt{b}+\sqrt{c}$, $-\sqrt{a}+\sqrt{b}+\sqrt{c}$

Özellik / Property

★ **$\sqrt{a\mp2\sqrt{b}}$ nin $\sqrt{m}\mp\sqrt{n}$ biçiminde yazılışı :**

$\sqrt{a\mp2\sqrt{b}}$ ifadesi için a, b $\in R^+$ olmak üzere,

$b = m\cdot n$, $a = m + n$ koşulunu sağlayan m ve n gibi iki doğal sayı bulunabiliyorsa,

$m > n$ olmak üzere, $\sqrt{a\mp2\sqrt{b}}=\sqrt{m}\mp\sqrt{n}$ şeklinde yazılabilir.

Writing the expression $\sqrt{a\mp2\sqrt{b}}$ in the form of $\sqrt{m}\mp\sqrt{n}$

For the expression $\sqrt{a\mp2\sqrt{b}}$,

If you can find two natural numbers m and n in a way that $b = m\cdot n$ and $a = m + n$ when $m > n$, $m,n \in N^+$.

We can write as follows: $\sqrt{a\mp2\sqrt{b}}=\sqrt{m}\mp\sqrt{n}$.

Önemli / Important

- $a,b\in R^+$

$$\left.\begin{array}{l} a=\dfrac{m+n}{2}\\ b=m\cdot n\end{array}\right\} \Rightarrow \sqrt{a\pm\sqrt{b}}=\sqrt{\frac{m}{2}}\pm\sqrt{\frac{n}{2}}$$

İÇ İÇE KÖKLER / NESTED RADICALS

Özellik / Property

★ $\sqrt[n]{\sqrt[m]{a}} = \sqrt[n \cdot m]{a}$

★ $\sqrt[n]{a^k \sqrt{b^x \sqrt{c}}} = \sqrt[n \cdot k \cdot x]{c \cdot b^x \cdot a^{k \cdot x}}$

Özellik / Property

★ $\sqrt{a+\sqrt{a+\sqrt{a+\ldots}}} = \dfrac{1+\sqrt{1+4 \cdot a}}{2}$

$\sqrt{a-\sqrt{a-\sqrt{a-\ldots}}} = \dfrac{-1+\sqrt{1+4 \cdot a}}{2}$

★ $a \in R^+,\ n \cdot (n+1) = a$

$\sqrt{a+\sqrt{a+\sqrt{a+\ldots}}} = n+1$

$\sqrt{a-\sqrt{a-\sqrt{a-\ldots}}} = n$

Özellik / Property

★ $y = \sqrt[n]{a \sqrt[n]{a \sqrt[n]{a \sqrt[n]{a \ldots}}}} = \sqrt[n-1]{a}$

Özellik / Property

★ $y = \sqrt[n]{a : \sqrt[n]{a : \sqrt[n]{a \ldots}}} = \sqrt[n+1]{a}$

1. $\sqrt[3]{-125}+\sqrt{(-3)^2}+\sqrt{16}-\sqrt[4]{(-7)^4}=?$

A) –12 B) –5 C) 3 D) 7 E) 9

2. $\sqrt{0,04}+\sqrt{0,16}+\sqrt{0,0009}+\sqrt[3]{0,027}=?$

A) 0,6 B) 0,63 C) 0,9 D) 0,93 E) 1,2

3. $\dfrac{\sqrt[3]{0,027}-\sqrt{0,64}}{\sqrt{0,04}}=?$

A) $\frac{-5}{2}$ B) $\frac{-1}{2}$ C) $\frac{1}{2}$ D) 1 E) $\frac{5}{2}$

4. $\dfrac{\sqrt{0,09}}{\sqrt{0,16}}+\dfrac{\sqrt{0,25}}{\sqrt{0,36}}=?$

A) $\frac{19}{4}$ B) $\frac{19}{6}$ C) $\frac{9}{5}$ D) $\frac{19}{12}$ E) $\frac{34}{19}$

5. $\left.\begin{aligned}a&=3-\sqrt{5}\\ b&=3+\sqrt{5}\end{aligned}\right\}\Rightarrow\sqrt{(a+b)^2+13}=?$

A) 5 B) 6 C) 7 D) 8 E) 9

6. $x>y>0$

$\Rightarrow\sqrt{(y-x)^2}+\sqrt{(x+y)^2}+\sqrt{x^2}=?$

A) 2x + y B) 2x C) 3x – 2y D) y + x E) 3x

7. $x<0<y$

$\Rightarrow\sqrt{(x-y)^2}+\sqrt{x^2}+\sqrt{y^2}=?$

A) 2x B) 2x C) 0 D) 2x – 2y E) 2y – 2x

8. $\sqrt{(\sqrt{5}-2)^2}+\sqrt{(\sqrt{5}-3)^2}=?$

A) $2\sqrt{5}-1$ B) $5-2\sqrt{5}$ C) $1-\sqrt{5}$ D) 1 E) 3

9. $\sqrt{20}+2\sqrt{45}-3\sqrt{245}=?$

A) $-15\sqrt{5}$ B) $-13\sqrt{5}$ C) $-12\sqrt{5}$ D) $-11\sqrt{5}$ E) $-10\sqrt{5}$

10. $\sqrt{1+\frac{7}{9}}+\sqrt{2-\frac{7}{16}}=?$

A) $\frac{5}{2}$ B) $\frac{31}{12}$ C) $\frac{8}{3}$ D) $\frac{11}{4}$ E) $\frac{17}{6}$

11. $(\sqrt{3}+\sqrt{2})^2+(\sqrt{3}-\sqrt{2})^2=?$

A) $4\sqrt{6}$ B) 6 C) $2\sqrt{6}$ D) 10 E) $4\sqrt{6}-10$

12. $(\sqrt{5}+2)\cdot(2\sqrt{5}-\sqrt{3})+\sqrt{3}(\sqrt{5}+2)=?$

A) $10-4\sqrt{5}$ B) $4\sqrt{5}-10$ C) $4\sqrt{5}+10$ D) $10+2\sqrt{5}$ E) $10-2\sqrt{5}$

v13. $\sqrt{7+2\sqrt{6}}=?$

A) $\sqrt{3}+\sqrt{2}$ B) $\sqrt{3}-\sqrt{2}$ C) $\sqrt{5}+1$ D) $\sqrt{6}+1$ E) $\sqrt{6}-1$

14. $\sqrt{5-2\sqrt{6}}+\sqrt{7-2\sqrt{12}}=?$

A) $2\sqrt{2}+2$ B) $\sqrt{3}-2$ C) $2\sqrt{3}-2$ D) $\sqrt{2}-2$ E) $2-\sqrt{2}$

15. $\sqrt{4+\sqrt{12}}+\sqrt{7-\sqrt{48}}=?$

A) $2\sqrt{3}-1$ B) $3-\sqrt{3}$ C) $\sqrt{3}$ D) 3 E) $\sqrt{3}-3$

16. $\sqrt{7+\sqrt{33}}=?$

A) $\sqrt{11}+\sqrt{3}$ B) $\sqrt{11}+3$ C) $\frac{\sqrt{11}+3}{2}$ D) $\frac{\sqrt{11}-3}{2}$ E) $\frac{\sqrt{11}+\sqrt{3}}{\sqrt{2}}$

YANITLAR / ANSWERS

1	**2**	**3**	**4**	**5**	**6**	**7**	**8**
B	D	A	D	C	E	E	D
9	**10**	**11**	**12**	**13**	**14**	**15**	**16**
B	B	D	C	D	E	D	E

1. $(x-3)^2+(y+4)^2+\sqrt{x^2+y^2-a}=0 \Rightarrow$ **a = ?**

A) 5 B) 9 C) 16 D) 25 E) 32

2. $\sqrt{18}+\sqrt{72}-\sqrt{288}=?$

A) $-3\sqrt{2}$ B) $\sqrt{2}$ C) $3\sqrt{2}$
D) $9\sqrt{2}$ E) $21\sqrt{2}$

3. $\dfrac{7}{3-\sqrt{2}}-\dfrac{7}{3+\sqrt{2}}=?$

A) $\sqrt{2}$ B) $2\sqrt{2}$ C) $6\sqrt{2}$
D) $9\sqrt{2}$ E) $12\sqrt{2}$

4. $\sqrt{8+2\sqrt{7}}\cdot(\sqrt{7}-1)=?$

A) 6 B) 7 C) 8 D) 9 E) 10

5. $\dfrac{\sqrt{3+\sqrt{5}}}{\sqrt{5}+1}=?$

A) $\dfrac{1}{\sqrt{2}}$ B) 1 C) $\sqrt{2}$ D) $2\sqrt{2}$ E) 4

6. $\sqrt{a+\sqrt{a+\sqrt{a+\ldots}}}=8 \Rightarrow$ **a = ?**

A) 56 B) 64 C) 72 D) 80 E) 88

7. $\sqrt{12+\sqrt{11+\sqrt{30-a}}}=4 \Rightarrow$ **a = ?**

A) 5 B) 10 C) 15 D) 20 E) 25

8. $\sqrt{x\cdot\sqrt{x\cdot\sqrt{x\cdot\ldots}}}=\sqrt{y:\sqrt{y:\sqrt{y:\ldots}}} \Rightarrow \dfrac{y}{x}=?$

A) x B) y C) x^2 D) y^2 E) $x\cdot y$

9. $\left.\begin{array}{l}\sqrt{3}=a\\ \sqrt{5}=b\end{array}\right\} \Rightarrow \sqrt{75}=?$

A) $a\sqrt{b}$ B) $b\sqrt{a}$ C) $\sqrt{ab}$
D) ab E) $a\cdot b^2$

10. $\sqrt{12+\sqrt{13\cdot\sqrt{13\cdot\ldots}}}=?$

A) 25 B) 5 C) 1 D) $\dfrac{1}{5}$ E) $\dfrac{1}{25}$

11. $\sqrt{1001 \cdot 997 + 4} = ?$

A) 997 B) 998 C) 999 D) 1000 E) 1001

12. $\frac{a}{b} + \frac{b}{a} = 4 \Rightarrow \sqrt{\frac{a^4+b^4}{a^2b^2} + 18} = ?$

A) $\sqrt{2}$ B) $2\sqrt{2}$ C) 11
D) $4\sqrt{2}$ E) 8

13. $\sqrt{9-\sqrt{17}} - \sqrt{9+\sqrt{17}} = ?$

A) –1 B) $-\sqrt{2}$ C) –2 D) $-2\sqrt{2}$ E) –4

14. $\sqrt{12+\sqrt{12+\sqrt{12+\ldots}}} = \sqrt{x:\sqrt{x:\sqrt{x:\ldots}}} \Rightarrow x = ?$

A) 4 B) 8 C) 16 D) 32 E) 64

15. $\frac{\sqrt{2}}{3+2\sqrt{2}} + \frac{\sqrt{2}}{3-2\sqrt{2}} = ?$

A) $\sqrt{2}$ B) $\sqrt{3}$ C) $\sqrt{6}$
D) 6 E) $6\sqrt{2}$

16. $\sqrt{24^2+32^2+96^2} + \sqrt{27^2+36^2+108^2} = ?$

A) 144 B) 169 C) 196 D) 221 E) 289

17. $\sqrt{a\sqrt[3]{\frac{1}{a}}} = 2 \Rightarrow \sqrt{a\sqrt{a\sqrt[3]{\frac{1}{a}}}} = ?$

A) 1 B) 2 C) 4 D) 8 E) 16

18. $a, b \in \mathbb{R}^+$

$\left.\begin{array}{l} a\sqrt{b} = 3x^2 \\ b\sqrt{a} = 9x \end{array}\right\} \Rightarrow b = ?$

A) 1 B) 3 C) $3\sqrt{2}$
D) 9 E) $9\sqrt{2}$

19. $\sqrt[a]{4^b} = \sqrt[b]{64^a} \Rightarrow \frac{a}{b} = ?$

A) 1 B) $\frac{\sqrt{3}}{3}$ C) $\sqrt{3}$ D) 3 E) $3\sqrt{3}$

YANITLAR / ANSWERS

1	2	3	4	5	6	7	8	9	10
D	A	B	A	A	A	A	C	B	B
11	12	13	14	15	16	17	18	19	
C	D	B	E	E	D	D	D	B	

1. $\left.\begin{aligned} a &= \sqrt{2} \\ b &= \sqrt{3} \end{aligned}\right\} \Rightarrow \sqrt{0,24} = ?$

A) $\frac{ab}{5}$
B) $\frac{a^2 \cdot b}{5}$
C) $\frac{a^3 \cdot b}{10}$
D) $\frac{a^2 \cdot b}{3}$
E) $\frac{a \cdot b}{2}$

2. $\frac{\sqrt{3}+\sqrt{5}}{\sqrt{12}+\sqrt{20}} = 2^{a+1} \Rightarrow a = ?$

A) 2
B) 1
C) 0
D) –1
E) –2

3. $\left.\begin{aligned} x &= \sqrt{3} \\ y &= \sqrt{2} \end{aligned}\right\} \Rightarrow \frac{3x^2 - 2y^2}{x - y} = ?$

A) $2(x - y)$
B) $5(x + y)$
C) $5(x - y)$
D) $x + 2y$
E) $2x - y$

4. $\left.\begin{aligned} y^2 &= \sqrt[3]{a^2 b} \\ y^3 &= \sqrt[4]{ab^3} \end{aligned}\right\} \Rightarrow b = ?$

A) a^{-3}
B) a
C) $2a$
D) a^2
E) a^3

5. $\sqrt{2} = a \Rightarrow \left(\frac{\sqrt{3}}{\sqrt{6} - \sqrt{3}}\right)^2 = ?$

A) $2a - 1$
B) $3a - 1$
C) $3 + 2a$
D) $3a + 2$
E) $2a + 1$

6. $(\sqrt{3} - \sqrt{2})^2 + \sqrt{24} = ?$

A) $-\sqrt{6}$
B) $2\sqrt{6}$
C) $4\sqrt{6}$
D) 5
E) 6

7. $\left.\begin{aligned} a &= \sqrt{7 + \sqrt{18 + \sqrt{36}}} \\ b &= 1 - 2\sqrt{6} \end{aligned}\right\} \Rightarrow b = ?$

A) a
B) $a - 3$
C) $3a$
D) $3 - 2a$
E) $2a - 1$

8. $a + 1 = \sqrt{3} \Rightarrow \sqrt{8 - \sqrt{48}} = ?$

A) $a + 2$
B) $a - 2$
C) $\sqrt{a} + 2$
D) $\frac{a\sqrt{2}}{2}$
E) $a\sqrt{2}$

9. $\left.\begin{array}{l} a=\sqrt{4+2\sqrt{3}} \\ b=\sqrt{4-2\sqrt{3}} \end{array}\right\} \Rightarrow \sqrt[3]{(a\cdot b)^2} = ?$

A) 2 B) $\sqrt{2}$ C) $\sqrt[3]{4}$ D) $\sqrt{3}-1$ E) $\sqrt{3}+1$

10. $\sqrt{203\cdot 205 - 202\cdot 206} = ?$

A) $2\sqrt{3}$ B) $2\sqrt{2}$ C) 2 D) $\sqrt{2}$ E) $\sqrt{3}$

11. $\frac{1}{\sqrt{4^{1-x}}} + \sqrt{4^{x-1}} + 1 = 17 \Rightarrow x = ?$

A) 1 B) 2 C) 3 D) 4 E) 5

12. $a > 0,\ \sqrt[3]{a}\cdot\sqrt{a\sqrt[3]{a}} = 6 \Rightarrow a = ?$

A) $\sqrt[3]{6}$ B) $\sqrt{6}$ C) $\sqrt{5}$ D) 6 E) 16

13. $\left(\frac{1}{\sqrt{5}-2} + \frac{1}{\sqrt{5}+2}\right)\cdot\frac{1}{\sqrt{5}} = ?$

A) 5 B) 2 C) $2\sqrt{5}$ D) $\frac{\sqrt{5}}{3}$ E) $\frac{\sqrt{3}}{3}$

14. $\frac{15+\sqrt{2x-3}}{2x-\sqrt{3-2x}} = ?$

A) 0 B) 3 C) 5 D) 6 E) 7

15. $a^2 = 4 - 2\sqrt{3} \Rightarrow \frac{a-\sqrt{3}}{1-a\sqrt{3}} = ?$

A) $\sqrt{3}$ B) $\sqrt{3}-1$ C) 1
D) $\sqrt{3}+2$ E) $\sqrt{2}$

16. $\frac{\sqrt{2}}{\sqrt{5}-\sqrt{3}} + \frac{\sqrt{2}}{\sqrt{5}+\sqrt{3}} = ?$

A) $\sqrt{10}$ B) $\sqrt{5}$ C) 0 D) $\sqrt{3}$ E) $\sqrt{3}-\sqrt{5}$

17. $a > 0$,

$$\frac{\sqrt[6]{\frac{1}{a}}}{\sqrt[3]{a^{-2}\sqrt{\frac{1}{a}}}} = x \Rightarrow x^6 = ?$$

A) a B) a^2 C) a^3 D) a^4 E) a^5

YANITLAR / ANSWERS

1	2	3	4	5	6	7	8	9
C	E	B	E	C	D	D	E	C
10	11	12	13	14	15	16	17	
E	D	D	B	C	D	A	D	

1. $\frac{6}{\sqrt{3}}-12\sqrt{3}+\sqrt{12}+\sqrt{192}=?$

A) 0 B) $\sqrt{3}$ C) $2\sqrt{3}$ D) $3\sqrt{3}$ E) $4\sqrt{3}$

2. $\sqrt{1+\frac{5}{4}}+\sqrt{1+\frac{7}{9}}-\sqrt{5+\frac{5}{4}}=?$

A) 0 B) $\frac{1}{6}$ C) $\frac{1}{4}$ D) $\frac{1}{3}$ E) $\frac{1}{2}$

3. $\sqrt{5+2\sqrt{3-2\sqrt{2}}}=?$

A) $\sqrt{2}$ B) $\sqrt{3}$ C) $\sqrt{2}+1$ D) $\sqrt{3}-\sqrt{2}$ E) 1

4. $1-\frac{\frac{1}{\sqrt{3}}-\sqrt{3}}{\sqrt{2}-\frac{2}{\sqrt{3}}}-3=?$

A) $\sqrt{3}$ B) $2\sqrt{3}$ C) $\sqrt{6}$
D) $1+\sqrt{6}$ E) $2+\sqrt{6}$

5. $\sqrt[3]{\sqrt{2}}=(0,25)^{\frac{-x+1}{3}} \Rightarrow x=?$

A) $\frac{2}{5}$ B) $\frac{3}{5}$ C) $\frac{4}{5}$ D) 1 E) $\frac{5}{4}$

6. $\sqrt[4]{2+\sqrt{3}}\cdot\sqrt[8]{7-2\sqrt{12}}=?$

A) 1 B) $\sqrt{2}$ C) 3 D) $2\sqrt{3}$ E) 4

7. $\sqrt{\sqrt{6}-\sqrt{2}}\cdot\sqrt[3]{6+\sqrt{2}}\cdot\sqrt[6]{\sqrt{6}+\sqrt{2}}=?$

A) 1 B) 2 C) $\sqrt{2}$ D) $\sqrt[3]{2}$ E) $\sqrt[4]{2}$

8. $\sqrt{\frac{2,4}{0,08}+\frac{0,21}{0,07}+\frac{5}{0,5}+6}=?$

A) 3 B) 4 C) 5 D) 6 E) 7

9. $\sqrt{13+4\sqrt{3}}\cdot\sqrt{5+\sqrt{12+4\sqrt{3}}\cdot\sqrt{5-\sqrt{12+4\sqrt{3}}}} = ?$

A) 7 B) 8 C) 9 D) 10 E) 11

10. $\dfrac{2\sqrt{3}+2}{\sqrt{2+\sqrt{3}}}-\dfrac{\sqrt{2}+2}{\sqrt{2}+1} = ?$

A) $\sqrt{2}$ B) $\sqrt{3}$ C) 2 D) 3 E) $\sqrt{3}-\sqrt{2}$

11. $\dfrac{\sqrt{3}+1}{\sqrt{6}+\sqrt{3}+\sqrt{2}+1}-\sqrt{3+2\sqrt{2}} = ?$

A) –3 B) –2 C) –1 D) $\sqrt{3}-\sqrt{2}$ E) $\sqrt{6}$

12. $9\cdot\sqrt[3]{x\sqrt[3]{x\sqrt[3]{x\ldots}}} = 20+\sqrt{x\sqrt{x\sqrt{x\ldots}}}$

$\Rightarrow \sum x = ?$

A) 9 B) 16 C) 25 D) 31 E) 41

13. $\sqrt{7+\sqrt[3]{4\sqrt[3]{4\sqrt[3]{4\ldots}}}}+\sqrt[4]{x:\sqrt[4]{x:\sqrt[4]{x\ldots}}} = 5$

$\Rightarrow x = ?$

A) 4 B) 8 C) 16 D) 27 E) 32

14. $A=\sqrt{\sqrt{5}+\sqrt{3}}$

$B=\sqrt{\sqrt{5}+\sqrt{2}}+\sqrt{\sqrt{5}-\sqrt{2}} \Rightarrow B = ?$

A) 2A B) $2A^2$ C) $A\sqrt{2}$ D) $2\sqrt{A}$ E) $A^2\sqrt{2}$

15. $x, y \in R$,

$3^{\sqrt{x-6}}+7^{\sqrt{6-x}} = y \Rightarrow \sqrt[3]{x+y} = ?$

A) $\sqrt[3]{4}$ B) $\sqrt[3]{6}$ C) 1 D) 2 E) $\sqrt[3]{12}$

16. $\dfrac{\sqrt{21}-\sqrt{30}+3}{\sqrt{7}-\sqrt{10}+\sqrt{3}}\cdot\sqrt{15} = ?$

A) 3 B) 5 C) $3\sqrt{3}$ D) $4\sqrt{3}$ E) $3\sqrt{5}$

YANITLAR / ANSWERS

1	2	3	4	5	6	7	8
A	D	C	C	E	A	B	E
9	**10**	**11**	**12**	**13**	**14**	**15**	**16**
E	A	B	E	E	C	D	E

1. $\sqrt{0{,}36}-2\cdot\sqrt{0{,}64}+3\cdot\sqrt{1{,}21}-\sqrt{1{,}96}=?$

A) 0,12 B) 1,12 C) 0,9 D) 0,8 E) 2,1

2. $\sqrt{\frac{3}{7}}-\sqrt{189}+\sqrt{\frac{7}{3}}+\frac{11}{\sqrt{21}}=?$

A) $-2\sqrt{21}$ B) $-\sqrt{21}$ C) 0
D) $\sqrt{21}$ E) $4\sqrt{21}$

3. $\sqrt[3]{0{,}008}+\sqrt{0{,}36}-\sqrt[5]{0{,}00032}=?$

A) 2,2 B) 0,9 C) 0,8 D) 0,7 E) 0,6

4. $\sqrt{\frac{49}{81}+\frac{9}{16}-\frac{7}{6}}=?$

A) $\frac{1}{\sqrt{2}}$ B) $\frac{1}{6}$ C) $\frac{1}{12}$ D) $\frac{1}{36}$ E) $\frac{5}{36}$

5. $\frac{1}{\sqrt{5}-\sqrt{3}}+\frac{1}{\sqrt{5}+\sqrt{3}}+\sqrt{21-8\sqrt{5}}=?$

A) $4\sqrt{5}$ B) $3\sqrt{3}$ C) 3 D) 4 E) $\frac{9}{2}$

6. $\left.\begin{array}{l}A=\sqrt{8+\sqrt{15}}\\ B=\sqrt{8-\sqrt{15}}\end{array}\right\}\Rightarrow\frac{(A+B)^2}{A\cdot B}=?$

A) $\frac{7}{5}$ B) $\frac{15}{14}$ C) $\frac{14}{15}$ D) $\frac{30}{7}$ E) 4

7. $\frac{1}{\sqrt{5-\sqrt{21}}}-\frac{1}{\sqrt{5+\sqrt{21}}}=?$

A) $\frac{\sqrt{6}}{2}$ B) $\frac{\sqrt{7}}{2}$ C) $\frac{\sqrt{5}+\sqrt{7}}{2}$
D) $\frac{\sqrt{7}-\sqrt{5}}{2}$ E) $\sqrt{5}-1$

8. $x<|x|$ ve / and $y>0\Rightarrow\sqrt{x^2-4xy+4y^2}-|y-x|=?$

A) $2(x-y)$ B) x C) y
D) $2(x+y)$ E) $2(-x+y)$

9. $\sqrt{\dfrac{0,\overline{ab}+0,\overline{ba}}{0,\overline{a}+0,\overline{b}}}+\sqrt{\dfrac{0,\overline{3}+0,\overline{5}}{0,\overline{35}+0,\overline{53}}}=?$

A) 2 B) 3 C) 4 D) 5 E) 6

10. $\left.\begin{array}{l} a=\sqrt{8+\sqrt{60}} \\ b=\sqrt{8-\sqrt{60}} \end{array}\right\} \Rightarrow \dfrac{a}{b}-\dfrac{b}{a}=?$

A) 6 B) 12 C) $4\sqrt{3}$ D) $8\sqrt{5}$ E) $2\sqrt{15}$

11. $a<b<3 \Rightarrow \dfrac{\sqrt{a^2-6a+9}}{a-3}+\dfrac{\sqrt{a^2-2ab+b^2}}{b-a}=?$

A) 2a–b B) 2 C) 0 D) –2 E) b–2a

12. $9^a=5 \Rightarrow 3^{5a}=?$

A) $\sqrt{5}$ B) $1+\sqrt{5}$ C) $5\sqrt{5}-1$ D) $25\sqrt{5}$ E) $\dfrac{\sqrt{21}}{7}$

13. $\sqrt[3]{\dfrac{\sqrt{3}}{\sqrt[3]{729}}}=3^x \Rightarrow x=?$

A) $-\dfrac{1}{2}$ B) $-\dfrac{3}{2}$ C) $\dfrac{1}{2}$ D) 2 E) $-\dfrac{5}{9}$

14. $A=\dfrac{25}{81}+\dfrac{9}{25}-\dfrac{2}{3}$

$\Rightarrow \dfrac{1}{\sqrt{A}}=?$

A) $\dfrac{5}{9}$ B) $\dfrac{15}{4}$ C) $\dfrac{15}{2}$ D) $\dfrac{45}{2}$ E) $\dfrac{27}{2}$

15. $\sqrt[6]{a\sqrt[6]{a\sqrt[6]{a\sqrt[6]{a\ldots}}}}=3 \Rightarrow a=?$

A) 3 B) 9 C) 27 D) 81 E) 243

16. $\sqrt{a\sqrt{a}}=125 \Rightarrow \sqrt{a+\sqrt{a+26}}=?$

A) 24 B) 25 C) 26 D) 27 E) 30

17. $\sqrt{2664^2-1776^2-888^2}=?$

A) 444 B) $888\sqrt{2}$ C) 1776 D) $444\sqrt{3}$ E) 1332

YANITLAR / ANSWERS

1	2	3	4	5	6	7	8	9
C	A	E	D	D	D	A	C	A
10	11	12	13	14	15	16	17	
E	C	D	A	D	E	C	C	

1. $\sqrt{7+\sqrt[3]{6+\sqrt[4]{7+\sqrt{81}}}}=?$

A) 9 B) 5 C) 4 D) 3 E) 2

2. $\sqrt[6]{(-5)^6}+\sqrt[7]{(-4)^7}+\sqrt{(2-\sqrt{7})^2}+\sqrt[3]{(2-\sqrt{7})^3}=?$

A) -9 B) $-5-2\sqrt{7}$ C) $5-2\sqrt{7}$

D) 0 E) 1

3. $\dfrac{\sqrt{150}\cdot\sqrt{72}}{\sqrt{300}}=?$

A) 10 B) 9 C) 8 D) 7 E) 6

4. $\sqrt{7-\sqrt{13}}-\sqrt{7+\sqrt{13}}=?$

A) $-\sqrt{2}$ B) $\sqrt{2}$ C) 2

D) $2+\sqrt{13}$ E) $2-\sqrt{13}$

5. $\dfrac{1}{\sqrt{6}-1}-\dfrac{1}{\sqrt{6}+1}=?$

A) $\dfrac{1}{\sqrt{6}}$ B) $\dfrac{\sqrt{6}}{2}$ C) $\dfrac{1}{6}$

D) 3 E) $\dfrac{2}{5}$

6. $\dfrac{\sqrt{5}-\dfrac{1}{\sqrt{5}}}{\sqrt{5}+\dfrac{1}{\sqrt{5}}}=?$

A) $\dfrac{2}{3}$ B) $\dfrac{1}{3}$ C) 1

D) 2 E) 5

7. $9^y.\sqrt[4]{4^x+4^x+4^x+4^x}=0,\overline{4}\Rightarrow x+y=?$

A) -1 B) 0 C) 2 D) 3 E) 5

8. $\left.\begin{array}{l}a=\sqrt{2,5}-\sqrt{0,1}\\ b\cdot\sqrt{1,6}=1\end{array}\right\}\Rightarrow a\cdot b=?$

A) 4 B) 2 C) $\dfrac{5}{4}$ D) 1 E) $\dfrac{4}{5}$

9. $\sqrt{7-\sqrt{40}}-\sqrt{7+\sqrt{40}} = ?$

A) $2\sqrt{5}$ B) $-2\sqrt{2}$ C) $2\sqrt{2}$ D) 0 E) $\sqrt{2}$

10. $\sqrt{6\sqrt{6\sqrt{6...}}} + a = \sqrt{56+\sqrt{60+\sqrt{16}}} \Rightarrow a = ?$

A) 2 B) 3 C) 4 D) 5 E) 6

11. $x > x^2 \Rightarrow \sqrt{x^2-2x+1}-\sqrt{x^2} = ?$

A) –1 B) 2x C) 1 – 2x D) 1 – x E) 1

12. $x = \sqrt{5} \Rightarrow \dfrac{\sqrt{45}+\sqrt{20}-\sqrt{80}}{5+\sqrt{5}} = ?$

A) $\dfrac{-x}{x^2+2^x}$ B) $\dfrac{x}{x+2}$ C) $\dfrac{-1}{x^2+x}$ D) $\dfrac{1}{x+1}$ E) $\dfrac{x}{x^2+1}$

13. $0 < a < 4 \Rightarrow \sqrt{\dfrac{a^2}{16}+\dfrac{16}{a^2}-2} = ?$

A) $\dfrac{16-a^2}{a}$ B) $\dfrac{16+a^2}{4a}$ C) $\dfrac{4-a}{4a}$ D) $\dfrac{16-a^2}{4a}$ E) $\dfrac{4-a}{a}$

14. $\left.\begin{aligned} x &= \sqrt{2+\sqrt{3}} \\ y &= \sqrt{2-\sqrt{2+\sqrt{3}}} \\ z &= \sqrt{2+\sqrt{2+\sqrt{3}}} \end{aligned}\right\} \Rightarrow x.y.z = ?$

A) $-\sqrt{5}$ B) –1 C) 1 D) $\sqrt{2}$ E) 2

15. $\sqrt[4]{81.1321-25.81} = ?$

A) 54 B) 36 C) 27 D) 18 E) 9

16. $\dfrac{y-\sqrt{xy}}{x-y}-\dfrac{\sqrt{x}}{\sqrt{x}+\sqrt{y}} = ?$

A) x B) y C) 1 D) 0 E) –1

17. $\dfrac{\sqrt{5}-\sqrt{2}}{\sqrt{15}+\sqrt{10}-\sqrt{6}-2} = ?$

A) 1 B) $\sqrt{5}+\sqrt{2}$ C) $\sqrt{5}-\sqrt{2}$ D) $\sqrt{3}+\sqrt{2}$ E) $\sqrt{3}-\sqrt{2}$

YANITLAR / ANSWERS

1	2	3	4	5	6	7	8	9
D	E	E	A	E	A	C	D	B
10	11	12	13	14	15	16	17	
A	C	D	D	C	D	E	E	

1. $\sqrt{a}+\sqrt{b}=13$ ve / and $a-b=39 \Rightarrow$ **a = ?**

A) $\sqrt{2}$ B) 2 C) $2\sqrt{2}$ D) 8 E) 64

2. $\sqrt{\frac{144}{25}+\frac{25}{144}-2}=?$

A) $\frac{49}{60}$ B) $-\frac{79}{60}$ C) $\frac{119}{60}$ D) $-\frac{119}{60}$ E) $\frac{119}{120}$

3. $\sqrt{502\cdot 592-622\cdot 472}=?$

A) 20 B) 30 C) 40 D) 50 E) 60

4. $\frac{2+\sqrt{2}+\sqrt{30}+\sqrt{6}}{\sqrt{2}+1}=?$

A) $\sqrt{3}+2$ B) $\sqrt{3}$ C) $\sqrt{2}+\sqrt{3}$ D) $2+\sqrt{2}$ E) $\sqrt{2}$

5. $\sqrt[6]{\sqrt{5}-\sqrt{2}}\,.\,\sqrt{\sqrt{5}+\sqrt{2}}\,.\,\sqrt[3]{\sqrt{5}-\sqrt{2}}=?$

A) $\sqrt{2}$ B) $\sqrt{3}$ C) $2\sqrt{3}$ D) 2 E) 3

6. $\sqrt{27+2\sqrt{x}}-\sqrt{27-2\sqrt{x}}=6 \Rightarrow$ **x = ?**

A) 50 B) 98 C) 162 D) 244 E) 256

7. $\frac{x-2}{\sqrt[3]{x^2}+\sqrt[3]{2x}+\sqrt[3]{4}}=2\sqrt[3]{2} \Rightarrow$ **x = ?**

A) 27 B) 36 C) 54 D) 81 E) 162

8. $\sqrt{\sqrt{2}+\sqrt{3}-2\sqrt[4]{6}}+\sqrt[4]{2}=?$

A) $2\sqrt{2}$ B) $\sqrt{3}-\sqrt{2}$ C) $\sqrt{3}$ D) $\sqrt[4]{3}$ E) $\sqrt{3}+2$

9. $0 < y < x$

$$\left.\begin{array}{l}\sqrt{x+y-2\sqrt{x.y}} = 3\\ x-y = 15\end{array}\right\} \Rightarrow \mathbf{x = ?}$$

A) 1 B) 2 C) 4 D) 16 E) 25

10. $\sqrt{3} + \sqrt{6} + \sqrt{9} + \dots + \sqrt{57} + \sqrt{60} = a$

$\Rightarrow \mathbf{1 + \sqrt{2} + \sqrt{3} + \sqrt{4} + \sqrt{5} + \dots + \sqrt{19} + \sqrt{20} = ?}$

A) $\frac{a\sqrt{3}}{3}$ B) $a\sqrt{3}$ C) a D) 3a E) $3\sqrt{3}a$

11. $m > 0$

$\sqrt{m+1+\sqrt{4m}} = 5 \Rightarrow \mathbf{m = ?}$

A) 16 B) 14 C) 12 D) 8 E) 4

12. $a = \sqrt{7} - \sqrt{5}$

$b = \sqrt{5} - \sqrt{7} + 2$

$\Rightarrow \mathbf{4b - 4 + a^2 - b^2 = ?}$

A) $2\sqrt{7}$ B) $2\sqrt{5}$ C) $-2\sqrt{5}$ D) 0 E) $-2\sqrt{7}$

13. $\sqrt{13+\sqrt[3]{n+\sqrt{13+\sqrt[3]{n+\dots}}}} = 4 \Rightarrow \mathbf{n = ?}$

A) 21 B) 23 C) 25 D) 26 E) 27

14. $\sqrt{16+x^7} - \sqrt{x^7} = 8$

$\Rightarrow \mathbf{\sqrt{16+x^7} + \sqrt{x^7} = ?}$

A) 2 B) $2\sqrt{2}$ C) $2\sqrt{3}$ D) $\sqrt{3}+1$ E) $\sqrt{2}+1$

15. $x + \sqrt{x} = 3$

$\Rightarrow \mathbf{\dfrac{3+\sqrt{x^3}}{\sqrt{x}} = ?}$

A) 1 B) 2 C) 3 D) 4 E) 5

16. $x\sqrt{x} - 15\sqrt{x} = 4 \Rightarrow \mathbf{x + 4\sqrt{x} = ?}$

A) 16 B) 12 C) $2\sqrt{3}$ D) 1 E) –1

YANITLAR / ANSWERS

1	2	3	4	5	6	7	8
E	C	E	C	B	C	C	D
9	**10**	**11**	**12**	**13**	**14**	**15**	**16**
D	A	A	D	B	A	D	E

1. $x=\sqrt{a\sqrt[3]{a}},\ y=\sqrt[3]{a\sqrt{a}}$

$\Rightarrow$ **x·y = ?**

A) $\sqrt[6]{a}$ B) $\sqrt[6]{a^5}$ C) a D) $a.\sqrt[6]{a}$ E) 21

2. $\sqrt{x\sqrt[3]{x\sqrt[3]{\frac{1}{x}}}}=\sqrt[4]{\sqrt[3]{\sqrt{x^n}}}\ \Rightarrow$ **n = ?**

A) 11 B) 12 C) 13 D) $\frac{44}{3}$ E) $\frac{52}{3}$

3. $A, x \in R$

$A=\sqrt{x-5}+\sqrt{5-x}+x^2-4x+4$

$\Rightarrow$ $\boldsymbol{\sqrt{A}=?}$

A) $\sqrt{2}$ B) $\sqrt{3}$ C) 1 D) 2 E) 3

4. $\sqrt{A+2012}+\sqrt{A-2010}=2011$

$\sqrt{A+2012}-\sqrt{A-2010}=B$

$\Rightarrow$ $\boldsymbol{B^2+3=?}$

A) 6 B) 7 C) 9 D) 12 E) 15

5. $A=\sqrt[6]{3-|x-4|}$

$A\in R \Rightarrow \sum x=?$

A) 21 B) 26 C) 27 D) 28 E) 30

6. $$\frac{1}{\sqrt{2}+\frac{1}{1+\frac{1}{\sqrt{2}}}}=?$$

A) $\sqrt{2}$ B) $2\sqrt{2}$ C) $\sqrt{2}-1$ D) $\sqrt{2}+1$ E) $\frac{1}{2}$

7. $$\frac{2+\sqrt{2}}{\sqrt{10}+\sqrt{6}+\sqrt{5}+\sqrt{3}}\cdot\sqrt{16+4\sqrt{15}}=?$$

A) 1 B) 2 C) 3 D) 4 E) 6

8. $$\frac{\sqrt{a-2b+1}-a+3b+\sqrt{2b-a-1}}{a+b+4}=?$$

A) $\frac{1}{3}$ B) $\frac{1}{2}$ C) $\frac{2}{5}$ D) $\frac{2}{3}$ E) $\frac{3}{2}$

9. $\sqrt{3+\sqrt{x-\sqrt{3+\sqrt{x-\sqrt{3+\sqrt{x...}}}}}} = 3$

$\Rightarrow$ **x = ?**

A) 9 B) 18 C) 27 D) 39 E) 100

10. $A = \sqrt{24-\sqrt{42-\sqrt{42...}}}$
$B = \sqrt{27+\sqrt{72+\sqrt{72...}}}$

$\Rightarrow$ $\mathbf{A^2 - B = ?}$

A) 9 B) 12 C) 18 D) 20 E) 24

11. $\dfrac{\mathbf{a-b}}{\mathbf{\sqrt{a}+\sqrt{b}}} + \mathbf{\sqrt{a}+\sqrt{b} = ?}$

A) $2\sqrt{a}$ B) $2\sqrt{b}$ C) 2a D) 2b E) 0

12. $a = \sqrt[3]{\frac{1}{2}},\ b = \sqrt[5]{\frac{1}{3}},\ c = \sqrt[15]{\frac{1}{6}}$

$\Rightarrow$ **? < ? < ?**

A) a < b < c B) a < c < b C) c < a < b D) c < b < a E) b < a < c

13. $A = \dfrac{5+\sqrt{5}}{\sqrt{3}-1},\ B = \dfrac{3+\sqrt{3}}{\sqrt{5}} \Rightarrow$ **B = ?**

A) $\dfrac{\sqrt{6}A}{10}$ B) $\dfrac{\sqrt{15}A}{3}$ C) $\dfrac{\sqrt{A}}{3}$ D) $\dfrac{\sqrt{15}A}{10}$ E) $\dfrac{\sqrt{15}A}{6}$

14. $\sqrt{x-13} = \sqrt{y-12} + \sqrt{12-y}$

$\Rightarrow$ $\mathbf{\sqrt{x+y} + \sqrt{x-y} = ?}$

A) 5 B) 6 C) 8 D) $5\sqrt{2}$ E) $6\sqrt{2}$

15. $\sqrt{2\sqrt[3]{2\sqrt[4]{\frac{1}{2}}}} = \sqrt[4]{\sqrt{y}} \Rightarrow$ **y = ?**

A) 16 B) 24 C) 32 D) 48 E) 64

YANITLAR / ANSWERS

1	2	3	4	5	6	7	8
D	D	E	B	D	E	B	A
9	10	11	12	13	14	15	
D	B	A	A	D	B	C	

BÖLÜM 5
CHAPTER 5

TARİHSEL NOT / HISTORICAL NOTE

Atiyah, Sir Michael Francis [1929 –]

Topoloji, Geometri, Analiz, cebirsel değişimlerin transdantal teorisi, diferansiyel, işlemler ve kuantum alan teorisine önemli katkıları olan ingiliz matematikçidir. 1966 da Field Medal, 2004 da Abel ödülü ile ödüllendirilmiştir.

British mathematician who made important contributions in topology, geometry, analysis, the transcenolantal theory of algebraic varieties, differential, operations and quantum field theory. he was awarded the Fields Madel in 1966 and the Abel Prize in 2004.

ÖZDEŞLİKLER ve ÇARPANLARA AYIRMA
IDENTITIES and FACTORING

Bu bölüm, 150 test sorusu içermektedir.

This chapter includes 150 test questions.

BÖLÜM / CHAPTER 5

ÖZDEŞLİKLER ve ÇARPANLARA AYIRMA / IDENTITIES and FACTORING

- Özdeşlikler ve Çarpanlara Ayırma / Identities and Factoring .... 109 – 132

BÖLÜM

05

CHAPTER

ÖZDEŞLİKLER ve ÇARPANLARA AYIRMA
IDENTITIES and FACTORING

ÖZDEŞLİKLER / IDENTITIES

Tanım: Bilinmeyenlerinin her gerçek değeri için doğru olan eşitliklere ***özdeşlik*** denir.

Definition: The equations that hold true for all of their unknowns' real values are called ***identities***.

Örneğin / For example:

$(x + 3)^2 = x^2 + 6x + 9$ eşitliğinde / in equality,

x = 1 için / for: $(1 + 3)^2 = 1^2 + 6 \cdot 1 + 9$

$4^2 = 1 + 6 + 9$

$16 = 16$

x = –2 için / for: $(-2 + 3)^2 = (-2)^2 + 6\cdot(-2) + 9$

$(1)^2 = 4 - 12 + 9$

$1 = 1$

Burada x'e verilen her değer için eşitlik sağlanır. O halde, bu eşitlik bir özdeşliktir.

For all values assigned to x, the equality holds true. Therefore, this equality is an identity.

TAMKARE İFADELER / PREFECT SQUARE EXPRESSIONS

▼ $\mathbf{(a + b)^2 = a^2 + 2ab + b^2}$

▼ $\mathbf{(a - b)^2 = a^2 - 2ab + b^2}$

Özellik / Property

★ $(a + b)^2 = a^2 + 2ab + b^2 \Rightarrow \mathbf{a^2 + b^2 = (a + b)^2 - 2ab}$

★ $(a - b)^2 = a^2 - 2ab + b^2 \Rightarrow \mathbf{a^2 + b^2 = (a - b)^2 + 2ab}$

Özellik / Property

★ $\left(ax + \frac{b}{x}\right)^2 = a^2x^2 + \frac{b^2}{x^2} + 2ab$

★ $\left(ax - \frac{b}{x}\right)^2 = a^2x^2 + \frac{b^2}{x^2} - 2ab$

Özellik / Property

★ $(a + b)^2 = (a - b)^2 + 4ab$

★ $(a - b)^2 = (a + b)^2 - 4ab$

Özellik / Property

★ $(a + b + c)^2 = a^2 + b^2 + c^2 + 2\cdot(a\cdot b + a\cdot c + b\cdot c)$

★ $(a + b - c)^2 = a^2 + b^2 + c^2 + 2\cdot(a\cdot b - a\cdot c - b\cdot c)$

Özellik / Property

★ $a^2 + b^2 = 0 \Rightarrow \mathbf{a = 0,\ b = 0}$

İKİ KARE FARKI İFADELER / DIFFERENCE of TWO SQUARES EXPRESSIONS

▼ $\mathbf{x^2 - y^2 = (x + y)\cdot(x - y)}$

Önemli / Important

- $x, y \in Z^+$, a bir asal sayı olmak üzere,
  Provided that a is a prime number,
  $x^2 - y^2 = a \Rightarrow \mathbf{x + y = a,\ x - y = 1}$

İKİ KÜP TOPLAMI ve FARKI / SUM and DIFFERENCE of TWO CUBES

▼ $\mathbf{a^3 - b^3 = (a - b)\cdot(a^2 + ab + b^2)}$

▼ $\mathbf{a^3 + b^3 = (a + b)\cdot(a^2 - ab + b^2)}$

TAM KÜP İFADELER / PERFECT CUBE EXPRESSIONS

- $(a + b)^3 = a^3 + 3a^2 b + 3ab^2 + b^3$
- $(a - b)^3 = a^3 - 3a^2 b + 3ab^2 - b^3$

Özellik / Property

★ $a^3 + b^3 = (a + b)^3 - 3 \cdot a \cdot b(a + b)$

★ $a^3 - b^3 = (a - b)^3 + 3 \cdot a \cdot b(a - b)$

ÇARPANLARA AYIRMA / FACTORING

- Bir cebirsel ifadeyi, iki veya daha fazla ifadenin çarpımı biçiminde yazmaya ***çarpanlara ayırma*** denir.
- The operation of writing an algebraic expression as multiplication of two or more than two expressions is called ***factorization(or factoring).***

ORTAK ÇARPAN PARANTEZİNE ALMA / COMMON FACTOR BRACKETING

- Cebirsel bir ifadenin tüm terimlerinde ortak ifade varsa, bu ortak terim ifadenin çarpanıdır.
- If all the terms of an algebraic expression contain a common factor, then that term can be factored out of the expression.

I. $Ax + Ay = A \cdot (x + y)$

GRUPLANDIRMA YOLUYLA ÇARPANLARA AYIRMA / FACTORIZATION by GROUPING

- Her terimde ortak olan bir ifade yoksa, terimler ikişer ikişer veya üçer üçer gruplandırılarak ortak çarpan parentezine alınır ve çarpanlarına ayrılır.
- If there is no single common expressions then expressions are grouped to groups of two's or three's and common expression factorization is done.

I. $Ax + By + Bx + Ay = A(x + y) + B(x + y)$

$= (x + y) \cdot (A + B)$

$Ax^2 + Bx + C$ ŞEKLİNDEKİ İFADELERİN ÇARPANLARA AYRILMASI FACTORIZATION of THE EXPRESSIONS in $Ax^2 + Bx + C$ FORM

▼ $A = 1, \quad m \cdot n = C, \quad m + n = B$

$x^2 + Bx + C = (x + m)\cdot(x + n)$ dir.

$$\begin{array}{ccc} \downarrow & \downarrow & \downarrow \\ x & m+n & m \\ x & & n \end{array}$$

▼ $A = 1, \quad m \cdot n = C, \quad p \cdot q = A, \quad m \cdot q + n \cdot p = B$

$Ax^2 + Bx + C = (px + m)\cdot(qx + n)$

$$\begin{array}{cc} \downarrow & \downarrow \\ px & m \\ qx & n \end{array}$$

TERİM EKLEYİP – ÇIKARARAK ÇARPANLARA AYIRMA / FACTORIZATION by TERM ADDING – SUBTRACTING

▼ Verilen bir ifade, terim ekleyip çıkartılarak, bilinen bir özdeşliğin açılımına dönüştürülür.

▼ A given expression can convert to expantion of a known identity by adding and subtracting a certain term.

RASYONEL İFADELERDE SADELEŞTİRME / SIMPLIFICATION in RATIONAL EXPRESSIONS

▼ Pay ve paydadaki harfli ifadeler, çarpım durumunda değilse önce çarpanlarına ayrılır, sonra sadeleştirme yapılır.

▼ If the letters in denominator and numerator are not in the multiplication form, first factorization and then simplification are done.

1. $\left.\begin{array}{l} a+b=8 \\ a\cdot b=12 \end{array}\right\} \Rightarrow \mathbf{a^2+b^2=?}$

A) 30 B) 36 C) 40 D) 49 E) 64

2. $\left.\begin{array}{l} a-b=7 \\ a\cdot b=5 \end{array}\right\} \Rightarrow \mathbf{a^2+b^2=?}$

A) 39 B) 42 C) 47 D) 49 E) 59

3. $\left.\begin{array}{l} a^2+b^2=48 \\ a+b=8 \end{array}\right\} \Rightarrow \mathbf{a\cdot b=?}$

A) 4 B) 7 C) 8 D) 9 E) 12

4. $\left.\begin{array}{l} x-3y=4 \\ x\cdot y=6 \end{array}\right\} \Rightarrow \mathbf{x^2+9y^2=?}$

A) 36 B) 42 C) 49 D) 52 E) 64

5. $\left.\begin{array}{l} x=4,168 \\ y=2,832 \end{array}\right\} \Rightarrow \mathbf{(x-y)^2+4xy=?}$

A) 36 B) 40 C) 49 D) 64 E) 72

6. $\left.\begin{array}{l} x^2+xy=12 \\ y^2+xy=24 \end{array}\right\} \Rightarrow \mathbf{|x+y|=?}$

A) 3 B) 4 C) 5 D) 6 E) 8

7. $\left.\begin{array}{l} a+b=6 \\ a\cdot b=4 \end{array}\right\} \Rightarrow \mathbf{a^3+b^3=?}$

A) 144 B) 180 C) 184 D) 186 E) 196

8. $\left.\begin{array}{l} a^3-b^3=100 \\ a-b=4 \end{array}\right\} \Rightarrow \mathbf{a\cdot b=?}$

A) 3 B) 4 C) 6 D) 8 E) 12

9. $x-\frac{1}{x}=4 \Rightarrow \mathbf{x^2+\frac{1}{x^2}=?}$

A) 12 B) 14 C) 16 D) 18 E) 20

10. $2x+\frac{3}{x}=5 \Rightarrow \mathbf{4x^2+\frac{9}{x^2}=?}$

A) 27 B) 25 C) 23 D) 21 E) 13

11. $x-\frac{1}{x}=6 \Rightarrow \mathbf{x^3-\frac{1}{x^3}=?}$

A) 234 B) 216 C) 200 D) 198 E) 196

12. $a+\frac{1}{a}=5 \Rightarrow \mathbf{\frac{a^4+1}{a^2}=?}$

A) 21 B) 22 C) 23 D) 24 E) 25

13. $a^2+3a-2=0 \Rightarrow \mathbf{a^2+\frac{4}{a^2}=?}$

A) 11 B) 13 C) 15 D) 17 E) 19

14. $a+\frac{1}{a}=4 \Rightarrow \mathbf{a^4+\frac{1}{a^4}=?}$

A) 142 B) 164 C) 194 D) 196 E) 254

15. $\left.\begin{array}{l} x+y=7 \\ x\cdot y=8 \end{array}\right\} \Rightarrow \mathbf{|x-y|=?}$

A) $\sqrt{15}$ B) 4 C) $\sqrt{17}$ D) $3\sqrt{2}$ E) $\sqrt{19}$

16. $\left.\begin{array}{l} x=4,372 \\ y=3,628 \end{array}\right\} \Rightarrow \mathbf{(x-y)^2+4xy=?}$

A) 7 B) 8 C) 49 D) 64 E) 100

17. $\left.\begin{array}{l} x^3+3xy^2=106 \\ 3x^2y+y^3=42 \end{array}\right\} \Rightarrow \mathbf{x-y=?}$

A) 4 B) 5 C) 6 D) 7 E) 8

YANITLAR / ANSWERS

1	2	3	4	5	6	7	8	9
C	E	C	D	C	D	A	A	D
10	11	12	13	14	15	16	17	
E	A	C	B	C	C	D	A	

1. $a \cdot b = 2$

$\frac{1}{a^2} + \frac{1}{b^2} = 6 \Rightarrow \mathbf{a + b = ?}$

A) $\sqrt{5}$ B) $2\sqrt{5}$ C) $3\sqrt{5}$ D) $\sqrt{7}$ E) $2\sqrt{7}$

2. $\sqrt{x} - \sqrt{y} = 6$

$\Rightarrow \dfrac{xy\sqrt{y} - y\sqrt{x^3}}{y} = ?$

A) $-6x$ B) $-4x$ C) $4x$ D) $6x$ E) $8x$

3. $x + y = 4$

$z - x = 3$

$\Rightarrow \mathbf{x^2 + xy - xz - yz = ?}$

A) -16 B) -12 C) -6 D) 6 E) 12

4. $a \neq b,$

$a^2 - 2ab + b^2 + 3a - 3b = 0$

$\Rightarrow \mathbf{a - b = ?}$

A) -5 B) -4 C) -3 D) 3 E) 4

5. $x^2 = x + 1$

$\Rightarrow \dfrac{x^3 + 3}{x^2 + 1} = ?$

A) $\frac{1}{2}$ B) 1 C) $\frac{3}{2}$ D) 2 E) $\frac{5}{2}$

6. $a > 1$

$\Rightarrow \dfrac{\sqrt{a^3 + a^2 + a}}{\sqrt{a - 1}} \cdot \dfrac{a - 1}{\sqrt{a^3 - 1}} = ?$

A) $\frac{\sqrt{a}}{a}$ B) $\sqrt{a}$ C) a D) $\sqrt{a - 1}$ E) $\sqrt{a^2 + 1}$

7. $a = 8^{\frac{1}{3}} + 8^{-\frac{1}{3}}$

$b = 8^{\frac{1}{3}} - 8^{-\frac{1}{3}}$

$\Rightarrow \mathbf{a^3 - 3a^2b + 3ab^2 - b^3 = ?}$

A) 1 B) 2 C) 4 D) 8 E) 16

8. $\dfrac{a^2 + b^2}{b - a} - \dfrac{a}{1 - \frac{a}{b}} - \dfrac{b}{\frac{b}{a} - 1} = ?$

A) $-ab$ B) ab C) $a - b$ D) $b - a$ E) $a + b$

9. $a \neq b$

$$\frac{1-a+b}{a-b}+\frac{a-b+1}{b-a}=?$$

A) -2 B) -1 C) 2 D) $\frac{1}{a-b}$ E) $\frac{2}{a-b}$

10. $$\frac{\sqrt{x}}{\sqrt{x}+\sqrt{y}}-\frac{y-\sqrt{xy}}{x-y}=?$$

A) x B) y C) 0 D) 1 E) 2

11. $$\frac{x(y-x)^4-(x-y)^5}{(xy-y^2)^4}=?$$

A) $\frac{1}{y}$ B) $\frac{1}{y^2}$ C) $\frac{1}{y^3}$ D) $\frac{x}{y^2}$ E) $\frac{x}{y^3}$

12. $x \in R^+$,

$$\frac{x-4}{\sqrt{x}-2}-\frac{x-1}{\sqrt{x}+1}=?$$

A) 2 B) 3 C) $\sqrt{x}$ D) $x\sqrt{x}$ E) 4

13. $$\frac{a^3+3a^2}{a^{n-3}}:\frac{a^2+8a+15}{a^{n+1}+5a^n}=?$$

A) a^n B) a^{n+1} C) a^5 D) a^4 E) a^3

14. $$\left(a-\frac{2b^2}{a+b}\right)\cdot\left(1-\frac{b}{a+2b}\right)=?$$

A) 1 B) 2 C) 4 D) $a-b$ E) $a+b$

15. $5a+1=\sqrt{a} \Rightarrow \frac{25a^2+1}{a}=?$

A) -25 B) -12 C) -9 D) 9 E) 25

16. $$\frac{x^2+x+1+x^{-1}}{1-x^4}=?$$

A) $\frac{1}{x-1}$ B) $\frac{1}{1-x}$ C) $\frac{-1}{x(x+1)}$ D) $\frac{-1}{x(x-1)}$ E) $\frac{-2}{x-1}$

YANITLAR / ANSWERS

1	2	3	4	5	6	7	8
E	A	B	C	D	B	A	D
9	**10**	**11**	**12**	**13**	**14**	**15**	**16**
A	D	C	B	C	D	C	D

1. $(a^{-1} + b^{-1}) : (a + b) = ?$

A) $a \cdot b$
B) $\frac{1}{a \cdot b}$
C) $a + b$
D) $(a + b)^2$
E) 1

2. $\left(\frac{3}{x-y} - \frac{2}{y-x}\right) \cdot (4x - 4y) = ?$

A) 4
B) 8
C) 12
D) 16
E) 20

3. $x = 10{,}699$

$y = 1{,}301$

$\Rightarrow \frac{x^3 + y^3}{x^2 - xy + y^2} = ?$

A) 9
B) 10
C) 11
D) 12
E) 15

4. $\frac{x^2 - 16}{x^2 - 8x + 16} : \frac{2x + 8}{x - 4} = ?$

A) $\frac{1}{2}$
B) $\frac{2}{5}$
C) $\frac{4}{3}$
D) 3
E) 7

5. $\frac{x^2 - xy}{y^2 - xy} = ?$

A) $\frac{x}{y}$
B) $\frac{y}{x}$
C) $x \cdot y$
D) $\frac{x+y}{x-y}$
E) $-\frac{x}{y}$

6. $\frac{x^3 - 1}{x^2 - 1} : \frac{x^2 + x + 1}{x + 1} = ?$

A) $\frac{1}{x-1}$
B) $x - 1$
C) $\frac{x-1}{x+1}$
D) $x + 1$
E) 1

7. $\frac{a^2 - 64}{a^2 - 6a - 16} : \frac{a + 8}{a^2 + 10a + 16} = 15 \Rightarrow a = ?$

A) 5
B) 6
C) 7
D) 8
E) 16

8. $\frac{x^3 - 5x^2 - 6x}{x^2 - 4x - 12} \cdot \frac{x^2 - 4}{x^2 + x} = ?$

A) 1
B) $\frac{1}{x}$
C) $x - 2$
D) $x + 2$
E) $x + 1$

9. $\dfrac{a^3+b^3}{a^2-ab+b^2} : \dfrac{(a+b)}{4} = ?$

A) 0　B) 1　C) 2　D) 3　E) 4

10. $\dfrac{x^3+y^3}{(x-y)^2+xy} = ?$

A) $x-y$　B) $x+y$　C) $y+2x$　D) x^2-y^2　E) $2x$

11. $a^2-b^2=(a+b)(a-b) \Rightarrow$ **$2004 \cdot 1998 = ?$**

A) 2001^2-4^2　B) 2001^2-3^2　C) 2001^2-6^2　D) 2002^2-6^2　E) 1002^2-4^2

12. $\dfrac{x^2-yx-x+y}{x-1} = ?$

A) $y-x$　B) $y+1$　C) $x-y$　D) $x+2$　E) $x-1$

13. $x > 0$,

$4x^2+\dfrac{1}{x^2}=12 \Rightarrow 2x+\dfrac{1}{x} = ?$

A) 3　B) 4　C) 12　D) 16　E) 32

14. $\left(\dfrac{4-\dfrac{1}{x^2}}{2-\dfrac{1}{x}}\right)\cdot\left(\dfrac{x^2}{2x^2+x}\right) = ?$

A) $\dfrac{1}{x}$　B) x　C) x^2　D) $1-x$　E) 1

15. $\dfrac{1}{a-1}+\dfrac{2a-a^2}{1-a}=12 \Rightarrow$ **a = ?**

A) 10　B) 11　C) 12　D) 13　E) 14

16. $(x+2y)^2+(y-2)^2=0 \Rightarrow$ **$x \cdot y = ?$**

A) 10　B) 8　C) –6　D) –7　E) –8

17. $a, b \in Z^+$

$a^2-b^2=29 \Rightarrow \dfrac{a}{b} = ?$

A) $\dfrac{12}{17}$　B) $\dfrac{17}{13}$　C) $\dfrac{15}{14}$　D) $\dfrac{11}{9}$　E) $\dfrac{18}{17}$

YANITLAR / ANSWERS

1	2	3	4	5	6	7	8	9
B	E	D	A	E	E	C	C	E
10	11	12	13	14	15	16	17	
B	B	C	C	E	D	E	C	

1. $(a - b + c)^2 - (a + b - c)^2 = ?$

A) $2b - c$ B) $4a(b - c)$ C) $4a(c - b)$ D) $2b - a$ E) $c - b$

2. $\frac{a^2 + 3a + x}{(a-1)(a+1)} = \frac{a+y}{a+1} \Rightarrow x + y = ?$

A) 0 B) –1 C) –2 D) –3 E) –4

3. $\frac{8a^2 - 2b^2}{8a^2 - 8ab + 2b^2} = ?$

A) $\frac{b+2a}{-b}$ B) $\frac{a+b}{b-2a}$ C) $\frac{b+2a}{-b+2a}$ D) $\frac{b \cdot a}{b+a}$ E) $\frac{2a}{b}$

4. $x \neq 1 \Rightarrow \frac{x^2}{x-1} + \frac{3-2x}{1-x} = ?$

A) $x + 1$ B) $x - 2$ C) $x + 3$ D) $x + 2$ E) $x - 3$

5. $\left(1 + x + \frac{5 + x^2}{1 - x}\right) \cdot \left[(1 - x^2):(1 + x)\right] = ?$

A) 6 B) 8 C) $x + 1$ D) $x - 2$ E) x

6. $\left(\frac{1}{x-3} - \frac{x-2}{x-3}\right) : \frac{x^2 - 9}{x^3 - 2x^2 - 9x + 18} = ?$

A) 0 B) 1 C) $3 - x$ D) $2 - x$ D) x^2

7. $\frac{2^{2x} - 2^{-2x}}{2^x - 2^{-x}} = ?$

A) 2^{-x} B) $1 + 2^x$ C) $2^x - 2^{-x}$ D) $2^x + 2^{-x}$ E) $2^x + 2^{2x}$

8. $\frac{x^2 + 4}{x^2 - 3x - 4} = \frac{Ax}{x+1} + \frac{B}{x-4} \Rightarrow B + A = ?$

A) 2 B) 3 C) 4 D) 5 E) 6

9. $x^6 - x^4 - 2x^3 - (x^4 \cdot x^2 - x^3) = ?$

A) x^6 B) $-x^4 - x^2$ C) $-x^5$ D) $x^3 - x^4$ E) $-x^3 \cdot (x + 1)$

10. $\left.\begin{matrix} x^2 + xy = 4 \\ y^2 + xy = 12 \end{matrix}\right\} \Rightarrow \frac{x-y}{x+y} = ?$

A) -1 B) $-\frac{1}{2}$ C) $-\frac{2}{3}$ D $-\frac{3}{4}$ E) $-\frac{4}{5}$

11. $\frac{x}{y} - \frac{y}{x} = \sqrt{2} \Rightarrow \frac{x^4 + y^4}{x^2y^2} = ?$

A 2 B) $2\sqrt{2}$ C 4 D $4\sqrt{2}$ E 16

12. $x^2 + 2x + 4 = 0 \Rightarrow 3x + \frac{12}{x} = ?$

A) 0 B) –2 C) –4 D) –6 E) – 8

13. $\left(\frac{2}{a} - \frac{a}{2}\right)^2 - \left(\frac{a}{2} - \frac{2}{a}\right)^2 = ?$

A) 0 B) 1 C) 4a D) $\frac{8}{a}$ E) 32

14. $a^2 - 5a - 1 = 0 \Rightarrow a^2 + \frac{1}{a^2} = ?$

A) 13 B) 18 C) 25 D) 27 E) 36

15. $\left(\frac{1}{1 - \frac{1}{x + \frac{1}{x}}} : \frac{x^3 + x}{x^3 + 1}\right) - \frac{1}{x} = ?$

A) 1 B) x C) $x - 1$ D) $x + 2$ E) 2

16. $\left(\frac{2a}{1-a^2}\right)^2 - \left(\frac{1+a^2}{1-a^2}\right)^2 = ?$

A) a–1 B) 1 C) a + 1 D) 1 – a E) –1

17. $a = 2^x + 2^y$, $b = 2^x - 2^y$, $a^2 - b^2 = 64 \Rightarrow x + y = ?$

A) 1 B) 2 C) 3 D) 4 E) 5

YANITLAR / ANSWERS

1	2	3	4	5	6	7	8	9
C	A	C	C	A	D	D	D	E
10	11	12	13	14	15	16	17	
B	C	D	A	D	A	E	D	

1. $x^2 + y^2 + 4x - 6y + 13 = 0$

$\Rightarrow \mathbf{x \cdot y = ?}$

A) –6 B) –4 C) 4 D) 6 E) 9

2. $x - y = 3$

$y - z = 2$

$\Rightarrow \mathbf{xy + yz + z - y^2 - xz - x = ?}$

A) 0 B) 1 C) 4 D) 6 E) 9

3. $2x - 5 - \dfrac{\mathbf{3x^2 - 16x + 5}}{\mathbf{3x - 1}} = ?$

A) 1 B) 2 C) x D) 2x E) x – 1

4. $\left.\begin{aligned} m &= 1 + \frac{1}{x} \\ n &= 1 - \frac{1}{x} \end{aligned}\right\} \Rightarrow \dfrac{\mathbf{m^2x - n^2x}}{\mathbf{m + n}} = ?$

A) –2 B) –1 C) 1 D) 2 E) 4

5. $x^3 + x^2y - xy + y + 1 = 0 \Rightarrow \mathbf{x = ?}$

A) –y B) y C) y + 1

D) –y – 1 E) 1 – y

6. $x + \sqrt{x} = 1 \Rightarrow \mathbf{x^2 - 3x + 5 = ?}$

A) 2 B) 3 C) 4 D) 5 E) 6

7. $\left(2^{\frac{1}{2}} + 1\right) \cdot \left(2^{\frac{1}{4}} + 1\right) \cdot \left(2^{\frac{1}{8}} + 1\right) \cdot \left(2^{\frac{1}{16}} + 1\right) = A$

$\sqrt[8]{2} = x \Rightarrow \mathbf{A = ?}$

A) $\dfrac{1}{\sqrt{x} - 1}$ B) $\dfrac{1}{x - 1}$ C) x – 1

D) $x^2 - 1$ E) x

8. $x^2 + x + 1 = 0$

$A = x^{2011} + x^{2012} \Rightarrow \mathbf{A = ?}$

A) $-x^2$ B) –x C) –1 D) x E) x^2

9. $\mathbf{\frac{-3x^2+13x-14}{x^4-16}\cdot\frac{-3x^3+4x^2-12x+16}{-27x^2+99x-84}=?}$

A) $\frac{3x-5}{x-2}$ B) $\frac{-1}{3(x+2)}$ C) $\frac{1}{x+2}$

D) $\frac{2}{x+6}$ E $\frac{x}{3(x+2)}$

10. $x^2+5x+2=0$

$\Rightarrow \mathbf{x^2+\frac{4}{x^2}=?}$

A) 14 B) 18 C) 21 D) 24 E) 28

11. $x+3=\sqrt{x} \Rightarrow \mathbf{x+\frac{3}{\sqrt{x}}=?}$

A) −6 B) −4 C) −3 D) −2 E) −1

12. $\mathbf{\frac{\sqrt{ab}+\sqrt{b}}{1+\sqrt{a}+\sqrt{ab}+\sqrt{b}}\cdot\frac{\sqrt{b}+1}{\sqrt{b}}=?}$

A) $-\sqrt{a}$ B) $-\sqrt{b}$ C) 1 D) $\sqrt{a}$ E) $\sqrt{b}$

13. $x+y-z=12$

$xy-xz-yz=36$

$\Rightarrow \mathbf{x^2+y^2+z^2=?}$

A) 13 B) 26 C) 39 D) 48 E) 72

14. $x^2+x+1=0$

$\Rightarrow \mathbf{x^5+x^3-x^2+3=?}$

A) 1 B) 2 C) 3 D) 4 E) 5

15. $x>0$

$5\sqrt{x}=x+1$

$\Rightarrow \mathbf{\frac{x^2+1}{x}=?}$

A) 17 B) 20 C) 23 D) 25 E) 28

16. $x+\frac{1}{x}=a$

$x^3+x^2+\frac{1}{x^2}+\frac{1}{x^3}=-2 \Rightarrow \mathbf{a^2+a=?}$

A) 2 B) 3 C) 4 D) 6 E) 8

YANITLAR / ANSWERS

1	2	3	4	5	6	7	8
A	B	C	D	D	C	A	C
9	10	11	12	13	14	15	16
B	C	D	C	E	D	C	B

1. $\left(\frac{a}{b}+1\right)\cdot\left(\frac{b}{a}-1\right)=-3 \Rightarrow \frac{a^2-b^2}{a\cdot b}=?$

A) $\frac{2}{3}$ B) $\frac{1}{2}$ C $\frac{3}{2}$ D) 2 E) 3

2. $\dfrac{a+\frac{1}{a}+2}{1+\frac{1}{a}}=?$

A) a B) $\frac{1}{a}$ C) 2a D) a + 1 E) a − 1

3. $\left(1+\frac{2}{x}+\frac{1}{x^2}\right):\frac{x^2-1}{x^3-x^2}=?$

A) x − 1 B) x + 1 C) x D) $\frac{1}{x-1}$ E) $\frac{1}{x+1}$

4. $\frac{(a^2+a\cdot b)^2}{a^2-b^2}\cdot\frac{a^3-b^3}{a^3b+a^2b^2+ab^3}=?$

A) $\frac{b(a+b)}{a}$ B) $\frac{b^2}{a-b}$ C) $\frac{a(a+b)}{b}$ D) $\frac{a}{a+b}$ E) $\frac{a+b}{a-b}$

5. $a+1-\frac{a^2+1}{a+1}=?$

A) $\frac{2}{a+1}$ B) $\frac{a-1}{a+1}$ C) $\frac{2a}{a+1}$ D) $\frac{2a^2}{a+1}$ E) $\frac{2a}{a^2-1}$

6. $\left(\frac{x}{x+y}-\frac{x}{x-y}\right)\cdot\left(\frac{1}{x}-\frac{1}{y}\right)=?$

A) $-\frac{2}{x+y}$ B) $\frac{2}{x+y}$ C) $\frac{x-y}{x+y}$ D) $\frac{xy}{x+y}$ E) $-\frac{xy}{x+y}$

7. $x+\frac{1}{x}=4 \Rightarrow \frac{x^4+1}{x^2}=?$

A) 14 B) 16 C) 18 D) 20 E) 24

8. $\left(\frac{x+y}{x-y}-1\right):\left(\frac{y}{x+y}+\frac{y}{x-y}\right)=?$

A) $\frac{x-y}{y}$ B) $\frac{x+y}{y}$ C) $\frac{x+y}{x}$ D) $\frac{x-y}{x}$ E) $\frac{x}{y}$

9. $\dfrac{a^3}{a+b}+\dfrac{b^3}{b-a}-\dfrac{a^2+b^2}{\frac{b}{a}-\frac{a}{b}}=?$

A) a B) b C) a + b
D) a^2-b^2 E) a^2+b^2

10. $a^2-7a=9 \Rightarrow \dfrac{a^4+81}{a^2}=?$

A) 43 B) 59 C) 61 D) 67 E) 73

11. $x+y+z=-6,$
$xy+xz+yz=-64$
$\Rightarrow x^2+y^2+z^2=?$

A) 128 B) 144 C) 160 D) 164 E) 196

12. $\left(\dfrac{9}{a^2}-4b^2\right):\left(2b-\dfrac{3}{a}\right)+\left(\dfrac{3}{a}+3b\right)=?$

A) b B) 5b C) –b D) 3b E) 2b

13. $\left(\dfrac{1}{x}-\dfrac{1}{y}\right)\cdot\dfrac{x^2y+xy^2}{x^2-3xy+2y^2}=?$

A $\dfrac{x}{y}$ B) $\dfrac{2y-x}{x+y}$ C) $\dfrac{x+y}{2y-x}$
D) y + 2x E) x + y

14. $x^2-10x+y^2+6y+34=0 \Rightarrow x-y=?$

A) –3 B) –5 C) –8 D) 2 E) 8

15. $\dfrac{x^2-y^2+2x+1}{x-y+1}=?$

A) x + y B) x – y C) x – y – 1
D) x + y + 1 E) x – y + 1

16. $x^2-4x+6=0 \Rightarrow x^2+\dfrac{36}{x^2}=?$

A) 28 B) 20 C) 12 D) 4 E) 2

17. $\dfrac{a^3+b^3+a\cdot b(a+b)}{a^4-b^4}=?$

A) a – b B) a + b C) ab
D) $\dfrac{1}{a-b}$ E) $\dfrac{a+b}{a-b}$

YANITLAR / ANSWERS

1	2	3	4	5	6	7	8	9
E	D	B	C	C	B	A	C	E
10	11	12	13	14	15	16	17	
D	D	A	C	E	D	D	D	

1. $\dfrac{x^2-x-20}{x^2+2x-15}\cdot\dfrac{2x^2-50}{6x}\cdot\dfrac{3x}{x^2-10x+25}=?$

A) $x+4$
B) 1
C) $\dfrac{x+4}{x-3}$
D) $x-5$
E) $x+3$

2. $\left(x+\dfrac{1}{x}\right)^2=3 \Rightarrow \dfrac{x^3+\dfrac{1}{x^3}}{x+\dfrac{1}{x}}=?$

A) -2
B) -1
C) 0
D) 2
E) 3

3. $\left(\dfrac{a^3-2a^2+a-2}{a^3-a^2+a-1}\right)\cdot\left(\dfrac{a-2}{a-1}\right)^{-1}=?$

A) $a-1$
B) $\dfrac{a-1}{a-2}$
C) $\dfrac{a-2}{a-1}$
D) 1
E) -1

4. $\dfrac{8x^2}{2x-1}+\dfrac{1}{\dfrac{1}{2}-x}=?$

A) $4x+1$
B) $4x+2$
C) $x+1$
D) $x-1$
E) 2

5. $\dfrac{x^2+y^2-4-2xy}{x-y-2}=?$

A) $x-y$
B) $x-y-1$
C) $x-y+1$
D) $x-y+2$
E) $x+y+2$

6. $\left(\dfrac{xy+x^2}{x^2+y^2}\right)\cdot\left(\dfrac{x}{x+y}+\dfrac{y}{x-y}\right)=?$

A) $\dfrac{x}{y}$
B) $\dfrac{y}{x-y}$
C) $\dfrac{x}{x-y}$
D) $\dfrac{y}{x+y}$
E) $\dfrac{x}{x+y}$

7. $a \neq b \neq 0$

$\Rightarrow \dfrac{a^3}{a^2-ab}+\dfrac{b^3}{b^2-ab}=?$

A) a^2+ab+b^2
B) a^2-ab+b^2
C) $a+b$
D) $a-b$
E) 1

8. $\dfrac{\dfrac{1}{a}-1}{1+\dfrac{1}{a}}:\dfrac{a^2-2a+1}{a^2-a}=?$

A) $\dfrac{a}{a-1}$
B) $-\dfrac{a}{a+1}$
C) $\dfrac{a+1}{a}$
D) $a-1$
E) 1

9. $\dfrac{\left(\sqrt{x}-\dfrac{y}{\sqrt{x}}\right)\cdot\left(\sqrt{x}+\dfrac{y}{\sqrt{x}}\right)}{\dfrac{1}{x}-\dfrac{1}{y}} = ?$

A) $-xy$ B) x^2y C) $xy - y^2$ D) $y^2 - x$ E) $-xy - y^2$

10. $\dfrac{x^{n+3} - x^{m+3}}{x^m - x^n} : (x^{n+2} : x^n) = ?$

A) -1 B) $-x$ C) 1 D) x E) 2

11. $\dfrac{x}{y} = 99 \Rightarrow \dfrac{x^3 + y^3 + 3xy(x+y)}{x^3 + 2x^2y + xy^2} = ?$

A) 10^2 B) 10^3 C) 10^4 D) 10^6 E) 10^9

12. $\dfrac{x^2 + x - y^2 + y}{(x-y)^2 - 1} : \dfrac{x}{x-y-1} = ?$

A) $\dfrac{x+y}{x}$ B) $\dfrac{1}{x}$ C) $\dfrac{x-y}{x}$ D) $\dfrac{x+y-1}{x}$ E) 1

13. $\dfrac{\dfrac{1}{b+1}}{\dfrac{1}{a+1} - \dfrac{1}{b+1}} + \dfrac{a}{a-b} = ?$

A) $\dfrac{1}{a+b}$ B) $\dfrac{a}{a+1}$ C) $\dfrac{a-b}{a+b}$ D) $\dfrac{1}{b-a}$ E) $\dfrac{b}{a+1}$

14. $80^2 + 8^2 = ?$

A) $72^2 + 5 \cdot 8^2$ B) $72^2 + 6 \cdot 9^2$ C) $72^2 + 5 \cdot 16^2$ D) $72^2 + 6 \cdot 25^2$ E) $72^2 + 3 \cdot 36^2$

15. $\left.\begin{array}{r} x + y = 8 \\ y + m = 5 \end{array}\right\} \Rightarrow 2y^2 + 2my + 10x = ?$

A) 40 B) 60 C) 70 D) 80 E) 90

16. $\dfrac{1 - 16^x}{(1 + 4^x) \cdot (1 + 2^x)} = -7 \Rightarrow x = ?$

A) 1 B) 2 C) 3 D) 4 E) 5

17. $\dfrac{(x+y)^3 + 3xy(x-y)}{(x+y)^2 - 4xy} - \dfrac{3xy}{x-y} = ?$

A) $x+y$ B) $\dfrac{x+y^2}{x-y}$ C) $\dfrac{x+y}{x-y}$ D) $2xy$ E) $x-y$

YANITLAR / ANSWERS

1	2	3	4	5	6	7	8	9
C	C	D	B	D	C	C	B	E
10	11	12	13	14	15	16	17	
B	A	A	D	C	D	C	E	

1. $x = 9999 \Rightarrow \mathbf{x^2 + 2x = ?}$

A) $10^6 - 1$ B) $10^8 - 1$ C) $10^6 - 4$ D) $10^8 - 9$ E) $10^{10} - 16$

2. $x + y \neq 0$,
$x^3 + y^3 = xy(x + y) \Rightarrow \mathbf{x = ?}$

A) y^2 B) y C) $2y$ D) $\frac{3y}{2}$ E) $\frac{y}{2}$

3. $a^2 - b^2 + 2a + 6b - 8 = (a + b - 2)\cdot x \Rightarrow \mathbf{x = ?}$

A) $a + b + 4$ B) $a - b + 4$ C) $a - b$ D) $a + b + 2$ E) $a - b - 2$

4. $a^2 + 2a - 2 = 0 \Rightarrow \mathbf{\frac{1}{a} - \frac{1}{a^2} = ?}$

A) $-\frac{1}{2}$ B) $-\frac{1}{3}$ C) $\frac{1}{2}$ D) $\frac{1}{3}$ E) 1

5. $x^3 + y^3 = 189$,
$3xy(y + x) = 540 \Rightarrow \mathbf{x \cdot y = ?}$

A) 12 B) 15 C) 16 D) 18 E) 20

6. $a - b = b - c = 4 \Rightarrow \mathbf{a^2 + c^2 - 2a \cdot c = ?}$

A) 0 B) 4 C) 8 D) 16 E) 64

7. $\mathbf{\frac{x^6 - 1}{x^4 + x^2 + 1} : \left(\frac{1}{x} - 1\right) = ?}$

A) $-x - x^2$ B) $x + x^2$ C) x D) $x - 1$ E) $1 - x$

8. $x \in R^+$, $x + \frac{1}{x} = 4 \Rightarrow \mathbf{x^2 - \frac{1}{x^2} = ?}$

A) 6 B) $2\sqrt{3}$ C) 10 D) 12 E) $8\sqrt{3}$

9. $\dfrac{3^{12}-1}{3^8+3^4+1}=?$

A) 12 B) 27 C) 80 D) 81 E) 92

10. $\dfrac{89^2-27^2}{(87)^2+1160-(29)^2}=?$

A) $\frac{31}{39}$ B) $\frac{31}{34}$ C) $\frac{17}{27}$ D) $\frac{13}{36}$ E) $\frac{19}{24}$

11. $a^2+b^2+c^2+29=6a-4b+8c \Rightarrow \mathbf{a+b+c=?}$

A) 3 B) 4 C) 5 D) 6 E) 7

12. $a+\dfrac{1}{a+3}=2 \Rightarrow \mathbf{a^2+6a+9+\dfrac{1}{(a+3)^2}=?}$

A) 2 B) 4 C) 5 D) 23 E) 25

13. $\left.\begin{array}{l} a+b=4 \\ a^3+b^3=4ab \end{array}\right\} \Rightarrow \mathbf{a^2+b^2=?}$

A) 5 B) 6 C) 7 D) 8 E) 12

14. $x-z=z-y=3 \Rightarrow \mathbf{x^2+y^2-2z^2=?}$

A) 6 B) 9 C) 12 D) 15 E) 18

15. $(1+2^2)\cdot(1+2^4)\cdot(1+2^8)\cdot\ \dots\ \cdot(1+2^{256})=a$

$\Rightarrow \mathbf{4^{256}=?}$

A) 1 – 3a B) 3a + 1 C) 3a – 1

D) a^2 E) a^2-1

16. $\dfrac{x^4+3x^2+4}{x^2-x+2}=16 \Rightarrow \mathbf{x^2+x=?}$

A) 18 B) 16 C) 14 D) 12 E) 10

17. $\left.\begin{array}{l} x+y=2\sqrt{3}-1 \\ y-x=\sqrt{3}+1 \end{array}\right\} \Rightarrow \mathbf{x^2-y^2+2x+1=?}$

A) –6 B) $-\sqrt{6}$ C) 5

D) $4\sqrt{3}+1$ E) 12

YANITLAR / ANSWERS

1	2	3	4	5	6	7	8	9
B	B	B	A	E	E	A	E	C
10	11	12	13	14	15	16	17	
B	C	D	D	E	B	C	A	

1. $\dfrac{a}{a^2+4}-\dfrac{2(a-2)^2}{a^4-16}=?$

A) $a+2$ B) $\dfrac{1}{a+2}$ C) $a-2$ D) $\dfrac{1}{a-2}$ E) $a+3$

2. $x-y+2=0$

$\Rightarrow \dfrac{x^2-3x+3y-y^2}{x+y-3}=?$

A) -2 B) -1 C) 0 D) 1 E) 2

3. $\dfrac{x^2+ax+b}{3x^2-x-4}=\dfrac{mx+n}{3x-4}$

$\Rightarrow a-b=?$

A) -2 B) -1 C) 0 D) 1 E) 2

4. $a, b > 0,$

$6a^2-5ab-4b^2=0 \Rightarrow \dfrac{2b^2-a^2}{b^2}=?$

A) $\dfrac{2}{9}$ B) $\dfrac{1}{9}$ C) $\dfrac{3}{4}$ D) $\dfrac{2}{3}$ E) $\dfrac{1}{3}$

5. $x-2y-z=0$

$\Rightarrow \dfrac{x^2-(y-z)^2}{(x-y)^2-z^2}=?$

A) -3 B) -2 C) 0 D) 2 E) 3

6. $\dfrac{a^{n+1}-a^n}{a-a^3}\cdot\left(\dfrac{1}{a^{n-1}}+\dfrac{1}{a^{n-2}}\right)=?$

A) $-a$ B) -1 C) 1 D) a E) $a+1$

7. $A=4x^4-4y^4$

$B=3x^2+3y^2$

$C=x^4+2x^2y^2+y^4$

$\Rightarrow$ **OBEB (A, B, C) = ?**

A) $x+y$ B) x^2+y^2 C) $2(x+y)$ D) $2(x^2+y^2)$ E) x^2-y^2

8. $x=\sqrt[3]{4}-\sqrt[3]{2} \Rightarrow x^3+6x=?$

A) 6 B) 4 C) 3 D) 2 E) 1

9. $\dfrac{x\sqrt{y}-y\sqrt{x}}{\sqrt{x}-\sqrt{y}}+\dfrac{1-xy}{\sqrt{xy}-1}=?$

A) –2 B) –1 C) 1 D) 2 E) 3

10. $\dfrac{a(a-b^3)-b(b-a^3)}{a(1+b^2)+b(1+a^2)}=?$

A) a – b B) a + b C) a D) b E) a·b

11. $\dfrac{(1+3+5+\ldots+n)-\dfrac{m^2}{4}}{\dfrac{m+n+1}{4}}=?$

A) m + n – 1 B) m + n + 1 C) m – n + 1
D) n – m + 1 E) n – m

12. $\sqrt[3]{a}-\sqrt[3]{b}=4$

$a-b=58 \Rightarrow a\cdot b=?$

A) $\frac{-1}{8}$ B) $\frac{-1}{4}$ C) $\frac{-1}{2}$ D) $\frac{1}{2}$ E) $\frac{1}{8}$

13. $x+\sqrt{x}=5$

$\Rightarrow 3x+\dfrac{15}{\sqrt{x}}=?$

A) 6 B) 9 C) 12 D) 15 E) 18

14. $a-\dfrac{1}{a}=3 \Rightarrow a^3-\dfrac{1}{a^3}-30=?$

A) 3 B) 4 C) 5 D) 6 E) 9

15. $\dfrac{(x+y)^2-3(x+y)}{x^2-9+y^2+2xy}:\dfrac{x+y}{x+y+3}=?$

A) x + y B) x – y C) x + 2 D) x + 3 E) 1

16. $A=x^2-6x+15$

$B=-2x^2-4x-12$

$\Rightarrow \min(A)-\max(B)=?$

A) 12 B) 14 C) 16 D) 18 E) 20

YANITLAR / ANSWERS

1	2	3	4	5	6	7	8
B	A	D	A	E	B	B	D
9	**10**	**11**	**12**	**13**	**14**	**15**	**16**
B	A	D	A	E	D	E	C

BÖLÜM
CHAPTER
6

TARİHSEL NOT / HISTORICAL NOTE

Boole, George [1815 – 1864]

İngiliz matematikçi, matematiksel mantığın babalarından birisi olarak tanınır. İlk büyük çalışması, 1854'de yayımlanan "Düşünme kurallarının incelenmesi (Investigation of the Laws of Thought)"dir. Boolean cebirleri olarak bilinen sembolik çıkarım türünü geliştirmiştir. Onun De Morgan ve diğerleri ile olan çalışması, modern cebirin gelişmesini sağlamıştır.

British mathematician who was one of the founding fathers of mathematical logic. His major work, published in 1854, is his Investigation of the Laws of Thought. The kind of Symbolic argument that he developed led to the study of so-called Boolean algebras, which are of current significance in computing and algebra. His work, together with that of De Morgan and others, helped to pave the way for the development of modern formal algebra.

ORAN ve ORANTI
RATIOS and PROPORTIONS

Bu bölüm, 100 test sorusu içermektedir.

This chapter includes 100 test questions.

BÖLÜM / CHAPTER 6

ORAN ve ORANTI / RATIOS and PROPORTIONS

- Oran ve Orantı / Ratios and Proportions .... 133 – 150

BÖLÜM

06

CHAPTER

ORAN ve ORANTI
RATIOS and PROPORTIONS

ORAN ve ORANTI / RATIOS and PROPORTIONS

Tanım: Birimleri aynı olan iki niceliğin karşılaştırılmasına ***oran*** denir.
a'nın b'ye oranı a : b veya $\frac{a}{b}$ şeklinde yazılır.

Definition: Comparing two quantities with the same units is called ***ratio***.
The ratio of a to b is written as a : b or $\frac{a}{b}$.

$$\frac{3\,\text{kg}}{8\,\text{kg}}=\frac{3}{8},\ \frac{7\,\text{cm}}{5\,\text{cm}}=\frac{7}{5},\ \frac{3\,\text{lt}}{12\,\text{lt}}=\frac{1}{4}$$

Tanım: Oran katsayıları aynı olan iki veya daha fazla orandan oluşan eşitliklere ***orantı*** denir.

Definition: A **proportion** is an equation consisting of two or more expressions with the same ratio.

$$\frac{a}{b}=k,\ \frac{c}{d}=k \Rightarrow \frac{a}{b}=\frac{c}{d}=k$$

▼ Bu orantıda, a ve b sayıları, c ve d sayıları ile orantılıdır.
a, b, c, d sayılarına ***orantının terimleri*** denir, k ye **orantı sabiti** denir.

▼ In this proportion, the numbers a and b are proportional to the numbers c and d.
a, b, c, d numbers are called ***terms of proportion***, k is called **proportionality constant.**

1. terim (dış)
1st term(extreme)

3. terim (iç)
3rd term(mean)

$$\frac{a}{b}=\frac{c}{d}$$

2. terim (iç)
2nd term(mean)

4. terim (dış)
4th term(extreme)

dışlar
extremes

$\frac{a}{b}=\frac{c}{d}$ orantısı (proportion) $a:b=c:d$ biçiminde gösterilmiştir (is shown by).

içler
means

İç terimlerin çarpımı dış terimlerin çarpımına eşittir.
The product of the means equals the product of the extremes.

dışlar
extremes

$$\mathbf{a\cdot d=b\cdot c}$$

içler
means

Özellik / Property

★ $\frac{a}{c}=\frac{b}{d}\Leftrightarrow a:b=c:d$

$\frac{a}{d}=\frac{b}{e}=\frac{c}{f}\Leftrightarrow a:b:c=d:e:f$

★ $\frac{a}{b}=\frac{c}{d}\Longrightarrow$

I. $a\cdot d=b\cdot c$

II. $\frac{a}{c}=\frac{b}{d}$

III. $\frac{d}{b}=\frac{c}{a}$

IV. $\frac{b}{a}=\frac{d}{c}$

★ $\frac{a}{b}=\frac{c}{d}=k\Rightarrow$

I. $\frac{a+c}{b+d}=k$

II. $\frac{m\cdot a}{m\cdot b}=\frac{t\cdot c}{t\cdot d}=\frac{m\cdot a+t\cdot c}{m\cdot b+t\cdot d}=k$

★ $\frac{a}{b}=\frac{c}{d}=k$

$\Rightarrow\frac{a\cdot c}{b\cdot d}=k^2$

★ $\frac{a}{b}=\frac{c}{d}=k$

$\Rightarrow\frac{a^n}{b^n}=\frac{c^n}{d^n}=k^n$

ORANTI ÇEŞİTLERİ / PROPORTION TYPES

DOĞRU ORANTI / DIRECT PROPORTION

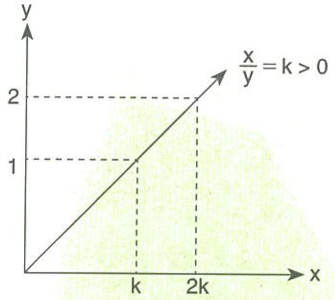
▼ Orantılı iki çokluktan biri artarken diğeri aynı oranda artıyorsa yada biri azalırken diğeri de aynı oranda azalıyorsa bu iki çokluk doğru orantılıdır.

$\frac{x}{y} = k$, $k \in R^+$ ifadesine doğru orantının denklemi denir. x ile y çokluklarının doğru orantılı olduğu grafik aşağıdaki gibidir.

▼ If in two proportional quantities increasing the value of one of them results in increasing the value of the other one with the same ratio, or decreasing the value of one of them results in decreasing the other one with the same ratio, these two quantities are called directly proportional quantities.

$\frac{x}{y} = k$, $k \in R^+$ is called the equation of the direct proportion.

The graph of directly proportional quantities x and y is like the figure shown below.



TERS ORANTI / INVERSE PROPORTION

▼ Orantılı iki çokluktan biri artarken diğeri aynı oranda azalıyorsa yada biri azalırken diğeri aynı oranda artıyorsa bu iki çokluk ters orantılıdır.

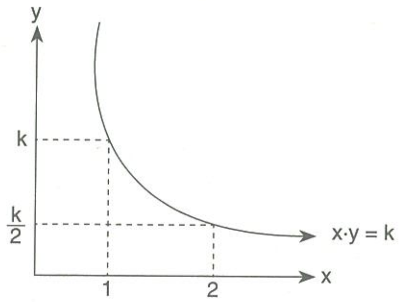
$x \cdot y = k$, $k \in R^+$ ifadesine ters orantının denklemi denir.

x ile y çokluklarının ters orantılı olduğu grafik aşağıdaki gibidir.

▼ If in two proportional quantities increasing the value of one of them results in decreasing the value of the other one with the same ratio, or decreasing the value of one of them results in increasing the other one with the same ratio, these two quantities are called inversely proportional quantities.

$x \cdot y = k$, $k \in R^+$ is called the equation of the inverse proportion.

The graph of inversely proportional quantities x and y is like the figure shown below.



ORTALAMA ÇEŞİTLERİ / TYPES of MEAN

ARİTMETİK ORTALAMA / ARITHMETIC MEAN

▼ n tane sayının aritmetik ortalaması bu n sayının toplamının n' ye bölünmesidir.

Buna göre a_1, a_2, a_3, a_4, ..., a_n sayılarının aritmetik ortalaması $\frac{a_1+a_2+a_3+\ldots+a_n}{n}$ dir

▼ The arithmetic mean of n numbers is division of the sum of these numbers into n.

Accordingly, the arithmetic mean of numbers a_1, a_2, a_3, a_4, ..., a_n is $\frac{a_1+a_2+a_3+\ldots+a_n}{n}$.

GEOMETRİK ORTALAMA / GEOMETRIC MEAN

▼ n tane sayının geometrik ortalaması bu n sayının n. dereceden kök içerisinde çarpılmasıdır. Buna göre a_1, a_2, a_3, a_4, ..., a_n sayılarının geometrik ortalaması $\sqrt[n]{a_1 \cdot a_2 \cdot a_3 \cdot \ldots \cdot a_n}$ dir.

▼ The geometric mean of n numbers is the n'th root of the product of these numbers. Accordingly, the geometric mean of numbers a_1, a_2, a_3, a_4, ..., a_n is $\sqrt[n]{a_1 \cdot a_2 \cdot a_3 \cdot \ldots \cdot a_n}$

1. $\frac{a}{3} = \frac{b}{2} = \frac{c}{5} \Rightarrow \frac{a^2 + b \cdot c}{c^2 + b^2} = ?$

A) $\frac{9}{25}$ B) $\frac{19}{20}$ C) $\frac{10}{21}$ D) $\frac{16}{3}$ E) $\frac{19}{29}$

2. $3a = 2b = 4c \Rightarrow \frac{2a + 3b}{2b + c} = ?$

A) $\frac{4}{5}$ B) $\frac{13}{15}$ C) $\frac{14}{15}$ D) 1 E) $\frac{26}{15}$

3. $\frac{x}{2} = \frac{y}{3} = \frac{z}{7},\ 2x - y + z = 32 \Rightarrow y = ?$

A) 3 B) 6 C) 9 D) 12 E) 15

4. $\frac{a}{b} = \frac{2}{3} \Rightarrow \frac{a^2 - ab + b^2}{a^2 + ab - b^2} = ?$

A) $\frac{3}{5}$ B) $\frac{4}{7}$ C) 7 D) 10 E) 13

5. $\frac{a}{b} = \frac{c}{d} = 5 \Rightarrow \frac{a}{a+b} : \frac{c-d}{d} = ?$

A) $\frac{1}{8}$ B) $\frac{1}{6}$ C) $\frac{5}{24}$ D) $\frac{1}{4}$ E) $\frac{7}{24}$

6. $x, y \in Z^+$,

$\frac{x}{2} = \frac{y}{3},\ x^2 + y^2 = 468 \Rightarrow x - y = ?$

A) –5 B) –6 C) 6 D) 8 E) 10

7. $\frac{a}{b} = \frac{2}{3},\ \frac{b}{c} = \frac{3}{4} \Rightarrow \frac{2a + 5c}{2c} = ?$

A) 1 B) $\frac{4}{3}$ C) $\frac{3}{2}$ D) 3 E) 4

8. $\left.\begin{array}{l} \frac{a+b}{2} = 36,\ \frac{a+c}{3} = 18 \\ b + c = 76 \end{array}\right\} \Rightarrow ? < ? < ?$

A) $a < b < c$ B) $b < a < c$ C) $c < a < b$ D) $b < c < a$ E) $a < c < b$

9. $\frac{a+b}{12}=\frac{a-b}{15} \Rightarrow \frac{a}{b}=?$

A) –9 B) –5 C) 4 D) 6 E) 7

10. $\frac{a}{b}=\frac{b}{c}=\frac{c}{d}=\frac{3}{4} \Rightarrow \frac{a}{d}=?$

A) $\frac{3}{4}$ B) $\frac{3}{2}$ C) $\frac{3}{8}$ D) $\frac{9}{10}$ E) $\frac{27}{64}$

11. $\frac{a-b}{3}=\frac{a+b}{5}=\frac{a+b+c}{7} \Rightarrow \frac{a}{?}=\frac{b}{?}=\frac{c}{?}$

A) 1, 3, 5 B) 4, 5, 7 C) 4, 1, 2 D) 4, 2, 1 E) 3, 5, 7

12. $\frac{a}{b}=\frac{c}{d}=2 \Rightarrow \frac{a}{a+b}\cdot\frac{c-d}{d}=?$

A) $\frac{2}{3}$ B) 4 C) $\frac{4}{5}$ D) $\frac{5}{4}$ E) 1

13. $\frac{a}{b}=\frac{c}{d}=3 \Rightarrow \left(\frac{a-b}{b}\right):\left(\frac{c-d}{d}\right)=?$

A) 1 B) 2 C) 3 D) 4 E) 5

14. $a,b,c \in R^+$

$$\left.\begin{aligned} a\cdot c &= \frac{1}{2} \\ b\cdot c &= 3 \\ a\cdot b &= \frac{5}{4} \end{aligned}\right\} \Rightarrow\ ?<?<?$$

A) $a<b<c$ B) $b<c<a$ C) $a<c<b$ D) $b<a<c$ E) $c<b<a$

15. $ax = by = cz = 15,$

$\frac{1}{a}+\frac{1}{b}+\frac{1}{c}=\frac{1}{3} \Rightarrow x+y+z+?$

A) 1 B) 2 C) 3 D) 4 E) 5

16. $a+\frac{1}{5b}=3,\ b+\frac{1}{5a}=6 \Rightarrow \frac{b^2-a^2}{ab}=?$

A) $\frac{1}{4}$ B) $\frac{1}{2}$ C) 1 D) $\frac{3}{2}$ E) $\frac{5}{2}$

17. $$\left.\begin{aligned} \frac{b\cdot c}{a} &= 3 \\ \frac{c\cdot a}{b} &= 2 \\ \frac{a\cdot b}{c} &= 1 \end{aligned}\right\} \Rightarrow a^2+b^2+c^2=?$$

A) 8 B) 9 C) 10 D) 11 E) 12

YANITLAR / ANSWERS

1	2	3	4	5	6	7	8	9
E	E	D	C	C	B	D	E	A
10	11	12	13	14	15	16	17	
E	C	A	A	C	E	D	D	

1. $\frac{a}{b}=\frac{c}{d}=\frac{e}{f}=\frac{2}{3} \Rightarrow \frac{\mathbf{2a-5c+6e}}{\mathbf{2b-5d+6f}}=\mathbf{?}$

A) $\frac{1}{4}$ B) $\frac{1}{2}$ C) $\frac{1}{3}$ D) $\frac{2}{3}$ E) $\frac{3}{4}$

2. $\frac{1}{x-\frac{1}{y}}=3,\ \frac{1}{y-\frac{1}{x}}=\frac{1}{2} \Rightarrow \frac{\mathbf{y}}{\mathbf{x}}=\mathbf{?}$

A) 6 B) 5 C) 4 D) 3 E) 2

3. $a-\frac{1}{b}=5,\ b-\frac{1}{a}=3,$

$(a+b)=x(a-b) \Rightarrow \mathbf{x=?}$

A) 1 B) 2 C) 3 D) 4 E) 5

4. $\left.\begin{array}{l} a,b\in Z^{+} \\ \frac{a}{2}=\frac{b}{7}=k \end{array}\right\} \Rightarrow \mathbf{\sqrt{2a}+\sqrt{7b}=?}$

A) $9\sqrt{k}$ B) $\sqrt{k}$ C) $14\sqrt{k}$

D) 9 E) 6

5. $\frac{0,03}{0,12}=\frac{0,09}{x} \Rightarrow \mathbf{x=?}$

A) 0,26 B) 0,36 C) 0,42

D) 0,63 E) 0,74

6. $\frac{\frac{1}{3}}{\frac{3}{5}}=\frac{\frac{2}{3}}{x} \Rightarrow \mathbf{x=?}$

A) $\frac{3}{4}$ B) $\frac{5}{6}$ C) $\frac{6}{5}$ D) $\frac{4}{3}$ E) $\frac{2}{3}$

7. $\frac{x+y}{x}=2 \Rightarrow \frac{\mathbf{x+y}}{\mathbf{y}}=\mathbf{?}$

A) 0 B) 1 C) 2 D) 3 E) 4

8. $a:b:c=3:4:6 \Rightarrow \frac{\mathbf{a+3b}}{\mathbf{3c-2b}}=\mathbf{?}$

A) $\frac{3}{2}$ B) $\frac{3}{4}$ C) $\frac{1}{2}$ D) $\frac{1}{4}$ E) $\frac{4}{3}$

9. $\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = \frac{3}{4} \Rightarrow \mathbf{\frac{c+2e}{d+2f} = ?}$

A) $\frac{1}{4}$ B) $\frac{3}{4}$ C) $\frac{2}{3}$ D) $\frac{1}{3}$ E) $\frac{2}{5}$

10. $\left.\begin{aligned} a+\frac{1}{b} &= 2 \\ b+\frac{1}{a} &= 4 \end{aligned}\right\} \Rightarrow \mathbf{\frac{a}{b} = ?}$

A) $\frac{1}{4}$ B) $\frac{1}{2}$ C) 1 D) 2 E) 4

11. $\frac{x}{y} = \frac{3}{4}, \frac{y}{z} = \frac{2}{3} \Rightarrow \mathbf{x:y:z = ?:?:?}$

A) 5 : 6 : 10 B) 4 : 5 : 6 C) 2 : 3 : 5

D) 3 : 4 : 6 E) 3 : 5 : 10

12. $\frac{x}{y} = \frac{1}{2}, \frac{y}{z} = \frac{1}{4}, \frac{z}{t} = 1$ ve / and

$x+y+z+t = 3800 \Rightarrow \mathbf{z = ?}$

A) 1600 B) 1500 C) 1200

D) 1000 E) 800

13. $\left.\begin{aligned} & a,b,c \in Z^+ \\ & \frac{0,0001}{a} = \frac{0,00027}{b} = \frac{\sqrt{2}}{c} \end{aligned}\right\} \Rightarrow \mathbf{? < ? < ?}$

A) a < b < c B) b < a < c C) c < b < a

D) b < c < a E) c < a < b

14. $\left.\begin{aligned} \frac{a}{b} = \frac{c}{d} &= k \\ \frac{a-4c}{3b-12d} &= \frac{k}{x} \end{aligned}\right\} \Rightarrow \mathbf{x = ?}$

A) 1 B) 2 C) 3 D) 4 E) 5

15. $\left.\begin{aligned} \frac{a}{0,2} &= \frac{b}{0,5} \\ c &= \frac{b}{0,4} \end{aligned}\right\} \Rightarrow \mathbf{a:b:c = ?:?:?}$

A) 2 : 4 : 8 B) 2 : 5 : 12 C) 4 : 10 : 25

C) 3 : 10 : 25 E) 2 : 5 : 4

16. $\left.\begin{aligned} \frac{x}{a+b} = \frac{y}{c-b} &= \frac{z}{a-c} \\ x+y+z &= 2 \end{aligned}\right\} \Rightarrow \mathbf{y = ?}$

A) $\frac{c-b}{a-c}$ B) $\frac{a}{a+b}$ C) $\frac{c-b}{a}$ D) $\frac{a}{a+c}$ E) $\frac{a}{c-b}$

17. $\frac{1}{axy} = \frac{1}{byz} = \frac{1}{czx} \Rightarrow \mathbf{a:b:c = ?:?:?}$

A) z : x : y B) x : y : z C) y : z : x

D) $\frac{1}{y}:\frac{1}{x}:\frac{1}{z}$ E) $\frac{1}{x}:\frac{1}{y}:\frac{1}{z}$

YANITLAR / ANSWERS

1	2	3	4	5	6	7	8	9
D	A	D	A	B	C	C	A	B
10	11	12	13	14	15	16	17	
B	D	A	A	C	C	C	A	

1. $\frac{a}{2} = \frac{b}{3}$, $a + b = 40 \Rightarrow \mathbf{b - a = ?}$

A) 5 B) 6 C) 7 D) 8 E) 9

2. $\frac{a}{2} = \frac{b}{4} = \frac{c}{5}$

$2a + 4b + c = 125 \Rightarrow \mathbf{2a + b - c = ?}$

A) –18 B) –9 C) 5 D) 7 E) 15

3. $a : b : c = 2 : 3 : 4$,

$\sqrt{a+b+c} = 9 \Rightarrow \sqrt{\mathbf{a \cdot b}} = \mathbf{?}$

A) $2\sqrt{3}$ B) 4 C) $9\sqrt{6}$ D) 21 E) 27

4. $a + \frac{1}{b} = \frac{2}{b} \Rightarrow \sqrt{\mathbf{a \cdot b}} = \mathbf{?}$

A) 0 B) 1 C) 2 D) 3 E) 4

5. $\frac{a}{b} = \frac{5}{2} \Rightarrow \mathbf{\frac{5a + 2b}{a + b} = ?}$

A) $\frac{5}{3}$ B) $\frac{11}{2}$ C) $\frac{29}{7}$ D) $\frac{8}{3}$ E) $\frac{2}{5}$

6. $3a = 4b = 5c$,

$\frac{1}{a} + \frac{1}{b} + \frac{1}{c} = \frac{2}{3} \Rightarrow \mathbf{\frac{a}{5} + \frac{b}{3} + c = ?}$

A) $\frac{21}{2}$ B) $\frac{35}{8}$ C) $\frac{3}{10}$ D) $\frac{63}{10}$ E) $\frac{15}{10}$

7. $\frac{3}{x} = \frac{4}{y} = \frac{5}{z}$

$x + 2y - z = 3 \Rightarrow \mathbf{z - x = ?}$

A) –2 B) 0 C) 1 D) 2 E) 3

8. $a, b, c \in R^+$

$\frac{1}{a} = \frac{2}{b} = \frac{4}{c}$

$b^2 + a \cdot c = 32 \Rightarrow \mathbf{c - b = ?}$

A) $\frac{1}{2}$ B) $\frac{3}{5}$ C) $\frac{3}{2}$ D) 4 E) 6

9. $ax = by = cz$,

$a : b : c = 2 : 3 : 4$,

$x + y = 25 \Rightarrow \mathbf{y = ?}$

A) 9 B) 10 C) 30 D) 40 E) 45

10. $ax = by = cz = 18$,

$\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 2 \Rightarrow \mathbf{a + b + c = ?}$

A) 11 B) 18 C) 22 D) 36 E) 40

11. $\frac{a}{x} = \frac{b}{y} = \frac{c}{z} = \frac{2}{3}$,

$3a + b - 2c = 2x \Rightarrow \mathbf{\frac{3y}{16z} = ?}$

A) $\frac{1}{3}$ B) $\frac{2}{5}$ C) $\frac{3}{8}$ D) $\frac{8}{13}$ E) $\frac{3}{2}$

12. $\frac{2x + 3y}{11} = \frac{2y + x}{15} = \frac{z}{4}$,

$y + x = 12 \Rightarrow \mathbf{z = ?}$

A) –12 B) –3 C) 11 D) 15 E) 21

13. $a : b : c : = 2 : 4 : 5$,

$\frac{a^2 + b^2 + c^2}{a + b + c} = 135 \Rightarrow \mathbf{a = ?}$

A) 66 B) 110 C) 220 D) 270 E) 275

14. $x, y, z \in R^+$

$\frac{x}{3} = \frac{y}{8} = \frac{z}{12}$,

$y^2 + x \cdot z = 100 \Rightarrow \mathbf{z - y = ?}$

A) x B) $\frac{x}{y}$ C) x+1 D) z–2 E) 2y

15. $x + \frac{2}{y} = 3$, $y + \frac{2}{x} = 5$, $\frac{y}{z} = \frac{1}{3} \Rightarrow \mathbf{\frac{2x}{z} = ?}$

A) $\frac{1}{3}$ B) $\frac{3}{2}$ C) $\frac{2}{5}$ D) $\frac{8}{3}$ E) $\frac{7}{2}$

16. $x, y \in R^+$

$\frac{x}{3} = \frac{2}{y}$, $\frac{x + y}{x - y} = 3 \Rightarrow \mathbf{y = ?}$

A) $2\sqrt{2}$ B) 3 C) $\sqrt{3}$ D) 6 E) 9

17. $\frac{a}{2} = \frac{b}{3} = \frac{c}{4} \Rightarrow \mathbf{\frac{c \cdot b}{(2a + 3b) \cdot c} = ?}$

A) $\frac{1}{3}$ B) $\frac{3}{13}$ C) $\frac{12}{19}$ D) $\frac{9}{4}$ E) 5

YANITLAR / ANSWERS

1	2	3	4	5	6	7	8	9
D	E	C	B	C	D	C	D	B
10	11	12	13	14	15	16	17	
D	C	A	A	C	C	C	B	

1. $\left.\begin{array}{l} x+\dfrac{1}{y}=14 \\ y+\dfrac{1}{x}=\dfrac{7}{24} \end{array}\right\} \Rightarrow \dfrac{x+y}{x-13y}=?$

A) $\frac{7}{5}$ B) $\frac{3}{7}$ C) $\frac{5}{7}$ D) $\frac{10}{7}$ E) 7

2. $\left.\begin{array}{l} \dfrac{a\cdot b}{a+b}=30 \\ \dfrac{a}{b}=\dfrac{3}{5} \end{array}\right\} \Rightarrow a+b=?$

A) 72 B) 96 C) 112 D) 128 E) 144

3. $\left.\begin{array}{l} \dfrac{1}{a}-\dfrac{1}{b}=x \\ a-b=y \end{array}\right\} \Rightarrow a\cdot b=?$

A) xy B) $\frac{x}{y}$ C) $\frac{y}{x}$ D) $-\frac{y}{x}$ E) $-\frac{x}{y}$

4. $b>0$,

$\dfrac{a+b}{7}=\dfrac{a-b}{9}$ ve / and $a.b=-72 \Rightarrow a=?$

A) 48 B) 36 C) 12 D) –3 E) – 24

5. $\left.\begin{array}{l} \dfrac{a}{b}=\dfrac{4}{7} \\ \dfrac{b}{c}=\dfrac{2}{3} \\ \dfrac{c}{d}=\dfrac{3}{5} \end{array}\right\} \Rightarrow a:b:c:d=?:?:?:?$

A) 5, 7, 14, 15 B) 10, 21, 35, 63
C) 10, 21, 14, 35 D) 8, 14, 21, 35
E) 14, 35, 21, 10

6. $\left.\begin{array}{l} \dfrac{a-b}{b}=\dfrac{3}{4} \\ \dfrac{b-c}{c}=\dfrac{5}{7} \end{array}\right\} \Rightarrow \dfrac{a+c}{4c-b}=?$

A) $\frac{5}{3}$ B) $\frac{7}{4}$ C) $\frac{3}{2}$ D) $\frac{4}{5}$ E) $\frac{3}{4}$

7. $\left.\begin{array}{l} \dfrac{a}{b}=\dfrac{3}{4} \\ \dfrac{b}{c}=\dfrac{9}{7} \end{array}\right\}$

$a+b+c=182 \Rightarrow a=?$

A) 54 B) 56 C) 72 D) 110 E) 128

8. $\dfrac{a+b}{b}=\dfrac{7}{4}$ ve / and $\dfrac{b}{c}=9 \Rightarrow \dfrac{a}{c}=?$

A) $\frac{27}{4}$ B) $\frac{25}{3}$ C) 9 D) 8 E) $\frac{17}{2}$

9. $\left.\begin{array}{l}\frac{a}{b}=\frac{3}{5}\\ \frac{b}{c}=\frac{7}{9}\end{array}\right\}$ ve / and $a+b+c=606 \Rightarrow$ **c = ?**

A) 126 B) 210 C) 270 D) 304 E) 320

10. $\frac{a}{b}=\frac{c}{d}=k$ ve / and $\frac{9+4c}{p.b+4d}=k \Rightarrow$ **p = ?**

A) $\frac{a}{9}$ B) $\frac{9}{b}$ C) 9b D) 9a E) $\frac{9}{a}$

11. $\frac{u}{u+v}=\frac{4}{5}$,
$\frac{v}{v+t}=\frac{2}{3} \Rightarrow \mathbf{\frac{u+t}{t-v}=?}$

A) –10 B) –9 C) –8 D) –7 E) –6

12. $\frac{x-2}{3}=\frac{y+2}{4}=\frac{z}{5}$, $x+3y=6 \Rightarrow$ **3z = ?**

A) 5 B) 7 C) 10 D) 14 E) 16

13. $\frac{x}{y}=\frac{m}{n}=\frac{a}{b}=5$,

$3x-m+a=35$,

$n-3y=4 \Rightarrow$ **b = ?**

A) 12 B) 11 C) 9 D) 7 E) 4

14. $\frac{x+y}{15}=\frac{x-y}{13}=\frac{x\cdot y}{56}$
$\Rightarrow \mathbf{\frac{x-11y}{9}=?}$

A) $\frac{7}{9}$ B) $\frac{5}{9}$ C) $\frac{10}{3}$ D) 3 E) $\frac{4}{3}$

15. $x, y, z, t \in R^+$
$\left.\begin{array}{l}\frac{x}{y}=\frac{y}{z}=\frac{z}{t}\\ \frac{x\cdot z-y\cdot t}{y\cdot t}=143\end{array}\right\} \Rightarrow \mathbf{\frac{y+6z}{3z}=?}$

A) 2 B) 4 C) 6 D) 9 E) 15

16. $\frac{x}{y}=\frac{y}{m}=\frac{m}{n}=4 \Rightarrow \mathbf{\frac{x^3+y^3+m^3}{y^3+m^3+n^3}=?}$

A) 12 B) 16 C) 32 D) 64 E) 72

17. $x+y=8xy$,
$y+z=3xy \Rightarrow \mathbf{\left(\frac{1}{z}-\frac{1}{x}\right)\cdot\frac{z}{y}=?}$

A) 2 B) 3 C) 5 D) 7 E) 11

YANITLAR / ANSWERS

1	2	3	4	5	6	7	8	9
A	D	D	E	D	B	A	A	C
10	11	12	13	14	15	16	17	
E	B	C	B	E	C	D	C	

1. $a = 3b = 4c,$

$\Rightarrow \dfrac{\mathbf{2a-b+3c}}{\mathbf{a+2b-3c}} = \mathbf{?}$

A) $\dfrac{29}{11}$ B) $\dfrac{17}{9}$ C) $\dfrac{15}{7}$ D) $\dfrac{13}{4}$ E) $\dfrac{11}{2}$

2. $\dfrac{a}{b} = \dfrac{c}{d} = \dfrac{e}{f} = \dfrac{m}{n} = 2$

$\Rightarrow \dfrac{\mathbf{m}}{\mathbf{a}} \cdot \dfrac{\mathbf{b}}{\mathbf{n}} \cdot \dfrac{\mathbf{e}}{\mathbf{c}} \cdot \dfrac{\mathbf{d}}{\mathbf{f}} = \mathbf{?}$

A) $\dfrac{1}{4}$ B) $\dfrac{1}{2}$ C) 1 D) 2 E) 4

3. $\dfrac{a}{b} = \dfrac{3}{4},$

$5b = 2c \Rightarrow$ **a : b : c = ?**

A) 3 : 4 : 10 B) 10 : 4 : 3 C) 5 : 3 : 2 D) 5 : 2 : 3 E) 2 : 3 : 5

4. $\dfrac{a}{x} = \dfrac{b}{y} = \dfrac{c}{z} = \dfrac{7}{8},$

$2a + 3b + 5c = 70$

$2x + 3y = 20 \Rightarrow$ **z = ?**

A) 5 B) 7 C) 9 D) 10 E) 12

5. $a, b, c, d \in \mathbb{R}^-,$

$\dfrac{a}{b} = \dfrac{c}{d}, \dfrac{a}{3} = \dfrac{c}{15} = \dfrac{9}{b} = \dfrac{d}{5} \Rightarrow$ **a = ?**

A) –9 B) –6 C) –3 D) –2 E) –1

6. $\dfrac{x-4y}{3y} = \dfrac{y-5z}{z} = 2 \Rightarrow \dfrac{\mathbf{x-5y}}{\mathbf{7z}} = \mathbf{?}$

A) 3 B) 4 C) 5 D) 6 E) 7

7. $\dfrac{x^2}{4} = \dfrac{9}{y^2} = A \Rightarrow \dfrac{\mathbf{y+2}}{\mathbf{x+3}} = \mathbf{?}$

A) A^2 B) A C) $\dfrac{1}{A}$ D) $\sqrt{A}$ E) $\dfrac{\sqrt{A}}{A}$

8. $a, b, c \in \mathbb{R}^+,$

$\dfrac{a+b+c}{45} = \dfrac{3}{a} = \dfrac{2}{b+c} \Rightarrow$ **a = ?**

A) 3 B) 6 C) 9 D) 12 E) 15

9. $x, y \in R^+$,

$\frac{2x+3y}{7}=\frac{5}{y}=\frac{x}{2}$

$\Rightarrow$ **$x^2+y^2=?$**

A) 12 B) 16 C) 18 D) 25 E) 27

10. $\frac{a}{x}=\frac{b}{y}=\frac{c}{z}=\frac{4}{5}$

$a\cdot x+b\cdot y+c\cdot z=20 \Rightarrow$ **$x^2+y^2+z^2=?$**

A) 16 B) 18 C) 20 D) 24 E) 25

11. $\frac{1}{2x}=\frac{1}{3y}=\frac{1}{4z}$

$z+y-x-\frac{1}{6}=0 \Rightarrow$ **$x=?$**

A) $\frac{1}{8}$ B) $\frac{1}{6}$ C) $\frac{1}{4}$ D) $\frac{1}{2}$ E) 1

12. $x+x\cdot y=\frac{1}{3}$

$y-x=3$

$y^2+y=\frac{2}{3} \Rightarrow$ **$\frac{y^2-x}{x}=?$**

A) 10 B) 11 C) 12 D) 14 E) 15

13. $\frac{x+y}{2}=\sqrt{x\cdot y} \Rightarrow$ **$\frac{2x^2+y^2}{2x^2-y^2}=?$**

A) 3 B) 4 C) 6 D) 9 E) 12

14. $\frac{a+3x}{y}=\frac{a-2x}{z}=\frac{3}{2}$

$z-y=10 \Rightarrow$ **$x=?$**

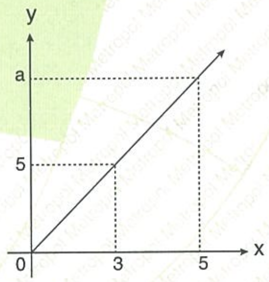
A) –3 B) –2 C) –1 D) 2 E) 3

15. $\frac{a}{x}=\frac{b}{y}=\frac{3}{5}$

$a^2+b^2=18 \Rightarrow$ **$x^2+y^2=?$**

A) 45 B) 50 C) 55 D) 60 E) 65

16.



$\Rightarrow$ **$a=?$**

A) $\frac{20}{3}$ B) 8 C) $\frac{25}{3}$ D) 10 E) $\frac{35}{3}$

YANITLAR / ANSWERS

1	2	3	4	5	6	7	8
A	C	A	E	A	C	E	C
9	**10**	**11**	**12**	**13**	**14**	**15**	**16**
D	E	E	B	A	A	B	C

1. $\frac{x+3y}{y}=\frac{7}{2}$,

$\frac{y-2z}{z}=\frac{1}{2} \Rightarrow \mathbf{\frac{2z-x}{z}=?}$

A) $\frac{1}{2}$ B) $\frac{2}{3}$ C) $\frac{3}{4}$ D) $\frac{4}{5}$ E) $\frac{5}{6}$

2. $\frac{x}{y}=\frac{3}{7}$, $\frac{z}{x}=\frac{5}{9}$

$2x-y+z=6 \Rightarrow \mathbf{y=?}$

A) 21 B) 35 C) 49 D) 56 E) 63

3. $x:y:z=5:3:4$

$2x-3y+\frac{5z}{4}=78 \Rightarrow \mathbf{y=?}$

A) 27 B) 30 C) 33 D) 36 E) 39

4. $xy+yz+xz=3xyz$

$a \cdot x=b \cdot y=c \cdot z=4$

$\Rightarrow \mathbf{a+b+c=?}$

A) 6 B) 8 C) 10 D) 12 E) 15

5. $\frac{2xy}{3}=\frac{5yz}{4}=\frac{6xz}{5}$

$x=5\sqrt{2} \Rightarrow \mathbf{3xz=?}$

A) 50 B) 60 C) 70 D) 80 E) 90

6. $\frac{x+y+z}{7}=\frac{x+y}{5}=\frac{x-y}{3}$

$\Rightarrow \mathbf{x:y:z=?}$

A) 2 : 3 : 4 B) 7 : 5 : 3 C) 4 : 3 : 2 D) 6 : 4 : 3 E) 4 : 1 : 2

7. $\frac{x}{3}=\frac{y}{5}=\frac{z}{7}=\frac{4x-3y+2z}{3a-4}$

$\Rightarrow \mathbf{a=?}$

A) 3 B) 4 C) 5 D) 6 E) 7

8. $\frac{a}{2}=\frac{b}{5}=\frac{c}{4} \Rightarrow \mathbf{\frac{a^2-2b^2+bc}{ab+ac-c^2}=?}$

A) –13 B) –11 C) –9 D) –8 E) –7

9. $\frac{a}{b} = \frac{c}{d} = 4 \Rightarrow \left(\frac{\mathbf{a+b}}{\mathbf{b}}\right) \cdot \left(\frac{\mathbf{c+d}}{\mathbf{c}}\right) = \mathbf{?}$

A) $\frac{20}{3}$ B) 7 C) $\frac{22}{3}$ D) 8 E) $\frac{25}{4}$

10. $\frac{x+z}{y} = 3,$

$\frac{z-y}{x} = 4 \Rightarrow \frac{\mathbf{x}}{\mathbf{y}} = \mathbf{?}$

A) $\frac{1}{5}$ B) $\frac{2}{5}$ C) $\frac{3}{5}$ D) $\frac{5}{3}$ E) $\frac{5}{2}$

11. $\frac{1}{x} = \frac{-2}{y} = \frac{3}{z}$

$3x + 2y + z = 14 \Rightarrow \mathbf{x = ?}$

A) 4 B) 5 C) 6 D) 7 E) 9

12. $\frac{a^2 + 2b^2 + 3ab}{a^2 + b^2 - 4ab} = 2 \Rightarrow \frac{\mathbf{a}}{\mathbf{b}} = \mathbf{?}$

A) 9 B) 10 C) 11 D) 14 E) 16

13. $\frac{x^2}{4} = \frac{y^2}{9} = \frac{x \cdot y}{6}$

$x + y = 15 \Rightarrow \mathbf{x \cdot y = ?}$

A) 48 B) 54 C) 60 D) 66 E) 72

14. $\frac{x \cdot y}{5} = \frac{x-2y}{3} = \frac{x+y}{4}$

$\Rightarrow \mathbf{x = ?}$

A) 5 B) 10 C) 15 D) 20 E) 25

15. $\frac{a+1}{b} = \frac{b+2}{c} = \frac{c+3}{a} = 7$

$\Rightarrow \mathbf{a + b + c = ?}$

A) –3 B) –2 C) –1 D) 1 E) 2

16. $\frac{x+y}{7} = \frac{x+z}{9} = \frac{y+z}{12}$

$\Rightarrow \frac{\mathbf{x}}{\mathbf{y}} = \mathbf{?}$

A) $\frac{1}{5}$ B) $\frac{2}{5}$ C) $\frac{3}{5}$ D) $\frac{4}{5}$ E) 1

YANITLAR / ANSWERS

1	2	3	4	5	6	7	8
C	E	E	D	D	E	C	A
9	**10**	**11**	**12**	**13**	**14**	**15**	**16**
E	B	D	C	B	C	D	B

BÖLÜM
CHAPTER
7

TARİHSEL NOT / HISTORICAL NOTE

Abel, Niels Henrik [1802 – 1829]

Norveçli matematikçi 19 yaşında dörtten büyük dereceli denklemlerin çözülemeyeceğini göstermiştir. Başka bir deyişle, bu denklemlerin ikinci dereceden denklemler gibi formülünün olmadığını kanıtlamıştır. Aynı zamada cebirsel fonksiyonlar teorisindeki temel gelişmeleri yanıtlamıştır.

Norwegian mathematician who at the age of 19, proved that the general equation of degree greater than 4 cannot be solved algebraically. In other words, there can be no formula for the roots of such an equation similar to the familiar formula for the quadratic equation. He was also responsible for fundamental developments in theory of algebraic functions.

BİRİNCİ DERECEDEN DENKLEMLER
FIRST DEGREE EQUATIONS

Bu bölüm, 99 test sorusu içermektedir.

This chapter includes 99 test questions.

BÖLÜM / CHAPTER 7

BİRİNCİ DERECEDEN DENKLEMLER / FIRST DEGREE EQUATIONS

- Birinci Dereceden Denklemler / First Degree Equations .... 151 – 168

BÖLÜM
07
CHAPTER

BİRİNCİ DEREDECEN DENKLEMLER
FIRST DEGREE EQUATIONS

BİRİNCİ DERECEDEN BİR BİLİNMEYENLİ DENKLEMLER / FIRST DEGREE EQUATIONS in ONE VARIABLE

Tanım: a ve b reel sayı ve $a \neq 0$ olmak üzere,

$ax + b = 0$ ifadesine ***birinci dereceden bir bilinmeyenli denklem*** denir. Bu denklemi sağlayan, x reel sayısına da ***denklemin kökü*** denir. Denklemi sağlayan x değerlerinin kümesine ***çözüm kümesi*** denir ve ***ÇK*** ile gösterilir.

Definition: A first degree equation in one variable x is an equation that can be written in the form $ax + b = 0$, where a and b are real numbers, and $a \neq 0$. The real number x which satisfy the equation is called the *root of this equation*. The set of the values of x which satisfy the equation is called *solution set* and shown by *SS*.

$$ax + b = 0 \Rightarrow ax = -b$$

$$x = -\frac{b}{a} \Rightarrow ÇK(SS) = \left\{-\frac{b}{a}\right\}$$

$ax + b = 0$

I. $a = 0,\ b = 0 \Rightarrow$ **ÇK(SS) = R**

II. $a = 0,\ b \neq 0 \Rightarrow$ **ÇK(SS) = Ø**

Önemli / Important

- $ax + b = cx + d$

  I. $a = c,\ b = d \Rightarrow$ **ÇK(SS) = R**

  II. $a = c,\ b \neq d \Rightarrow$ **ÇK(SS) = Ø**

Önemli / Important

- Birinci dereceden bir bilinmeyenli denklemlerin çözümü yapılırken bilinmeyen yok edildiğinde kalan sayısal değerler birbirine eşit ise çözüm kümesi reel sayılar eşit değil ise çözüm kümesi boş kümedir.
- When solving first degree equations with one unknown, if the numerical values after dissipating the unknown are equal, the solution set is all real numbers and if not, the solution set is empty set.

Özellik / Property

★ $a, b, c \in R$

$a = b \Rightarrow$ $\mathbf{a \mp c = b \mp c}$

Özellik / Property

★ $a, b, c \in R$

$a = b \Rightarrow$ $\mathbf{a \cdot c = b \cdot c}$

Özellik / Property

★ $a, b, c \in R, \; c \neq 0$

$a = b \Rightarrow \mathbf{a : c = b : c}$

BİRİNCİ DERECEDEN İKİ BİLİNMEYENLİ DENKLEM SİSTEMLERİ
SYSTEMS of FIRST DEGREE EQUATIONS with TWO UNKNOWNS

Tanım: $a, b, c \in R$ ve $a \neq 0$ ve $b \neq 0$ olmak üzere, $ax + by + c = 0$ biçiminde verilen denklemlere ***birinci dereceden iki bilinmeyenli denklem*** denir.

Definition: Provided that $a, b, c \in R$, $a \neq 0$, $b \neq 0$, the equations in the $ax + by + c = 0$ form are called ***first degree equations with two unknowns***.

Tanım: Denklemi sağlayan x ve y değerlerine ***denklemin kökleri*** denir. Birinci dereceden iki bilinmeyenli denklemlerin çözüm kümesi ikililerden oluşur. Birinci dereceden iki bilinmeyenli denklem, koordinat sisteminde bir doğru belirtir.

$a \neq 0, b \neq 0, a, b, c, d, e, f \in R \Rightarrow$

$ax + by = c$

$dx + ey = f$

denklemlerine ***birinci dereceden iki bilinmeyenli denklem sistemi*** denir.

Definition: x and y values which satisfy the equation are called the ***roots of equation***. The equations in that form have infinite roots and define a line in coordinate system.

$a \neq 0, b \neq 0, a, b, c, d, e, f \in R \Rightarrow$

$ax + by = c$

$dx + ey = f$

System in a form as shown above are called *system of first degree equations with two unknowns.*

Bu denklem sisteminde,

1. $\frac{a}{d} = \frac{b}{e} = \frac{c}{f}$ ise çözüm kümesi **reel sayılar**dır.

2. $\frac{a}{d} = \frac{b}{e} \neq \frac{c}{f}$ ise çözüm kümesi **boş küme**dir.

3. $\frac{a}{d} \neq \frac{b}{e}$ ise çözüm kümesi **bir elemanlı**dır.

In this equation system,

1. If $\frac{a}{d} = \frac{b}{e} = \frac{c}{f}$, then the solution set is **real numbers**.

2. If $\frac{a}{d} = \frac{b}{e} \neq \frac{c}{f}$, then the solution set is an **empty set**.

3. If $\frac{a}{d} \neq \frac{b}{e}$, then the solution set has **one element**.

BİRİNCİ DERECEDEN İKİ BİLİNMEYENLİ DENKLEM SİSTEMLERİNİN ÇÖZÜMÜ
SOLUTION of SYSTEMS of FIRST DEGREE EQUATIONS with TWO UNKNOWNS

YOK ETME YÖNTEMİ / THE ELIMINATION METHOD

Bu yöntemde değişkenlerden biri, diğer denklemde yer alan aynı değişkenin aynı büyüklükte katsayılı, fakat zıt işaretli değeri ile çarpılarak değişkenlerden biri yok edilir.

- Bir değişkeni yok etmek için verilen denklemleri toplayın veya çıkarın.
- Kalan değişken için elde edilen denklemi çözün.
- Bilinmeyen değişkenin değerini bulmak için, ana denklemlerden herhangi birinde bulunan değişkenin değerini yazın.

In the method of elimination, we eliminate (remove) one of the variables by making the coefficients of one of variables the same size but opposite in sign in two equations and then adding the equations.

- Add or subtract the given equations to eliminate one variable.
- Solve the resulting equation for the remaining variable.
- Replace the value of the known variable in one of the original equations to find the value of the other unknown variable.

YERİNE KOYMA YÖNTEMİ / THE SUBSTITUTION METHOD

Bu yöntemde değişkenlerden biri diğer değişken cinsinden yazılır. Seçilen değişken diğer denklemde yazılarak bir bilinmeyenli denklem haline getirilir.

- İki denklemden birinde, bir değişkeni diğeri cinsinden yazınız.
- Bu değişkeni, diğer denklemde yerine yazınız.
- Bir değişkenin değerini bu denklemden bulunuz.
- İkinci adımda bulunan denklemdeki sonuca, bulunan değeri yazın ve diğer değişkeni bulunuz.

In this method one of the variables is written in terms of the other variable. The chosen variable gets written in the other equation and form an equation with one unknown.

- In either equation, solve for one variable in terms of the other.
- Substitute for that variable in the other equation.
- Solve the resulting equation for the value of one variable.
- Substitute the result from step 2 in either equation. Solve for the other variable.

ÖZEL DENKLEMLER / SPECIAL EQUATIONS

Birkaç değişkenden oluşan bir denklem sistemini çözmek için, toplama veya çıkarma ile değişkenlerden birini yok etmeye çalışılır. Eğer iki denklemde de değişkenlerin katsayıları birbirine eşitse, bu değişkenler toplama veya çıkarma yoluyla yok edilir.

Üç bilinmeyenli üç denklemi çözmek için aşağıdaki adımlar izlenir:

- ▼ İki denklem kullanılarak, bir değişken yok edilir. Elde edilen denklem, iki değişken içeren yeni bir denklemdir.
- ▼ Verilen denklemlerin kombinasyonları kullanılarak aynı değişkenler yok edilir. Sonuçta iki değişkenden oluşan yeni denklem elde edilir.
- ▼ 1. ve 2. adımda elde edilen iki yeni denklem çözülür.
- ▼ Değişkenlerden birinin değeri bulunup diğerlerinde yerine yazılarak diğer değişkenler elde edilir.

In solving a system of equations containing several unknowns, we try to eliminate one unknown by addition or subtraction. If the coefficients of one variable are numerically equal in two equations, then that variable may be eliminated by adding or subtracting the two equations.

To solve a system of three equations with three unknowns, we have the following steps:

- ▼ Eliminate one variable using two of the equations. The result is a new equation containing, at most, two variables.
- ▼ Eliminate the same variable using another set or combination of the given equations. The result is another equation containing, at most, two variables.
- ▼ Solve the two new equations obtained in Steps 1 and 2.
- ▼ When the value of one variable has been found, work backward through the equations to find the other variables.

1. $3(4 - x) + 5x = 3 - x \Rightarrow$ **ÇK(SS) = ?**

A) $\{-4\}$ B) $\{-3\}$ C) $\{-2\}$ D) $\{-1\}$ E) $\{0\}$

2. $6x - (2 - 3(x - 2)) = 4x - (1 - x)$

$\Rightarrow$ **ÇK(SS) = ?**

A) $\left\{-\frac{4}{7}\right\}$ B) $\{-1\}$ C) $\left\{\frac{4}{7}\right\}$ D) $\left\{\frac{7}{4}\right\}$ E) $\varnothing$

3. $(x - 3)\cdot(x + 4) = (x - 3) \Rightarrow$ **ÇK(SS) = ?**

A) $\{-4\}$ B) $\{-3\}$ C) $\{4, -3\}$

D) $\{-4, 3\}$ E) $\{-3, 3\}$

4. $4x^2 - 16 = 2x + 4 \Rightarrow$ **ÇK(SS) = ?**

A) $\{-2\}$ B) $\{-2, 2\}$ C) $\{2\}$

D) $\left\{-2, \frac{5}{2}\right\}$ E) $\varnothing$

5. $3x + 2\cdot(x - 1) = 4\cdot(x - 3) + x$

$\Rightarrow$ **ÇK(SS) = ?**

A) $\{-1\}$ B) $\{-1, 2\}$ C) $\{1\}$

D) $\varnothing$ E) R

6. $2\cdot(2x - 4) + 3 = 4x - 5$

$\Rightarrow$ **ÇK(SS) = ?**

A) $\{0\}$ B) $\{0, 2\}$ C) $\{1\}$

D) $\varnothing$ E) R

7. $\frac{x+1}{2} - \frac{x-3}{3} = \frac{11}{6} \Rightarrow$ **x = ?**

A) 5 B) 4 C) 3 D) 2 E) 1

8. $\frac{x}{x+3} - \frac{1}{1 + \frac{2}{x+1}} = -\frac{1}{5} \Rightarrow$ **x = ?**

A) 2 B) 1 C) $\frac{1}{2}$ D) $\frac{3}{5}$ E) $\frac{1}{4}$

9. $\frac{1}{x-a+2}+\frac{1}{x+2}+\frac{1}{x-5}=\frac{13}{36}$

$x_1 = 7 \Rightarrow$ **a = ?**

A) –5 B) 0 C) 5 D) 9 E) 13

10. $\frac{2}{x}-\frac{x}{x+2}+\frac{x-2}{x}=\frac{6}{5} \Rightarrow$ **ÇK(SS) = ?**

A) {1} B) {0, 2} C) $\left\{\frac{1}{2}\right\}$

D) $\left\{-\frac{1}{3}\right\}$ E) $\left\{0, -\frac{1}{3}\right\}$

11. $(x^2-4)\cdot(x+3)=0$

$\Rightarrow$ **ÇK(SS) = ?**

A) {–3, –2, 2} B) {–3, 2} C) {–3, –2}

D) {–2, 2} E) ∅

12. $x^3+3x^2-x-3=0$

$\Rightarrow$ **ÇK(SS) = ?**

A) {–3, 1} B) {–3} C) {–2, 1}

D) {–1, 1} E) {–3, –1, 1}

13. $x, y \in Z^+$

$x^2-y^2=13$

$\Rightarrow \frac{x}{x+2}+\frac{1}{-2-\frac{1}{x-2}}=?$

A) $\frac{3}{2}$ B) $\frac{4}{3}$ C) 1 D) $\frac{1}{2}$ E) $\frac{1}{3}$

14. $|x-y+z-6|+|x+y-z-16|=0 \Rightarrow$ **x = ?**

A) 8 B) 9 C) 10 D) 11 E) 12

15. $(2x-y-6)^2+(3x+6y-34)^2=0 \Rightarrow$ **x + y = ?**

A) 3 B) $\frac{17}{5}$ C) 4 D) $\frac{24}{5}$ E) 8

16. $\frac{x+5}{x-4}=\frac{x+3}{x-2} \Rightarrow$ **x = ?**

A) 3 B) $\frac{2}{3}$ C) –1 D) $-\frac{1}{2}$ E) $-\frac{3}{4}$

17. $a \neq b$

$\frac{1}{a}-\frac{1}{b}=\frac{1}{a^2}-\frac{1}{b^2}=8 \Rightarrow$ **b = ?**

A) $\frac{7}{2}$ B) 2 C) $-\frac{2}{7}$ D) $-\frac{5}{2}$ E) $-\frac{7}{2}$

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1	2	3	4	5	6	7	8	9
B	D	E	D	D	E	D	A	E
10	11	12	13	14	15	16	17	
D	A	E	B	D	E	D	C	

1. $3x + 2(5 - 2x) + 9 = 12x - (3 - 5x) + 4x \Rightarrow \mathbf{x = ?}$

A) 1 B) 2 C) 3 D) 4 E) 5

2. $x + \dfrac{3x-5}{2} = 8 - \dfrac{2x-16}{3} \Rightarrow \mathbf{x = ?}$

A) 1 B) 2 C) 3 D) 4 E) 5

3. $0{,}37 \cdot x - 0{,}117 - 0{,}283 = 0{,}33x \Rightarrow \mathbf{x = ?}$

A) 8 B) 10 C) 12 D) 16 E) 18

4. $0{,}5 \cdot x + 0{,}5 \cdot y = 4$

$0{,}2 \cdot x + 0{,}4 \cdot z = 3{,}5$

$0{,}2 \cdot y + 0{,}3 \cdot z = 6{,}5$

$\Rightarrow \mathbf{x + y + z = ?}$

A) 9 B) 12 C) 18 D) 20 E) 24

5. $(4x + 10)^2 + (2y - 12)^2 = 0 \Rightarrow \mathbf{x + y = ?}$

A) $\frac{5}{2}$ B) 3 C) $\frac{7}{2}$ D) 4 E) $\frac{9}{2}$

6. $\left.\begin{array}{l} y - \dfrac{3y}{x} = 4 \\ 1 + \dfrac{x}{y} = 5 \end{array}\right\} \Rightarrow \mathbf{x = ?}$

A) 16 B) 17 C) 18 D) 19 E) 20

7. $\dfrac{x+1}{4} - \dfrac{x-3}{3} = 1 \Rightarrow \mathbf{x = ?}$

A) –1 B) 1 C) 2 D) 3 E) 4

8. $x^2 - y^2 = 40,\ x + y = 10 \Rightarrow \mathbf{(x, y) = (?, ?)}$

A) {7, 2} B) {7, 3} C) {6, 2}

D) $\{\sqrt{41}, 1\}$ E) $\{9, \sqrt{41}\}$

9. $A = \frac{1}{x}, B = \frac{x+1}{x-1} \Rightarrow$ **B = ?**

A) $\frac{1+A}{1-A}$ B) $\frac{A}{A+1}$ C) $\frac{1-A}{A}$

D) $\frac{1+A}{A}$ E) $\frac{A}{A-1}$

10. $y = \frac{1}{1-\frac{1}{\frac{x+1}{x}}} \Rightarrow$ **x = ?**

A) $\frac{y+1}{y}$ B) $\frac{y-2}{y}$ C) $2y+1$ D) $1-\frac{3}{2}y$ E) $y-1$

11. $\left.\begin{array}{l} 3x-4y-5z=13 \\ 4x-y+4z=24 \\ 2x-10y-6z=-13 \end{array}\right\} \Rightarrow$ **x + y + z = ?**

A) 10 B) 12 C) 16 D) 18 E) 24

12. $\left.\begin{array}{l} \frac{1}{x}+\frac{1}{y}=\frac{1}{28} \\ \frac{1}{y}+\frac{1}{z}=\frac{1}{2} \\ \frac{1}{x}+\frac{1}{z}=\frac{1}{4} \end{array}\right\} \Rightarrow$ **z = ?**

A) $\frac{5}{3}$ B) $\frac{14}{5}$ C) 1 D) $\frac{1}{2}$ E) $\frac{1}{4}$

13. $A = \frac{2a+4b}{a-3b} \Rightarrow$ **b = ?**

A) $\frac{a(A+4)}{3A-4}$ B) $\frac{a(2-A)}{3A+4}$ C) $\frac{a(A+4)}{3A+4}$

D) $\frac{a(A-2)}{3A-4}$ E) $\frac{a(A-2)}{3A+4}$

14. $\left.\begin{array}{l} x+y=8 \\ x-z=4 \\ x\cdot y+x\cdot z=20 \end{array}\right\} \Rightarrow$ **x = ?**

A) 4 B) 5 C) 6 D) 7 E) 8

15. $x + \frac{1}{1-\frac{1}{1-\frac{1}{x}}} = 1 \Rightarrow$ **ÇK(SS) = ?**

A) R B) R – {0} C) R – {0,1}

D) R – {–1} E) R – {–1, 0, 1}

16. $a, b, c \in Z^+$

$\left.\begin{array}{l} a\cdot b=24 \\ a\cdot c=42 \\ b\cdot c=28 \end{array}\right\} \Rightarrow$ **a + b + c = ?**

A) 22 B) 24 C) 20 D) 18 E) 17

17. $\left.\begin{array}{l} \frac{x}{y}=3 \\ 2x+3(x-y)+12=3x \end{array}\right\} \Rightarrow$ **x = ?**

A) –4 B) –8 C) –12 D) –15 E) –16

YANITLAR / ANSWERS

1	2	3	4	5	6	7	8	9
A	E	B	D	C	D	D	B	A
10	**11**	**12**	**13**	**14**	**15**	**16**	**17**	
E	A	B	E	B	C	E	C	

1. $x+\frac{x}{6}+\frac{4-x}{2}=\frac{5(x-4)}{6}+\frac{x}{2}+\frac{2(x+2)}{3}=?$

A) –3 B) –2 C) 1 D) 2 E) 3

2. $\frac{x-3}{x+3}=\frac{x+6}{x-6} \Rightarrow x=?$

A) $-\frac{1}{2}$ B) 0 C) $\frac{3}{7}$ D) $\frac{4}{9}$ E) 2

3. $\left.\begin{array}{l} a-b=12 \\ a-c=8 \\ b+c=24 \end{array}\right\} \Rightarrow a+b+c=?$

A) 46 B) 42 C) 36 D) 32 E) 28

4. $\left.\begin{array}{l} \frac{m+n}{n}=\frac{3}{5} \\ 3-\frac{n}{m}=\frac{11}{4}x \end{array}\right\} \Rightarrow x=?$

A) 1 B) 2 C) 3 D) 4 E) 5

5. $(x-y-8)^2+(2x+y-13)^2=0 \Rightarrow x+y=?$

A) 9 B) 8 C) 7 D) 6 E) 4

6. $\left(1+\frac{1}{2}\right)\cdot\left(1+\frac{1}{3}\right)\cdot\left(1+\frac{1}{4}\right)\cdot\ldots\cdot\left(1+\frac{1}{x}\right)=16$

$\Rightarrow x=?$

A) 32 B) 31 C) 30 D) 16 E) 15

7. $\left.\begin{array}{l} a+b=7 \\ a+c=10 \\ b+c=11 \end{array}\right\} \Rightarrow a\cdot b\cdot c=?$

A) 12 B) 21 C) 28 D) 42 E) 84

8. $\left.\begin{array}{l} a+2b=22 \\ -b-2c=8 \\ 2a+c=4 \end{array}\right\} \Rightarrow a+b+c=?$

A) 8 B) 7 C) 6 D) 5 E) 4

9. $m-\frac{x}{m}=n-\frac{x}{n} \Rightarrow$ **x = ?**

A) $-mn$ B) mn C) $m-n$

D) $m+n$ E) $\frac{m}{n}$

10. $\left.\begin{array}{l} a-4b+c=35 \\ 3a+8b+3c=57 \end{array}\right\} \Rightarrow$ **a + b + c = ?**

A) 19 B) 21 C) 23 D) 24 E) 27

11. $\frac{1}{x}+\frac{1}{y}-\frac{1}{z}=\frac{4}{9},$

$x \cdot y \cdot z=36$

$\Rightarrow$ **xy − xz − yz = ?**

A) −16 B) −12 C) −8 D) 4 E) 10

12. $\left.\begin{array}{l} \frac{2}{x}-\frac{1}{y}=\frac{5}{12} \\ \frac{1}{x}+\frac{2}{y}=\frac{5}{6} \end{array}\right\} \Rightarrow$ **x.y = ?**

A) 12 B) 18 C) 21 D) 28 E) 48

13. $2-\dfrac{2}{1-\dfrac{1}{2+\dfrac{1}{x}}}+\frac{3}{2}=0 \Rightarrow$ **x = ?**

A) $\frac{1}{3}$ B) 3 C) $\frac{11}{3}$ D) $\frac{9}{2}$ E) 5

14. $x, y \in Z$

$\left.\begin{array}{l} 4x+m-y=6 \\ x^2-y^2=11 \end{array}\right\} \Rightarrow$ **m = ?**

A) −13 B) −9 C) $-\frac{7}{2}$ D) 4 E) 12

15. $\left.\begin{array}{l} 6x+4y-2a+6b=74 \\ 4x+4y-4a=-48 \\ y-2b=8 \end{array}\right\} \Rightarrow$ **x + y + a + b = ?**

A) 93 B) 84 C) 69 D) 63 E) 57

16. $\left.\begin{array}{l} a+b+c=12 \\ a \cdot (b+c)=36 \\ b-c=4 \end{array}\right\} \Rightarrow$ **a · b · c = ?**

A) −24 B) −30 C) 18 D) 30 E) 42

17. $x, y, z \in Z$

$\left.\begin{array}{l} x^2-y^2=9 \\ x+y+z=18 \\ x(y+z)=65 \end{array}\right\} \Rightarrow$ **y = ?**

A) 3 B) 4 C) 5 D) 6 E) 9

YANITLAR / ANSWERS

1	2	3	4	5	6	7	8	9
E	B	A	B	D	B	E	C	A
10	11	12	13	14	15	16	17	
C	A	A	B	A	C	D	B	

1. $2x - [3x - (5x - 3) + 5] = 7 - [4 - (x + 2)] \Rightarrow$ **x = ?**

A) $\frac{8}{3}$ B) $\frac{10}{3}$ C) $\frac{11}{3}$ D) $\frac{13}{3}$ E) $\frac{14}{3}$

2. $\frac{1}{x+2} + \frac{1}{x-2} = \frac{12}{4-x^2} \Rightarrow$ **x = ?**

A) –6 B) –4 C) –2 D) 4 E) 6

3. $\frac{7}{x-2} = \frac{a}{a-b}$

$\frac{5}{x-2} = \frac{b}{a-b} \Rightarrow \frac{a}{b} = ?$

A) $\frac{6}{5}$ B) $\frac{7}{5}$ C) $\frac{8}{5}$ D) 6 E) 7

4. $\left(\frac{x-m}{n^2} - \frac{x-n}{m^2}\right) : \left(\frac{x}{m} - \frac{x}{n}\right) = \frac{-1}{x}$

$\Rightarrow$ **x = ?**

A) $m - n$ B) $m + n$ C) $m^2 + n^2$ D) $m^2 - n^2$ E) $m \cdot n$

5. $ax + 6y + 4 = 0$

$9x + 12y - b = 0,$

ÇK(SS) = R $\Rightarrow$ **a·b = ?**

A) –36 B) –18 C) –9 D) 18 E) 36

6. $\dfrac{2}{1 + \dfrac{1}{1 + \dfrac{1+x}{1 + \dfrac{1}{2}}}} = 1 \Rightarrow$ **x = ?**

A) –2 B) –1 C) 1 D) 2 E) 3

7. $x - y = y - z = 5$

$x + y + z = 42 \Rightarrow$ **x = ?**

A) 5 B) 10 C) 13 D) 15 E) 19

8. $\frac{1}{a} + \frac{1}{b} + \frac{1}{c} = 1$

$\frac{1}{a} - \frac{1}{b} + \frac{1}{c} = \frac{1}{3}$

$\frac{1}{a} - \frac{1}{c} = -\frac{1}{3} \Rightarrow$ **a + b + c = ?**

A) 9 B) 10 C) 11 D) 12 E) 13

9. $4a + 6b - 2c = 10$

$a + 2b - 2c = 6 \Rightarrow \mathbf{a + b + c = ?}$

A) –4 B) –2 C) –1 D) 1 E) 2

10. $4x - y = 10 + 2a$

$x - 2y = 6 - a \Rightarrow \mathbf{10y - 12x = ?}$

A) –44 B) –33 C) –22 D) 11 E) 22

11. $3(x - 1) + 2(2x - 3) - 9(x - 1) = -2x \Rightarrow$ **ÇK(SS) = ?**

A) ∅ B) {–2} C) {–1} D) {0} E) R

12. $6xy = x + y$

$3yz = y + z$

$\Rightarrow \mathbf{\frac{1}{z} - \frac{1}{x} = ?}$

A) –5 B) –3 C) –2 D) 1 E) 3

13. $2x + y = 4$

$x - y = 11$

$2x - y + a = -1$

ÇK(SS) = {(x, y)} ⇒ **a = ?**

A) –25 B) –23 C) –17 D) 23 E) 25

14. $\frac{x+3}{x+2} - \frac{x^2+2x-3}{x^2+x-2} = 0 \Rightarrow$ **ÇK(SS) = ?**

A) ∅ B) {0} C) {1, 2} D) R – {–2, 1} E) R

15. $\frac{x-1}{y-2} = \frac{2}{3}$

$\frac{y+1}{x-2} = 1 \Rightarrow$ **ÇK(SS) = ?**

A) {–7, –10} B) {–7, –5} C) {(–7, –5)} D) {(–5, –7)} E) {(–7, –10)}

16. $x > 0$,

$\frac{x}{2} + \frac{x}{3} = \frac{3}{x} + \frac{2}{x} \Rightarrow \mathbf{x = ?}$

A) $\sqrt{6}$ B) $\sqrt{3}$ C) 2 D) 3 E) 6

YANITLAR / ANSWERS

1	2	3	4	5	6	7	8	9
D	A	B	B	A	B	E	C	C
10	**11**	**12**	**13**	**14**	**15**	**16**		
A	E	B	C	D	E	A		

1. $xy + y = 15$

$\frac{1}{x+1} + \frac{1}{y} = \frac{3}{5} \Rightarrow \mathbf{x + y = ?}$

A) 7 B) 8 C) 9 D) 10 E) 12

2. $\frac{x}{x-y} + \frac{2}{x} = \frac{1}{3}$

$\frac{y}{y-x} - \frac{1}{x} = \frac{1}{2} \Rightarrow \mathbf{x = ?}$

A) –12 B) –8 C) –6 D) –3 E) –2

3. $2x + (a + 2)y = 4$

$(b - 3)x - 4y + 8 = 0$

$ÇK(SS) = R \Rightarrow \mathbf{a + b = ?}$

A) –2 B) –1 C) 0 D) 1 E) 2

4. $3x + y - 3z = 12$

$2x + y - 2z = -1 \Rightarrow \mathbf{y = ?}$

A) –27 B) –22 C) 22 D) 26 E) 28

5. $(4x - y - 10)\cdot m + (2x + 3y - 12)n = 0$

$m, n \in R \Rightarrow \mathbf{x + y = ?}$

A) 1 B) 2 C) 3 D) 4 E) 5

6. $\frac{4}{x} + \frac{3}{y} = 2$

$\frac{2}{x} - \frac{1}{y} = 6 \Rightarrow \mathbf{y = ?}$

A) –2 B) $-\frac{3}{2}$ C) –1 D) $-\frac{1}{2}$ E) 1

7. $\dfrac{\frac{x+1}{1-x} + \frac{x-1}{x+1}}{\frac{x+1}{1-x} - 1} = \frac{-3}{x-14} \Rightarrow \mathbf{x = ?}$

A) 3 B) 4 C) 5 D) 6 E) 7

8. $\frac{x}{y} + 1 = 0$

$\frac{x}{y} + \frac{2xz}{x-y} = 5 \Rightarrow \mathbf{z = ?}$

A) 3 B) 4 C) 5 D) 6 E) 8

9. $\frac{x}{3}+\frac{y}{6}=\frac{2}{3}$

$\frac{2x}{5}+\frac{y}{4}=\frac{1}{5} \Rightarrow \mathbf{x+y=?}$

A) –6 B) –4 C) –2 D) 4 E) 6

10. $3x + 2y = -1$

$5x + (3 - a)y = 5$

$ÇK(SS) = \varnothing \Rightarrow \mathbf{a = ?}$

A) $\frac{-1}{6}$ B) $\frac{-1}{4}$ C) $\frac{-1}{3}$ D) $\frac{1}{3}$ E) $\frac{1}{6}$

11. $\frac{m(x-m)}{n}+\frac{n(x-n)}{m}=x$

$\Rightarrow \mathbf{x = ?}$

A) m – n B) m + n C) n – m D) m·n E) $m^2 \cdot n^2$

12. $a + 3b + 2c = 5$

$2a + b - c = 8$

$3a + 4b - 3c = 7 \Rightarrow \mathbf{c = ?}$

A) 0 B) $\frac{1}{2}$ C) 1 D) $\frac{3}{2}$ E) 2

13. $2x^3+\frac{1}{x-2}=16-\frac{1}{2-x}$

$\Rightarrow$ **ÇK(SS) = ?**

A) {–2} B) {2} C) {–2, 2} D) {4} E) ∅

14. $mx + ny = m + n$

$mnx + mny = m^2 + n^2 \Rightarrow \mathbf{x = ?}$

A) $\frac{m}{n}$ B) $\frac{n}{m}$ C) m.n D) 1 E) 0

15. $\frac{1}{a}+\frac{3}{b}=15$

$\frac{1}{b}+\frac{3}{c}=17$

$\frac{1}{c}+\frac{3}{a}=12$

$ab + ac + bc = m(a \cdot b \cdot c) \Rightarrow \mathbf{m = ?}$

A) 11 B) 18 C) 22 D) 27 E) 33

16. $3-\cfrac{2}{2+\cfrac{x+2}{x+\cfrac{x}{4}}}=2 \Rightarrow \mathbf{x = ?}$

A) –6 B) –4 C) –2 D) –1 E) 1

YANITLAR / ANSWERS

1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
B	C	B	A	E	D	C	D	B
10	**11**	**12**	**13**	**14**	**15**	**16**		
C	B	D	E	B	A	C		

1. $\left.\begin{array}{l} c = 5a \\ 3a = 4b \\ 2ab = 3c \end{array}\right\} \Rightarrow \mathbf{a = ?}$

A) 2 B) 4 C) 8 D) 9 E) 10

2. $\left.\begin{array}{l} x + \frac{1}{y} = 3 \\ y + \frac{1}{x} = 6 \end{array}\right\}$ ve / and $2x+3y = 16 \Rightarrow \mathbf{x = ?}$

A) 1 B) 2 C) 3 D) 4 E) 5

3. $x, y, z \in Z^+$

$\left.\begin{array}{l} x \cdot y = 14 \\ y \cdot z = 7 \\ x \cdot z = 2 \end{array}\right\} \Rightarrow \mathbf{x \cdot y \cdot z = ?}$

A) 16 B) 15 C) 14 D) 12 E) 11

4. $x, y, z \in R$

$\left.\begin{array}{l} x^2 \cdot z = 4 \\ z^2 \cdot y = 8 \\ x \cdot y^2 = 2 \end{array}\right\} \Rightarrow \mathbf{x \cdot y \cdot z = ?}$

A) 3 B) 4 C) 5 D) 6 E) 8

5. $\dfrac{2}{1 + \dfrac{x-1}{x+3}} = 2 \Rightarrow \mathbf{x = ?}$

A) –2 B) –1 C) 0 D) 1 E) 2

6. $\dfrac{\dfrac{1}{1 - \dfrac{1}{x}}}{1 + \dfrac{1}{2}} = 1 \Rightarrow \mathbf{x = ?}$

A) 0 B) 1 C) 2 D) 3 E) 4

7. $\dfrac{xy + z}{z} + z - x = \dfrac{yz + x}{x}$

$\Rightarrow \mathbf{x = ?}$

A) $\dfrac{x+y}{x-y}$ B) $\dfrac{x.y}{x+y}$ C) $\dfrac{x+y}{x.y}$ D) $\dfrac{x-y}{x+y}$ E) $\dfrac{x.y}{x-y}$

8. $\left(1 - \frac{1}{4}\right) \cdot \left(1 - \frac{1}{9}\right) \cdot \left(1 - \frac{1}{16}\right) \cdot \ldots \cdot \left(1 - \frac{1}{x^2}\right) = \frac{4}{7}$

$\Rightarrow \mathbf{x = ?}$

A) 5 B) 7 C) 8 D) 9 E) 11

9. $\left.\begin{array}{l} 4ab = 5c \\ 2ac = 3b \\ 3bc = 24a \end{array}\right\} \Rightarrow \mathbf{a \cdot b \cdot c = ?}$

A) 12 B) 14 C) 15 D) 16 E) 18

10. $\left.\begin{array}{l} 3x + y = -2y \\ 2x + 3y = xy \end{array}\right\} \Rightarrow \mathbf{(x, y) = (?, ?)}$

A) {−1, 1} B) {−1, 2} C) {−1, −2} D) {1, −1} E) {2, −1}

11. $\left.\begin{array}{l} 4x + 5y = x^3 \\ 5x + 4y = y^3 \end{array}\right\} \Rightarrow \mathbf{x.y = ?}$

A) −2 B) −3 C) −4 D) −5 E) −6

12. $\left.\begin{array}{l} z \cdot (x + y) + z^2 = 62 \\ y \cdot (x + z) + y^2 = 51 \\ x \cdot (y + z) + x^2 = 143 \end{array}\right\} \Rightarrow \mathbf{x + y + z = ?}$

A) 12 B) 14 C) 16 D) 18 E) 24

13. $\left.\begin{array}{l} ab^2 = 9 \cdot (a + 8) \\ ab - 6 = 3a \end{array}\right\} \Rightarrow \mathbf{b = ?}$

A) 9 B) 10 C) 12 D) 14 E) 16

14. $\left.\begin{array}{l} a + b = 1 \\ 4 = 3a - c^2 + ab \end{array}\right\} \Rightarrow \mathbf{a = ?}$

A) 2 B) 3 C) 4 D) 5 E) 6

15. $a^2 + 4ac - 9b^2 + 6bc + 3c^2 = 48$

$3c - 3b = 4 - a$

$\Rightarrow \mathbf{a + c + 3b = ?}$

A) 10 B) 12 C) 16 D) 24 E) 48

16. $\left.\begin{array}{l} 2ab = a + b \\ 5ac = a + c \\ 9bc = b + c \end{array}\right\} \Rightarrow \mathbf{b = ?}$

A) $\frac{1}{3}$ B) 3 C) $\frac{7}{2}$ D) 5 E) 6

YANITLAR / ANSWERS

1	2	3	4	5	6	7	8	9
E	B	C	B	D	D	E	B	C
10	**11**	**12**	**13**	**14**	**15**	**16**		
D	D	C	A	A	B	A		

BÖLÜM
CHAPTER
8

TARİHSEL NOT / HISTORICAL NOTE

Aristarchus of Samos [about 270 BC]

Yunan gök bilimci, Dünya'nın Güneş etrafındaki dönüşünü açıklayan ilk astronomlardan biridir. Güneş ve Ay'ın Dünya'ya olan uzaklıklarını bulmak için geometrik yöntemler kullanmıştır.

Greek astronomer, noted for being the first to affirm that the Earth rotates and travels around the Sun. He treated astronomy mathematically and used geometrical methods to calculate the relative sizes of the Sun and Moon and their relative distances from the Earth.

BİRİNCİ DERECEDEN EŞİTSİZLİKLER
FIRST DEGREE INEQUALITIES

Bu bölüm, 50 test sorusu içermektedir.
This chapter includes 50 test questions.

BÖLÜM / CHAPTER 8

BİRİNCİ DERECEDEN EŞİTSİZLİKLER / FIRST DEGREE INEQUALITIES

- Birinci Dereceden Eşitsizlikler / First Degree Inequalities .... 169 – 180

BÖLÜM
08
CHAPTER

BİRİNCİ DERECEDEN EŞİTSİZLİKLER
FIRST DEGREE INEQUALITIES

BİRİNCİ DERECEDEN EŞİTSİZLİKLER / FIRST DEGREE INEQUALITIES

Tanım: $a \neq 0$, a ve b birer reel sayı olmak üzere,
$ax + b > 0$, $ax + b \geq 0$, $ax + b < 0$, $ax + b \leq 0$ biçimindeki ifadelere ***birinci dereceden eşitsizlikler*** denir. Eşitsizliği sağlayan sayıların oluşturduğu kümeye ***çözüm kümesi*** denir. Eşitsizliklerin çözüm kümeleri reel sayı aralıkları ile gösterilir.

Definition: Provided that $a \neq 0$, a and b are real numbers, the expressions in $ax + b > 0$, $ax + b \geq 0$, $ax + b < 0$, $ax + b \leq 0$ forms are called *first degree inequations.* The set of all real numbers that make the *inequation* valid is the solution set of the *inequation*. The *solution set of inequation* are shown by intervals.

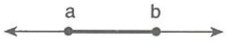
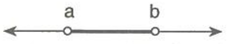
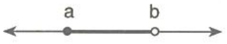
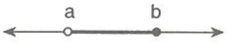
Sembol / Symbol	Anlamı / Meaning
=	eşit / is equal to
≠	eşit değil / is not equal to
<	küçük / is less than
>	büyük / is greater than
≤	küçük veya eşit / is less than or equal to
≥	büyük veya eşit / is greater than or equal to

Önemli / Important

- <, >, ≤, ≥ sembolleri, **eşitsizlik sembolleri** olarak adlandırılır.
  The symbols <, >, ≤, ≥ are called **inequality symbols.**

REEL SAYI DOĞRUSU ÜZERİNDE SINIRLI ARALIKLAR / BOUNDED INTERVALS on REAL NUMBER LINE

$a, b \in R$, $a < b$ olamak üzere / is

Gösterim Notation	Aralık türü Interval Type	Eşitsizlik Inequality	Şekilsel Gösterim Representation
[a, b]	kapalı / closed	$a \leq x \leq b$	
(a, b)	açık / open	$a < x < b$	
[a, b)	yarı açık / half open	$a \leq x < b$	
(a, b]	yarı açık / half open	$a < x \leq b$	

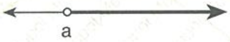
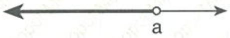
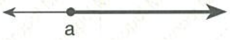
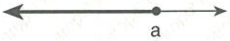
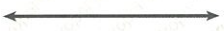
Önemli / Important

- (a,b) açık aralığı, koordinat sistemindeki x ekseni a, y ekseni b olan sıralı ikili ile karıştırılmamalıdır.
- The open interval (a,b), should not to be confused with a point in coordinate system with abscissa a and ordinate b.

REEL SAYI DOĞRUSU ÜZERİNDE SINIRSIZ ARALIKLAR / UNBOUNDED INTERVALS on the REAL NUMBER LINE

▼ a reel sayısını ele alalım. Reel sayı doğrusu üzerinde aşağıdaki aralıklar sınırsızdır.

▼ Let a be a real number. The following intervals on the real number line are unbounded.

Gösterim Notation	Aralık türü Interval Type	Eşitsizlik Inequality	Şekilsel Gösterim Representation
(a, ∞)	açık / open	$a < x$	
(−∞, a)	açık / open	$x < a$	
[a, ∞)	yarı açık / half open	$a \leq x$	
(−∞, a]	yarı açık / half open	$x \leq a$	
(−∞, ∞)	tüm reel eksen / entire real line		

Önemli / Important

- Bir kapalı aralık, bitim noktalarının ikisini de; yarı açık aralık, bitim noktalarının sadece birisini içerir. Açık aralık, bitim noktalarının hiçbirini içermez.
- A closed interval contains both of its endpoints, a half open interval contains only one of its endpoints, and an open interval does not contain either of its endpoints.

EŞİTSİZLİKLERİN ÖZELLİKLERİ / PROPERTIES of INEQUATIONS

Özellik / Property

★ **Üç hal özelliği:** a ve b reel sayıları için $a = b$, $a > b$, $a < b$ durumlarından birisi geçerlidir.

The Trichotomy property: For real numbers a and b, one and only one of the following is true: $a = b$, $a > b$, $a < b$

Özellik / Property

★ $a, b, c \in \mathbb{R}$,

$a < b \Rightarrow a \mp c < b \mp c$

Önemli / Important

- Toplama / Çıkarma özelliği, ≤ ve ≥ sembolleri için de her zaman geçerlidir.
- The Addition / Subtraction property, is also always true for an inequation that contains the symbol ≤ or ≥.

Önemli / Important

- Çarpma / Bölme özelliği, ≤ ve ≥ sembolleri için de her zaman geçerlidir.
- The multiplication / division property of inequations is also always true for the symbols ≤ and ≥.

Özellik / Property

★ **I.** $a, b \in R, \quad c \in R^+$

$a < b \Rightarrow a \cdot c < b \cdot c$

II. $a, b \in R, \quad c \in R^-$

$a < b \Rightarrow a \cdot c > b \cdot c$

Özellik / Property

★ $$\left.\begin{array}{l} a < b \\ c < d \end{array}\right\} \Rightarrow a \mp c < b \mp d$$

Önemli / Important

- Eşitsizliklerde taraf tarafa toplama ve çıkarma yapılabilir, taraf tarafa çarpma ve bölme yapılamaz.
- In equalities adding and subtracting respective sides can be done, but multiplying and dividing respective sides can not.

Önemli / Important

- Eşitsizliklerde taraf tarafa toplama aynı türden (ikisi de küçük veya ikisi de büyük) eşitsizliklerde yapabilir.
- In equalities adding and subtracting respective sides can be done if both the inequalities are from same type of inequalities, (both greater or both less).

Özellik / Property

★ **Geçişme özelliği:** $a < b$ ve $b < c \Rightarrow a < c$

The Transitive property: $a < b$ and $b < c \Rightarrow a < c$

Özellik / Property

★ **I.** $a, b \in R,\ a \cdot b > 0$

$a < b \Rightarrow \frac{1}{a} > \frac{1}{b}$

II. $a, b \in R,\ a \cdot b < 0$

$a < b \Rightarrow \frac{1}{a} < \frac{1}{b}$

Özellik / Property

★ n pozitif bir tamsayı ve $a < b$ ise $a^{2n+1} < b^{2n+1}$ dir.

If n is a positive integer and $a < b$ then $a^{2n+1} < b^{2n+1}$.

Önemli / Important

$x, y \in Z$

- x ve y en büyük ise, x + y en büyüktür.
- x ve y en küçük ise, x + y en küçüktür.
- x en küçük, y en büyük ise, x – y en küçüktür. Çünkü bu işlem çıkarma işlemidir.
- x en büyük, y en küçük ise, x – y en büyüktür. Çünkü bu işlem çıkarma işlemidir.

$x, y \in Z$

- The expression x + y is maximum if x and y are maximum.
- The expression x + y is minimum if x and y are minimum.
- The expression x – y is minimum if x is minimum, y is maximum. Because this operation is subtraction.
- The expression x – y is maximum if x is maximum, y is minimum. Because this operation is subtraction.

EŞİTSİZLİK SİSTEMLERİ / SYSTEMS of INEQUALITIES

▼ Eşitsizlik sistemleri çözülürken, eşitsizlikler ayrı ayrı çözülür, daha sonra bulunan çözüm kümelerinin kesişimi alınır.

▼ To solve systems of ineqalities, solve inequalities one by one then take the intersection of solutions.

1. $y < 0,\ \dfrac{x}{0,3} = \dfrac{y}{0,4} = 5z$

$\Rightarrow$ **? < ? < ?**

A) $y < x < z$

B) $x < z < y$

C) $x < y < z$

D) $z < x < y$

E) $z < y < x$

2. $\left.\begin{array}{l} a^3 \cdot b^2 > 0 \\ a \cdot c < 0 \\ 3b = 5c \end{array}\right\} \Rightarrow$ **?<?<?**

A) $a < b < c$

B) $b < a < c$

C) $b < c < a$

D) $a < c < b$

E) $c < b < a$

3. $a \in Z$

$2a - 5 < 6 \Rightarrow$ **max(a) =?**

A) 2 B) 3 C) 4 D) 5 E) 6

4. $\left.\begin{array}{l} x \in Z \\ 3 - x < \dfrac{x+1}{2} \\ 2x - 5 \leq 2 \end{array}\right\} \Rightarrow \sum x =$ **?**

A) 5 B) 6 C) 7 D) 8 E) 9

5. $\left.\begin{array}{l} x \in Z \\ -\dfrac{1}{6} \leq \dfrac{3-x}{24} < \dfrac{3}{8} \end{array}\right\} \Rightarrow \sum x =$ **?**

A) 11 B) 12 C) 13 D) 14 E) 15

6. $x + 2 \leq 2x + 4 < x + 8 \Rightarrow$ **ÇK(SS) = ?**

A) [–2, 3]

B) [–2, 4)

C) [–1, 5)

D) [–1, 4)

E) [0, 2)

7. $\dfrac{1}{a} < 0 < \dfrac{1}{b} < \dfrac{1}{c} \Rightarrow$ **? < ? < ?**

A) $a < b < c$

B) $a < c < b$

C) $b < c < a$

D) $b < a < c$

E) $c < b < a$

8. $-1 < x < y < 0 \Rightarrow$ **? < $\dfrac{y}{x}$ < ?**

A) $-1 < \dfrac{y}{x} < 0$

B) $-2 < \dfrac{y}{x} < -1$

C) $0 < \dfrac{y}{x} < 1$

D) $1 < \dfrac{x}{y} < 1$

E) $-3 < \dfrac{x}{y} < -2$

9. $$\left.\begin{array}{l} x, y \in Z \\ -2 < x < 5 \\ -4 < y < 6 \end{array}\right\} \Rightarrow \max(2y-3x) = ?$$

A) 10 B) 11 C) 12 D) 13 E) 14

10. $$\left.\begin{array}{l} 2b - a = 5 \\ -3 < b \leq 3 \end{array}\right\} \Rightarrow a \in ?$$

A) $(-6, 0)$ B) $(-9, 0)$ C) $(-9, 1]$ D) $(-10, 1)$ E) $(-11, 1]$

11. $$3 - \frac{a+1}{2} \leq 4 \Rightarrow a \geq ?$$

A) -3 B) -4 C) -5 D) -6 E) -7

12. $x \in Z$

$$-1 \leq \frac{3-2x}{4} \leq 5 \Rightarrow \sum x = ?$$

A) -30 B) -22 C) -16 D) -9 E) 0

13. $$\left.\begin{array}{l} -5 < x < 4 \\ -7 < y < -2 \end{array}\right\} \Rightarrow \max(x^2 + y^2) = ?$$

A) 70 B) 73 C) 75 D) 77 E) 80

14. $$\left.\begin{array}{l} a, b \in Z \\ -3 < a < 2 \\ 0 \leq b < 7 \end{array}\right\} \Rightarrow \max(a^2 - b) = ?$$

A) 1 B) 3 C) 4 D) 9 E) 12

15. $$\left.\begin{array}{l} x \in Z \\ 2x + 5y = 1 \\ 1 < y < 2 \end{array}\right\} \Rightarrow \sum y = ?$$

A) 3,2 B) 3,5 C) 3,8 D) 4,1 E) 4,8

16. $$\left.\begin{array}{l} -\frac{4}{5} < a < -\frac{1}{2} \\ x = \frac{3a+4}{a+1} \end{array}\right\} \Rightarrow ? < x < ?$$

A) $5 < x < 6$ B) $5 < x < 8$ C) $6 < x < 7$ D) $6 < x < 8$ E) $7 < x < 8$

17. $$\left.\begin{array}{l} x, y \in Z \\ -3 < x < 2 \\ -5 < y < 4 \end{array}\right\} \Rightarrow \max[x \cdot (x-y)] = ?$$

A) 6 B) 8 C) 9 D) 10 E) 12

YANITLAR / ANSWERS

1	2	3	4	5	6	7	8	9
A	C	D	A	C	B	B	C	D
10	**11**	**12**	**13**	**14**	**15**	**16**	**17**	
E	A	A	B	C	A	B	D	

1. $x \in Z$

$\frac{x}{3} - \frac{2x-1}{4} < 0 \Rightarrow$ **min(x) = ?**

A) –2 B) –1 C) 1 D) 2 E) 3

2. $\frac{3x-1}{2} - \frac{x+2}{3} \le \frac{2x+1}{3} \Rightarrow$ **ÇK(SS) = ?**

A) $(-\infty, -5]$ B) $[5, \infty)$ C) $(-\infty, 3)$ D) $[3, \infty)$ E) $(-\infty, 3]$

3. $\frac{2x-4}{x} + 1 \le 2 - \frac{1}{x} \Rightarrow$ **ÇK(SS) = ?**

A) ∅ B) [0, 3) C) (0, 3] D) {0} E) {3}

4. $\left.\begin{matrix} 2 < 3x-4 < 5 \\ -2 < 2-x < 1 \end{matrix}\right\} \Rightarrow$ **ÇK(SS) = ?**

A) (1, 4) B) (2, 4) C) (2, 3) D) (–1, 0) E) (–2, –1)

5. $x^2y^3 < 0,\ y^2z^3 < 0,\ x^3z^5 > 0$

$\Rightarrow$ **x, y, z = ? , ? , ?**

A) –, –, – B) + , – , + C) –, + , – D) – , – , + E) –, +, +

6. $x \in R$

$\left.\begin{matrix} 2 < \frac{x-1}{3} \le 4 \\ -1 \le \frac{2x-1}{5} \le 3 \end{matrix}\right\} \Rightarrow$ **ÇK(SS) = ?**

A) [7, 8) B) (7, 8] C) [4, 8) D) (4, 8) E) (4, 8]

7. $x - 1 \le -3x + 2 \le x + 6 \Rightarrow$ **ÇK(SS) = ?**

A) $\left(-1\frac{3}{4}\right)$ B) $\left(-1\frac{3}{4}\right]$ C) $(-1, 1)$ D) $\left[-1, \frac{3}{4}\right]$ E) $\left[-1, -\frac{3}{4}\right]$

8. $4x \le 6 - 2(1 - x) < 3x + 2 \Rightarrow$ **ÇK(SS) = ?**

A) [–1, 4) B) (–∞, –1) C) (4, ∞) D) R E) ∅

9. $\frac{x-5}{2}+x>3-\frac{4-x}{2} \Rightarrow$ **ÇK(SS) = ?**

A) $\left(-\infty,-\frac{7}{2}\right)$ B) $\left(-\infty,\frac{7}{2}\right)$ C) $\left(\frac{7}{2},\infty\right)$

D) R E) ∅

10. $\left.\begin{array}{r}4x-16<0\\ 6x+18>0\end{array}\right\} \Rightarrow$ **ÇK(SS) = ?**

A) $(-3, 4)$ B) $[-2, 2)$ C) $(0, 4)$

D) $(-3, 0)$ E) ∅

11. $3x-x^2 \in Z$

$-2<x\leq 4 \Rightarrow$ **$\min(4x-x^2)$ = ?**

A) –4 B) –11 C) –15 D) –18 E) –21

12. $y \neq 0$

$\frac{5x+y}{3y}=0,\ 4yz=0$

$4x-z=12 \Rightarrow$ **y = ?**

A) –5 B) –6 C) –10 D) –12 E) –15

13. $\sqrt{2-\frac{x+1}{3}} \Rightarrow$ **$x \in$?**

A) $x \geq 5$ B) $x \leq 5$ C) $x \geq 4$

D) $x \leq 4$ E) $-5 \leq x \leq 5$

14. $x \in Z$

$4-\frac{12}{x-2} \leq 0 \Rightarrow$ **$\sum x$ = ?**

A) 16 B) 14 C) 12 D) 6 E) 4

15. $\frac{(x^2+1)\cdot(x-2)}{(x+5)} \geq 0 \Rightarrow$ **ÇK(SS) = ?**

A) $[-5, 2]$

B) $(-\infty, -5) \cup [2, \infty)$

C) $(-\infty, -5] \cup [2, \infty)$

D) $[-\infty, -5] \cup (2, \infty)$

E) $(-5, 2]$

16. $\frac{(x+1)\cdot(-x+4)}{(x-2)^2} \geq 0 \Rightarrow$ **ÇK(SS) = ?**

A) $[1, 4]$ B) $[-\infty, -1] \cup [4, \infty)$ C) $(1, 4]$

D) $[-1, 4]-\{2\}$ E) $(-5, 2]$

17. $\frac{x-8}{|x^3-8|}<0 \Rightarrow$ **ÇK(SS) = ?**

A) $(-\infty, 2)$ B) $(2, \infty)$ C) $[-2, 2]$

D) $(-2, 8)$ E) $(-\infty, 8)-\{2\}$

YANITLAR / ANSWERS

1	2	3	4	5	6	7	8	9
D	E	C	C	A	B	D	E	C
10	11	12	13	14	15	16	17	
A	B	E	B	C	B	D	E	

1. $x, y \in R$,

$-4 < x < 2$,

$3 < y < 4 \Rightarrow$ **$? < 3x - 2y < ?$**

A) (–10, 8) B) (–10, 6) C) (–20, 8)
D) (–20, 6) E) (–20, 0)

2. $a, b \in Z$,

$-3 \leq a \leq 2$,

$-1 < b < 3 \Rightarrow$ **$\max(3a^2 - 2b^2) = ?$**

A) 12 B) 16 C) 24 D) 27 E) 32

3. $x, y, z \in R$,

$x^2 \cdot y < 0$,

$y^2 \cdot z < 0$,

$x \cdot y < 0$,

$5y = 7z \Rightarrow$ **$? < ? < ?$**

A) $x < y < z$ B) $z < y < x$ C) $y < z < x$
D) $y < x < z$ E) $z < x < y$

4. $x \in Z$,

$2(x + 1) + 3(x - 2) \leq 6 \Rightarrow$ **$\max(x) = ?$**

A) 0 B) 2 C) 3 D) 4 E) 6

5. $a, b \in R$,

$-2 < a < 4$,

$a + 2b = 2 \Rightarrow$ **$? < b < ?$**

A) (–1, 2) B) (–2, 1) C) (–3, 1)
D) (–3, –1) E) (–4, –2)

6. $x^2 < x$,

$6x + 1 = A, A \in Z$

$\Rightarrow$ **$\sum A = ?$**

A) 15 B) 18 C) 20 D) 24 E) 32

7. $2^a = 18$,

$3^b = 55$,

$5^c = 121 \Rightarrow$ **$? < ? < ?$**

A) $a < b < c$ B) $c < b < a$ C) $c < a < b$
D) $a < c < b$ E) $b < a < c$

8. $x, y \in R$,

$-5 \leq x \leq 1$,

$2 \leq y < 6$,

$x^2 + y^2 = A$,

$A \in Z \Rightarrow$ **$\max(A) = ?$**

A) 48 B) 54 C) 56 D) 60 E) 64

9. $x < x^3 < x^2$,

$\frac{3x+1}{5-y} = \frac{1}{2}$,

$y \in Z \Rightarrow \sum y = ?$

A) 25 B) 28 C) 30 D) 36 E) 42

10. $b \in Z$,

$-1 \le \frac{2a+1}{3} < 5$

$2a - 3b = -1 \Rightarrow \sum b = ?$

A) 8 B) 9 C) 10 D) 11 E) 12

11. $x, y \in R$,

$-3 < x < 2$,

$-4 < y < 3$,

$x^2 + 2y^2 = A$,

$A \in Z \Rightarrow$ **max(A) = ?**

A) 36 B) 38 C) 40 D) 42 E) 44

12. $7 < \frac{4a+21}{a} < 11$

$a \in Z \Rightarrow \sum a = ?$

A) 10 B) 12 C) 13 D) 15 E) 19

13. $2^{2x-5} = 15$,

$6x - y = 5$

$y \in Z \Rightarrow \sum y = ?$

A) 40 B) 41 C) 45 D) 48 E) 54

14. $x \in Z^+$,

$y, z \in R^+$,

$x < y < z$,

$\frac{8}{x} - \frac{3}{y} + \frac{5}{z} = \frac{1}{4} \Rightarrow$ **max(x) = ?**

A) 35 B) 36 C) 39 D) 40 E) 45

15. $c \in Z$,

$a, b \in R$,

$-2 \le a < 6$,

$0 < b \le 7$,

$2b - 3a + c = 5 \Rightarrow \sum c = ?$

A) 125 B) 128 C) 130 D) 133 E) 135

16. $x, y \in Z$,

$\frac{x+y}{y} = \frac{5}{4}$, $21 < x + y < 44$

$x - y = A \Rightarrow$ **min(A) = ?**

A) –24 B) –16 C) –8 D) 20 E) 24

YANITLAR / ANSWERS

1	2	3	4	5	6	7	8
E	D	C	B	A	C	B	D
9	**10**	**11**	**12**	**13**	**14**	**15**	**16**
C	B	C	D	B	C	D	A

BÖLÜM
CHAPTER
9

TARİHSEL NOT / HISTORICAL NOTE

Banach Stefan [1892 - 1945]

Leh matematikçi, Fonsiyonel analiz konusuna katkıları olan ilk kişilerdendir.

Polish mathematician who was a major contributor to the subject known as functional analysis.

MUTLAK DEĞER
ABSOLUTE VALUE

Bu bölüm, 73 test sorusu içermektedir.

This chapter includes 73 test questions.

BÖLÜM / CHAPTER 9

MUTLAK DEĞER / ABSOLUTE VALUE

- Mutlak Değer / Absolute Value .... 181 – 192

BÖLÜM

09

CHAPTER

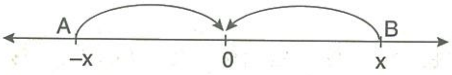
MUTLAK DEĞER
ABSOLUTE VALUE

MUTLAK DEĞER / ABSOLUTE VALUE

Tanım: Sayı doğrusu üzerinde, x sayısının sıfıra olan uzaklığına, x sayısının ***mutlak değeri*** denir ve **|x|** ile gösterilir.

Definition: *The **absolute value*** of a number is its distance from zero on the number line. The absolute value of the number x is denoted by **|x|**.

$$|0B| = |x| , \quad |0A| = |-x|$$



Buna göre,

$$|x| = \begin{cases} x, & x > 0 \text{ ise} \\ 0, & x = 0 \text{ ise} \\ -x, & x < 0 \text{ ise} \end{cases}$$

olarak tanımlanır.

According to that, |x| is defined.

Özellik / Property

★ Herhangi bir reel sayının mutlak değeri ya **pozitiftir** ya da **sıfırdır**.

The absolute value of any real number **is either positive** or **zero.**

★ $a, b \in R^{+}$, $a > b$,

$|a - b| = |b - a| = a - b$

★ $|a \cdot b| = |a| \cdot |b|$

★ $\left|\frac{a}{b}\right| = \frac{|a|}{|b|}$ $(b \neq 0)$

★ $|a + b| \leq |a| + |b|$

★ $A = |f(x)| \Rightarrow \min(A) = 0$

★ $a, b, c \in R$

$|a| + |b| + |c| = 0$

$\Rightarrow a = 0, \ b = 0, \ c = 0$

★ $a \in R^-$, n tek bir doğal sayı ise $|a^n| \neq a^n$, n çift bir doğal sayı ise $|a^n| = a^n$ dir.

$a \in R^-$, if n is odd number, then $|a^n| \neq a^n$, if n is even number, then $|a^n| = a^n$.

Özellik / Property

★ $\forall x \in R$ için / for $\sqrt{(f(x))^2} = |f(x)|$

MUTLAK DEĞERLİ DENKLEMLER / EQUATIONS CONTAINING ABSOLUTE VALUE

▼ $a \in R^+$, $|f(x)| = a \Rightarrow$ $f(x) = a$ **veya / or** $f(x) = -a$

▼ $|f(x)| = g(x)$

$$\begin{tikzcd} & {|f(x)| = g(x)} \arrow[dl] \arrow[dr] & \\ \text{I. } f(x) = g(x) & \text{veya / or} & \text{II. } f(x) = -g(x) \end{tikzcd}$$

▼ $|f(x)| = |g(x)| \Rightarrow$ $f(x) = g(x)$ **veya / or** $f(x) = -g(x)$

Önemli / Important

- $|f(x)| = g(x)$ denkleminin kökleri, mutlak değerli ifadeler sıfırdan küçük olmayacağı için denklemde yerine yazılarak denenmelidir.
- Roots of the equation $|f(x)| = g(x)$ should be tested by replacing in the equation as the absolute value expressions can not be less than zero.

MUTLAK DEĞERLİ EŞİTSİZLİKLER / INEQUALITIES of ABSOLUTE VALUE

▼ $a \in R^+$
$|f(x)| < a \Rightarrow$ $-a < f(x) < a$

▼ $a \in R^+$, $|f(x)| > a \Leftrightarrow$ $f(x) > a$ **veya / or** $f(x) < -a$

▼ $a, b \in R^+$, $a < b$,

$$\begin{tikzcd} & {a < |f(x)| < b} \arrow[dl] \arrow[dr] & \\ a < f(x) < b & \text{veya / or} & -b < f(x) < -a \end{tikzcd}$$

▼ $a, b \in R - \{0\}$, $|f(x)| < |g(x)| \Rightarrow$ $f^2(x) < g^2(x)$

1. $|\sqrt{5}-4|+|4+\sqrt{5}|=?$

A) 1
B) $1+2\sqrt{5}$
C) $1-2\sqrt{5}$
D) $7-2\sqrt{5}$
E) 8

2. $x<0$,

$\Rightarrow |-2x|+\sqrt{x^2}+|5-x|=?$

A) $5-4x$
B) 5
C) $2x-5$
D) $5-2x$
E) $5+2x$

3. $a<0<b$,

$\Rightarrow \sqrt{a^2-2ab+b^2}+|3a-2b|+4a-b=?$

A) $a-b$
B) $2a+b$
C) $2a-b$
D) $2b$
E) $8a-2b$

4. $|2x-6|+|3-x|=12 \Rightarrow$ **ÇK(SS) = ?**

A) $\{-1,7\}$
B) $\{1,7\}$
C) $\{-7,1\}$
D) $\{7\}$
E) $\{-1\}$

5. $a<|a|$,

$$\Rightarrow \frac{|a-3|+|2a|+3a}{|2-\sqrt{5}|+|\sqrt{5}-4|}=?$$

A) $-\frac{2}{3}$
B) $-\frac{3}{2}$
C) $\frac{2}{3}$
D) $\frac{3}{2}$
E) $\frac{5}{2}$

6. $x<2$,

$\Rightarrow \sqrt{x^2-4x+4}+|x-3|+2x=?$

A) $4x-5$
B) $2x-5$
C) 5
D) $2x+5$
E) $4x+5$

7. $a^2<a$,

$\Rightarrow \sqrt{a^2-6a+9}+|2a-2|=?$

A) $3a-2$
B) $5-a$
C) $a-4$
D) $3a-4$
E) $5-3a$

8. $|x+3|=2x-1 \Rightarrow x=?$

A) $-\frac{2}{3}$
B) 0
C) $\frac{2}{3}$
D) 3
E) 4

9. $\frac{1}{2}<x<3$,

$\Rightarrow \sqrt[2004]{(x-3)^{2004}}+|2x-1|=?$

A) $x+2$
B) $3x-4$
C) $x-4$
D) $2-x$
E) $4-x$

10. $|x-3|+y^2-4y+5=1 \Rightarrow x\cdot y=?$

A) -4
B) 0
C) 2
D) 3
E) 6

11. $|5 - 2x| = x - 3 \Rightarrow$ **ÇK(SS) = ?**

A) $\{2\}$ B) $\left\{\frac{2}{3}\right\}$ C) $\left\{2, \frac{2}{3}\right\}$

D) $\{2, 4\}$ E) $\varnothing$

12. $|2x + 3y - 12| + |2x - y - 4| = 0 \Rightarrow$ **$x \cdot y$ = ?**

A) –6 B) –4 C) 0 D) 6 E) 8

13. $|x - 3y| + (x + 2y - 10)^{2004} = 0 \Rightarrow$ **(x,y) = ?**

A) (–2, 3) B) (–2, 4) C) (2, –6)

D) (6, –2) E) (6, 2)

14. $|x - 3| = |2x + 1| \Rightarrow \sum$ **x = ?**

A) $-\frac{10}{3}$ B) $-\frac{5}{3}$ C) $\frac{5}{3}$ D) $\frac{10}{3}$ E) 4

15. $5 < |3x - 1| < 8 \Rightarrow$ **ÇK(SS) = ?**

A) $\left(-\frac{7}{3}, 3\right)$ B) $\left(-\frac{7}{2}, 2\right)$ C) (2, 3)

D) $\left(-\frac{7}{3}, -\frac{4}{3}\right) \cup (2, 3)$ E) $\left(-\frac{4}{3}, -\frac{2}{3}\right)$

16. $|16 - x^2| = |x - 4| \Rightarrow \sum$ **x = ?**

A) –8 B) –4 C) 0 D) 4 E) 8

17. $|x - 1| \le |x + 2| \Rightarrow$ **ÇK(SS) = ?**

A) $(-2, \infty)$ B) $\left[\frac{-1}{2}, \infty\right)$ C) $\left(\frac{1}{2}, \infty\right)$

D) $\left(-\infty, -\frac{1}{2}\right)$ E) $\left(-\infty, -\frac{1}{2}\right]$

18. $|x - 2|^{x^2 - 4} = 1 \Rightarrow \prod$ **x = ?**

A) –12 B) –6 C) –4 D) 3 E) 6

19. $(x - 1)^2 + |1 - x| - 2 = 0 \Rightarrow$ **ÇK(SS) = ?**

A) {0} B) {2} C) {0, 2}

D) {0, –2} E) {0, 4}

YANITLAR / ANSWERS

1	2	3	4	5	6	7	8	9	10
E	A	D	A	D	C	E	E	A	E
11	12	13	14	15	16	17	18	19	
E	D	E	A	D	B	B	B	C	

1. $3\cdot|x+3|+27=0 \Rightarrow$ **ÇK(SS) = ?**

A) Ø B) {6} C) {7} D) {12} E) {−12}

2. $\left|\dfrac{2x-3}{3}\right|=5 \Rightarrow \sum x = ?$

A) 3 B) 4 C) 5 D) 6 E) 9

3. $\left.\begin{array}{l}|x-1|=a\\|a-2|=4\end{array}\right\} \Rightarrow x_1\cdot x_2=?$

A) 12 B) −12 C) 35 D) −35 E) 24

4. $|x^2-3x-4|=|x-4| \Rightarrow \sum x = ?$

A) −2 B) 0 C) 2 D) 4 E) 6

5. $|2x-4|-|x+2|=-1 \Rightarrow \sum x = ?$

A) 13 B) 12 C) 6 D) 5 E) 0

6. $\left.\begin{array}{l}a<0\\\big|a+|2a-5|\big|+\sqrt{a^2}=7\end{array}\right\} \Rightarrow a=?$

A) −1 B) −2 C) −3 D) −4 E) −5

7. $\left.\begin{array}{l}x\in Z\\|5x-4|<12\end{array}\right\} \Rightarrow \max(8x-3)=?$

A) 13 B) 16 C) 19 D) 21 E) 22

8. $\big||2x-1|-5\big|=4 \Rightarrow \sum x = ?$

A) 0 B) 1 C) 2 D) 3 E) 4

9. $\sqrt{x^2+4x+4}=|2x+4|-18 \Rightarrow \sum x = ?$

A) 16 B) 6 C) 4 D) −4 E) −6

10. $|x^2-9|+2\,|x+3|=0 \Rightarrow \;\Rightarrow \sum x = ?$

A) 4 B) 2 C) 1 D) −3 E) −5

11. $\left.\begin{array}{l} x > -4 \\ \sqrt{x^2-4x+4} = 2-x \end{array}\right\} \Rightarrow \sum x = ?$

A) 3 B) 2 C) –3 D) –4 E) –5

12. $\left.\begin{array}{l} 2 < x < 7 \\ |2x-3| + |x-13| + 2x = 19 \end{array}\right\} \Rightarrow x = ?$

A) 2 B) 3 C) 4 D) 5 E) 6

13. $\left.\begin{array}{l} -1 < x < 3 \\ |x+4| - |x-5| - |x-7| = -2 \end{array}\right\} \Rightarrow x = ?$

A) 0 B) 1 C) 2 D) 3 E) 4

14. $\left.\begin{array}{l} a > 0 \\ |x-2a| = 3 \\ x_1.x_2 = 55 \end{array}\right\} \Rightarrow a = ?$

A) 3 B) 4 C) 5 D) 6 E) 8

15. $\left.\begin{array}{l} 3a + b > 0 \\ |x-a-b| = 3a + b \\ x_1 + x_2 = 8 \end{array}\right\} \Rightarrow a + b = ?$

A) 4 B) 6 C) 8 D) 12 E) 18

16. $x \in Z^-$

$x + 2 \cdot |x - 1| = 7 \Rightarrow x = ?$

A) –12 B) –9 C) –6 D) –5 E) –4

17. $x \in Z$

$|2x - 3| < 11 \Rightarrow \sum x = ?$

A) 15 B) 14 C) 13 D) 11 E) 10

18. $\left.\begin{array}{l} \left|\dfrac{2x+1}{2}\right| < 1 \\ a < x < b \end{array}\right\} \Rightarrow 2a^2 - b + 1 = ?$

A) – 1 B) $\frac{1}{3}$ C) $\frac{8}{3}$ D) 4 E) 5

19. $\big||2x+1| + 6\big| = 5 \Rightarrow$ ÇK(SS) = ?

A) {3} B) {4} C) {5} D) {8} E) ∅

YANITLAR / ANSWERS

1	2	3	4	5	6	7	8	9	10
A	A	D	C	C	A	D	C	D	D
11	12	13	14	15	16	17	18	19	
C	B	C	B	A	D	A	E	E	

1. $x^2 < x$, $|x \cdot y| = -x \cdot y$

$\Rightarrow$ **$|y - x| - |y| + |x| = ?$**

A) $-2x$ B) $-2y$ C) $2x$ D) $2y$ E) 0

2. $A \in \mathbb{R}$,

$A = |3x| + |2012 - 4x| - \sqrt{-x^2 + 6x - 9}$

$\Rightarrow$ **$A = ?$**

A) 2009 B) 2010 C) 2011 D) 2012 E) 2024

3. $a < 0$,

$\Rightarrow \dfrac{a}{|a|} + \dfrac{|-2a|}{a} + \dfrac{5|a|}{|5a|} = ?$

A) -4 B) -2 C) 0 D) 2 E) 4

4. $a < 0 < b$

$\Rightarrow$ **$|a - b| + |b - 2a| + |3a - b| - |b - 3a| = ?$**

A) $3a - 2b$ B) $2b - 3a$ C) $a - b$ D) $a + b$ E) 0

5. $a^2 < a$,

$\Rightarrow \dfrac{|-a|}{|a+1| - |a-1|} = ?$

A) $\frac{1}{2}$ B) 1 C) 2 D) a E) $2a$

6. $A = |3x - 2| + |5x - 3|$

$\Rightarrow$ **$\min(A) = ?$**

A) $\frac{1}{6}$ B) $\frac{1}{5}$ C) $\frac{1}{3}$ D) $\frac{1}{2}$ E) 1

7. $||2x + 1| - 5| = 6$

$\Rightarrow \prod x$ **?**

A) -18 B) -24 C) -28 D) -30 E) -32

8. $|2x - 5| + x = 1 \Rightarrow$ **ÇK(SS) = ?**

A) $\{2, 4\}$ B) $\{2\}$ C) $\{4\}$ D) $\mathbb{R}$ E) $\varnothing$

9. $4|x-2| + |2-x| = |x^2 - 7x + 10|$

$\Rightarrow \prod x = ?$

A) 0 B) 12 C) 15 D) 20 E) 30

10. $|x-2y+7| + |x+y+1| + |z+4| = 0$

$\Rightarrow$ **$x \cdot y \cdot z = ?$**

A) 4 B) 6 C) 12 D) 18 E) 24

11. $|3x-1| = |2x-19|$

$\Rightarrow \sum x = ?$

A) –18 B) –14 C) 14 D) 18 E) 22

12. $x < |x|$,

$x \in Z$,

$|2x+3| < 15 \Rightarrow \sum x = ?$

A) –24 B) –28 C) –30 D) –36 E) –42

13. $|x| + |x-2| = 12$

$\Rightarrow \sum x = ?$

A) 1 B) 2 C) 4 D) 6 E) 8

14. $|x+5| = x+5$

$|x-6| = 6-x,\ x \in Z$

$\Rightarrow \sum x = ?$

A) 0 B) 3 C) 4 D) 6 E) 9

15. $6 < \dfrac{30}{|x-5|}$

$x \in Z \Rightarrow \sum x = ?$

A) 32 B) 36 C) 40 D) 48 E) 52

16. $\dfrac{\big||3x+2|+5\big|+2x}{|x|+7} = 2$

$\Rightarrow \sum x = ?$

A) $\frac{7}{3}$ B) $-\frac{3}{4}$ C) $-\frac{2}{3}$ D) $\frac{3}{4}$ E) $\frac{4}{3}$

YANITLAR / ANSWERS

1	2	3	4	5	6	7	8
C	A	B	B	A	B	D	E
9	10	11	12	13	14	15	16
A	E	B	D	B	D	C	A

1. $x \cdot |x-5| = 6 \Rightarrow \sum x = ?$

A) 5 B) 8 C) 9 D) 10 E) 11

2. $|x+3| + |x-3| = 6 \Rightarrow$ **ÇK(SS) = ?**

A) $(-3, 3)$ B) $[-3, 3]$ C) $\{-3, 3\}$ D) $\varnothing$ E) $(-\infty, -3) \cup (3, +\infty)$

3. $a > b,\ |a| = |b|,\ \dfrac{2|b-a| + |ab-a^2|}{|a-b|} = 299 \Rightarrow$ **a = ?**

A) 294 B) 295 C) 296 D) 297 E) 301

4. $\left|3 - \dfrac{2x-2}{6}\right| = 2 \Rightarrow \sum x = ?$

A) 10 B) 14 C) 16 D) 18 E) 20

5. $|3x-12| + |4-x| = 16 \Rightarrow \sum x = ?$

A) 16 B) 12 C) 10 D) 8 E) 6

6. $\left.\begin{array}{l} -1 \le x \le 7 \\ |x-a| \le 4 \end{array}\right\} \Rightarrow$ **a = ?**

A) –4 B) –3 C) –2 D) 3 E) 4

7. $a < 0 < b \Rightarrow |a-b| - |a| - |-b| = ?$

A) –a B) –b C) 0 D) a E) a–b

8. $\left|\dfrac{x^2+2x}{x-2}\right| = \dfrac{x^2+2x}{2-x} \Rightarrow$ **ÇK(SS) = ?**

A) $[-2, 0]$ B) $(-\infty, -2] \cup [0, 2)$ C) $[-2, 0)$ D) $(2, +\infty)$ E) $[-2, 0] \cup [3, 4]$

9. $||x-4| - 12| = 9 \Rightarrow \sum x = ?$

A) 16 B) 17 C) 18 D) 19 E) 23

10. $3 < x < 6 \Rightarrow |x - |x-6| - |3-x| + 3| = ?$

A) 9 B) 2x – 6 C) x D) 12 E) 2x

11. $x \in \mathbb{Z}$,

$$\left|\frac{4x-1}{3}\right| \le 5 \Rightarrow \sum x = ?$$

A) 2 B) 3 C) 4 D) 5 E) 6

12. $x < -4$ ve / and $y < -3$

$\Rightarrow |-x-4| + |4-x| + |-y-3| + |3-y| = ?$

A) $2(x-y)$ B) $-2(x+y)$ C) -7 D) -1 E) -14

13. $|7x+21| = |7x-21| \Rightarrow |4x-16| = ?$

A) x B) –x C) 4 D) 8 E) 16

14. $|a-5| \le 2 \Rightarrow |a+3| + |9-a| + |1-a| - |8+a| = ?$

A) –6 B) –3 C) 3 D) 12 E) 21

15. $\left.\begin{array}{l} x, y \in \mathbb{N} \\ (x-y)^2 = x^2 + y^2 \\ y - x > 0 \end{array}\right\} \Rightarrow |3-x| + |2+y| - |x-y-3| = ?$

A) 0 B) 2 C) 4 D) 8 E) 12

16. $|4 - |5x-3|| = 9 \Rightarrow \sum x = ?$

A) $\frac{6}{5}$ B) 2 C) $\frac{12}{5}$ D) $\frac{16}{5}$ E) $\frac{18}{5}$

17. $\max\left(\frac{120}{|x+7| + |x+4| + |x-5|}\right) = ?$

A) 6 B) 7 C) 8 D) 9 E) 10

18. $x < 4 \Rightarrow \sqrt{x^2 - 15x + 60 + \sqrt{x^2 - 8x + 16}} + x - 4 = ?$

A) $2x-4$ B) $2x-12$ C) $12-2x$ D) 4 E) 12

19. $x^2 + 5|x| - 6 = 0 \Rightarrow x_1 \cdot x_2 = ?$

A) –36 B) –6 C) –1 D) 6 E) 36

YANITLAR / ANSWERS

1	2	3	4	5	6	7	8	9	10
E	B	D	E	D	D	C	B	A	C
11	12	13	14	15	16	17	18	19	
C	B	E	C	B	A	E	D	C	

BÖLÜM
CHAPTER 10

TARİHSEL NOT / HISTORICAL NOTE

Maryam Mirzakhani [1977 - 2017]

İranlı matematikçi ve Stanford Üniversitesi matematik profesörüdür. Araştırma konuları arasında Teichmüller teorisi, hiperbolik geometri, ergodik teori ve semlectic geometri yer almaktadır. 13 Ağustos 2014'te Fields Madalyası (matematiğin en prestijli ödülü) ile onurlandırıldı. Böylece, bu ödülle onurlandırılan ilk kadın ve ilk İranlı oldu. Ödül komitesi çalışmalarını "dinamizm ve geometride, Riemann yüzeylerinin ve moduli uzayları" konularında onurladı.

She was an Iranian mathematician and a professor of mathematics at Stanford University. Her research topics included Teichmüller theory, hyperbolic geometry, ergodic theory, and symplectic geometry. On August 13th 2014, she was honored with the Fields Medal (the most prestigious award in mathematics.) Thus, she became the first woman and the first Iranian to be honored with this award at the same time.The award committee cited her work in "the dynamics and geometry of Riemann surfaces and their moduli spaces".

MANTIK
LOGIC

Bu bölüm, 16 test sorusu içermektedir.

This chapter includes 16 test questions.

BÖLÜM / CHAPTER 10

MANTIK / LOGIC

- Mantık / Logic . . . . 193 – 200

BÖLÜM 10 CHAPTER

MANTIK
LOGIC

MANTIK / LOGIC

Mantık: Doğru düşünmenin kurallarını öğreten bir bilim dalıdır. Matematik bilimi de sistemli ve doğru düşünme yeteneğimizi geliştirmeyi amaçladığından mantık biliminden yararlanır.

Logic: Logic is a field instructing rules of righteous thinking. Mathematics also aims to improve our ability to think correctly and systematically and from this point of view is taking advantage of logic.

Tanım: Kesin olarak doğru ya da yanlış hüküm bildiren ifadelere **önerme** denir.
p, q, r, s, t, ... gibi harflerle gösterilir.

Definition: An expression declerating undoubted right or wrong is called a **proposition**. And it's shown by letters like p,q,r,s,t,.....

▼ Bir önermenin doğru ya da yanlış olmasına bu önermenin **doğruluk değeri** denir.

▼ To be true or false of a proposition is called its **truth-value**.

Doğru ise doğruluk değeri / If it's right, its truth-value : 1
Yanlış ise doğruluk değeri / If it's wrong, its truth-value : 0

yazılarak belirtilir.
is expressed by writing.

Önemli / Important

- Bir önermenin doğruluk değeri için : for truth-value of one proposition : $2^1 = 2$
- İki önermenin doğruluk değeri için / for truth-value of two propositions : $2^2 = 4$
- Üç önermenin doğruluk değeri için / for truth-value of three propositions : $2^3 = 8$
- n önermenin doğruluk değeri için / for truth-value of n propositions : 2^n farklı durum vardır. / 2^n different situations exist.

Tanım: Doğruluk değerleri aynı olan önermelere **denk önerme** denir ve $p \equiv q$ biçiminde gösterilir.

Definition: Propositions with the same truth-value are called **equivalent propositions** and shown by $p \equiv q$.

Tanım: Bir önermenin doğruluk değerini değiştirerek oluşturulan yeni önermeye bu **önermenin olumsuzu (değili)** denir. p önermesinin değili p′ sembolü ile gösterilir.

Definition: The new proposition composed by altering the truth-value of an proposition is called **adverse of that proposition**. Adverse of proposition p is shown by p′.

p	p′
1	0
0	1

▼ $(p')' \equiv p$

Tanım: İki ya da daha fazla önermenin birbirine "ve", "veya", "ise", "ancak ve ancak" bağlaçları ile bağlanması sonucu elde edilen önermelere **bileşik önerme** denir.

Definition: The new proposition composed by combining two or more propositions with connectives like: and, or, if, and if and only if is called **compound proposition**.

Bağlacın adı Name of Connective	Sembolü Symbol
veya / or	$\vee$
ve / and	$\wedge$
ise / if	$\Rightarrow$
ancak ve ancak / if and only if	$\Leftrightarrow$

BAĞLAÇLARIN DOĞRULUK TABLOSU / THE CORRECTNESS TABLE of CONNECTIVES

p	q	$p \vee q$	$p \wedge q$	$p \Rightarrow q$	$p \Leftrightarrow q$
1	1	1	1	1	1
1	0	1	0	0	0
0	1	1	0	1	0
0	0	0	0	1	1

şeklindedir.

is like the table above.

Özellik / Property

★ $p \vee p \equiv p$

$p \wedge p \equiv p$

★ $p \vee q \equiv q \vee p$

$p \wedge q \equiv q \wedge p$

★ $p \vee (q \vee r) \equiv (p \vee q) \vee r$

$p \wedge (q \wedge r) \equiv (p \wedge q) \wedge r$

★ $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$

$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$

★ $(p \vee q)' \equiv p' \wedge q'$

$(p \wedge q)' \equiv p' \vee q'$

★ $p \vee p' \equiv 1$ (totoloji / tautology)

$p \wedge p' \equiv 0$ (çelişki / contradiction)

★

p	$p \vee 1$	$p \wedge 1$	$p \vee 0$	$p \wedge 0$	$p \vee p'$	$p \wedge p'$
1	1	1	1	0	1	0
0	1	0	0	0	1	0

★ $p \Rightarrow q \equiv p' \vee q$

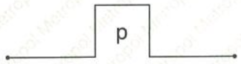
★ $p \vee (p \wedge q) \equiv p$

$p \wedge (p \vee q) \equiv p$

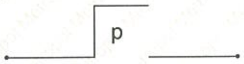
★ $p \Rightarrow q \equiv q' \Rightarrow p'$

★ $p \Leftrightarrow q \equiv (p \Rightarrow q) \wedge (q \Rightarrow p)$

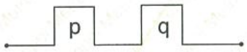
Önemli / Important

- 

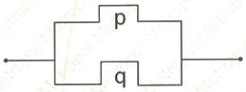
  doğruluk değeri / truth-value : $p \equiv 1$

- 

  doğruluk değeri / truth-value : $p \equiv 0$

- 

  seri bağlama / coupling in series : $p \wedge q$

- 

  paralel bağlama / coupling in paralel : $p \vee q$

Metropol

1. Aşağıdaki ifadelerden kaç tanesi bir önermedir?

How many of the following are propositions?

I. $2 = 3 + 3$

II. İyi günler. / Have a good day.

III. $x = 2$

IV. 812143245 sayısı 17 ile tam bölünür. / The number 812143245 by 17 is divisible.

V. $\sqrt{5} > 2$

A) 1 B) 2 C) 3 D) 4 E) 5

2. $p : 2^3 = 8$

$q : 3 + 2 = 7$

$r : 6 - (-4) = 10$

$s : 3 - 5 = 2$

ise aşağıdakilerden hangisi doğrudur?

Which of the following is true?

A) $p \not\equiv s$, $q \equiv r$

B) $p \not\equiv r$, $q \equiv s$

C) $p \equiv r$, $q \not\equiv s$

D) $p \equiv s$, $q \equiv r$

E) $p \equiv r$, $q \equiv s$

3. $1 \vee p \equiv q$

$p' \vee r \equiv 1$

$p \vee 0 \equiv r$

$\Rightarrow$ **(p, q, r) = (?, ?, ?)**

A) (1, 1, 1) B) (1, 0, 1) C) (1, 0, 0) D) (0, 0, 0) E) (0, 0, 1)

4. $(p \vee q) \wedge p' \equiv ?$

A) q B) q' C) $p \vee q'$ D) $p' \wedge q$ E) $p \wedge q'$

5. $(q \vee p) \vee p' \equiv ?$

A) p B) q C) p' D) 1 E) 0

6. $p \wedge (q \vee p') \equiv ?$

A) 0 B) 1 C) p D) q E) $p \wedge q$

7. $[(q' \vee q) \wedge p']' \equiv ?$

A) q B) q' C) p D) p' E) 1

8. $(p \wedge q) \Rightarrow (p \vee q) \equiv ?$

A) 1 B) 0 C) p D) q E) p'

9. $(p \Leftrightarrow 0) \vee p \equiv ?$

A) 0 B) 1 C) p D) q E) p'

10. **$(p \wedge q) \Rightarrow r \equiv 0$ ise / if $p \wedge (q \Leftrightarrow r) \equiv ?$**

A) 0 B) 1 C) p D) q E) r

11. **$\forall x \in R$ için / for $p(x)$: $|x+2| > 5$ ise / if $p'(x) \equiv ?$**

A) $\forall x \in R$ için / for $p'(x)$: $|x+2| < 5$

B) $\forall x \in R$ için / for $p'(x)$: $|x+2| \leq 5$

C) $\exists x \in R$ için / for $p'(x)$: $|x+2| \leq 5$

D) $\exists x \in R$ için / for $p'(x)$: $|x+2| = 7$

E) $\forall x \in R$ için / for $p'(x)$: $|x+2| \neq 7$

12. **"Sıfırdan farklı tüm reel sayıların karesi pozitiftir." önermesinin mantık dili ile karşılığı aşağıdakilerden hangisidir?**

Which of the following is equivalent to the proposition "The square of all real numbers except zero are positive." in language of logic?

A) $\forall x \in R$ için / for $x^2 > 0$

B) $\exists x \in R$ için / for $x^2 > 0$

C) $\forall x \in R - \{0\}$ için / for $x^2 > 0$

D) $\exists x \in R - \{0\}$ için / for $x^2 > 0$

E) $\forall x \in R - \{0\}$ için / for $x^2 \geq 0$

13. **Aşağıdaki bileşik önermelerden kaç tanesi totolojidir?**

How many of the compound propositions below are tautologies?

I. $p \vee p'$ **II.** $p \wedge p'$ **III.** $p \Leftrightarrow p'$ **IV.** $p \wedge 0$ **V.** $p \Rightarrow p'$

A) 1 B) 2 C) 3 D) 4 E) 5

14. **$p \equiv 1, q \equiv 0$ ise / if $(p \wedge q') \vee (q \wedge p')' \equiv ?$**

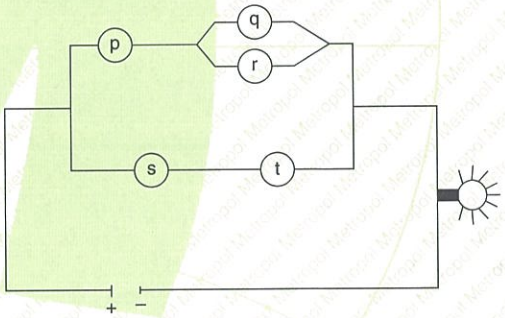
A) $p \wedge q$ B) $p' \vee q$ C) $p' \wedge q$ D) 1 E) 0

15. **$p \Rightarrow (r \vee q') \equiv 0$ ise / if $(p, q, r) \equiv (?, ?, ?)$**

A) (1, 1, 0) B) (1, 0, 1) C) (1, 1, 1)

D) (0, 1, 1) E) (0, 0, 1)

16.



Şekildeki elektrik devresine karşılık gelen bileşik önerme nedir?

What is the proposition equivalent to the electric circuit in the figure?

A) $[p \vee (q \wedge r)] \wedge (s \vee t)$

B) $[p \wedge (q \vee r)] \vee (s \wedge t)$

C) $[p \wedge (q \wedge r)] \wedge (s \vee t)$

D) $[p \wedge (q \vee r)] \wedge (s \wedge t)$

E) $[p \vee (q \wedge r)] \vee (s \wedge t)$

YANITLAR / ANSWERS

1	2	3	4	5	6	7	8
C	E	A	D	D	E	C	A
9	10	11	12	13	14	15	16
B	A	C	C	A	D	A	B

BÖLÜM CHAPTER 11

TARİHSEL NOT / HISTORICAL NOTE

Cantor, Georg (Ferdinand Ludwig Philipp) [1845 – 1918]

St. Petersburg'da doğmuş, hayatının büyük bir kısmını Almanya'da Halle Üniversitesi'nde geçirmiştir. 1873'de rasyonel sayılar kümesinin sayılamaz olduğunu göstermiştir. Aynı zamanda reel sayılar kümesinin de sayılamaz sonsuzlukta olduğunu göstermiştir. Daha sonra sonsuz kümeler kuramını geliştirmiştir.

He was born in St Petersburg, but spent most of his life at the University of Halle in Germany. In 1873, he showed that the set of rational numbers is non-denumerable. He also showed that the set of all real numbers is not. Later he fully developed his theory of infinite sets.

KÜMELER
SETS

Bu bölüm, 90 test sorusu içermektedir.
This chapter includes 90 test questions.

BÖLÜM / CHAPTER 11

KÜMELER / SETS

- Kümeler / Sets .... 201 – 216

BÖLÜM
11
CHAPTER

KÜMELER
SETS

KÜMELER / SETS

Tanım: Küme, nesnelerin açık olarak tanımlanmış bir listesidir. Kümeyi oluşturan nesnelere ***kümenin elemanları*** denir. Bir A kümesinin eleman sayısı **n(A)** veya **s(A)** ile gösterilir.

$\in$: kümenin elemanı

$\notin$: kümenin elemanı değil diye okunur.

A = {1, 3, a, b} kümesi için $3 \in A$, fakat $4 \notin A$ dır.

Definition: A set is a well defined collection of objects. Objects forming the set are called **elements or members of the set**. The number of elements of a set A is represented by **n(A)** or **s(A)**.

$\in$ reads: is an element of

$\notin$ reads: is not an element of

For the set A = {1, 3, a, b} we can say $3 \in A$, but $4 \notin A$.

Önemli / Important

- Kümede yer alan nesneler; harfler, sayılar ve insanlar olabilir.
- Kümeler, A,B,C gibi büyük harflerle gösterilir.
- Objects in sets can be letters, numbers, people, etc.
- Sets are represented by capital letters, such as A,B,C,... .

KÜMELERİN GÖSTERİMİ / DİSPLAYING of SETS

▼ **Liste yöntemi /** List method:

Bir kümenin elemanlarının küme parantezi içerisinde virgülle ayrılarak yazılmasına **liste yöntemi** denir.

Writing elements of a set inside curly braces separated with commas is called **list method**.

A = {1, 2, 3, 4, 5}

▼ **Ortak özellik yöntemi /** Common characteristic method**:**

Bir kümenin elemanlarının ortak özellikleriyle beraber küme parantezi içerisinde yazılmasına **ortak özellik yöntemi** denir.

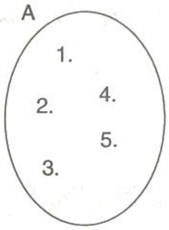
Writing elements of a set with their characteristics inside curly braces is called **common characteristic method**.

$A = \{x: x \leq 5,\ x \in N^{+}\}$

▼ **Venn şeması /** Venn diagram**:**

Bir kümenin elemanlarının kapalı bir geometrik şekil içerisinde gösterilmesine **Venn şeması** denir.

Displaying elements of a set inside an enclosed geometric figure is called **Vtenn diagram**.



Önemli / Important

- { } veya ∅ sembolleri **boş küme** olarak adlandırılır ve hiçbir eleman içermez.
- The symbols { } or ∅ are called the empty set and contain no elements.

  $A = \{\} \Rightarrow n(A) = 0$

  $A = \{\varnothing\} \Rightarrow n(A) = 1$

ALT KÜME / SUBSET

▼ Bir A kümesinin her elemanı, bir B kümesinin de elemanı ise **A'ya B'nin alt kümesi** denir. **$A \subset B$** biçiminde gösterilir.

▼ If every element of set A is also an element of set B, then we say that **set A is a subset of the set B**. It is represented by $A \subset B$.

Önemli / Important

- Bir küme sonlu sayıda elemana sahipse **sonlu küme** olarak adlandırılır. Sonsuz kümeler, sayılamayacak kadar çok elemana sahiptir. Örneğin; $Z = \{0, \pm1, \pm2, \ldots\}$ bir sonsuz kümedir.
- A set can have a finite number of elements and so is said to be a **finite set.** **Infinite sets** are sets which have infinitely many members. For example; $Z = \{0, \pm1, \pm2, \ldots\}$ is an infinite set.

Özellik / Property

Alt Kümenin Özellikleri / Properties of Subset

★ Her küme, kendisinin alt kümesidir ($A \subset A$).

★ Boş küme, her kümenin alt kümesidir ($\varnothing \subset A$).

★ $A \subset B$ ve $B \subset C \Rightarrow \mathbf{A \subset C}$

★ $A \subset B$ ve $B \subset A \Rightarrow \mathbf{A = B}$

★ n elemanlı bir kümenin alt kümelerinin sayısı $\mathbf{2^n}$ dir.

★ n elemanlı bir kümenin ($r \le n$) elemanlı alt kümelerinin sayısı $\mathbf{C(n,r) = \dfrac{n!}{r! \cdot (n-r)!}}$

★ Every set is subset of itself ($A \subset A$).

★ Empty set is subset of every set ($\varnothing \subset A$).

★ $A \subset B$ and $B \subset C \Rightarrow \mathbf{A \subset C}$

★ $A \subset B$ and $B \subset A \Rightarrow \mathbf{A = B}$

★ The number of subsets of a set with n elements is equal to $\mathbf{2^n}$.

★ The number of subsets which has r members, of a set which has n members can be found by the equation; $\mathbf{C(n,r) = \dfrac{n!}{r! \cdot (n-r)!}}$

ÖZ ALT KÜME / PROPER SUBSET

▼ Bir kümenin kendisinden farklı her alt kümesine **öz alt küme** denir.

▼ Each subset of a set in which does not equal to set itself is culled a **proper subset**.

Özellik / Property

★ n elemanlı bir kümenin öz alt kümelerinin sayısı $\mathbf{2^n - 1}$dir.

★ The number of proper subsets of a set which has n elements is $\mathbf{2^n - 1}$.

EVRENSEL KÜME / UNIVERSAL SET

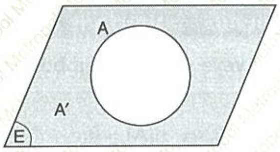
▼ Üzerinde işlem yapılan tüm kümeleri kapsayan kümeye **evrensel küme** denir. Evrensel küme **E** veya **U** ile gösterilir.

▼ A set comprising all sets, on which operations are being done, is called the **universal set** and represented by E or U.

KÜMELERDE TÜMLEME / COMPLEMENTATION IN SETS

▼ E kümesinin A kümesinde olmayan elemanlarının oluşturduğu kümeye **"A kümesinin tümleyeni"** denir. A kümesinin tümleyeni A′ ile gösterilir.

▼ **The complement of** A is the set of all elements of E which are not in A. The complement of A is denoted by A′.



Özellik / Property

★ $E' = \varnothing$

★ $\varnothing' = E$

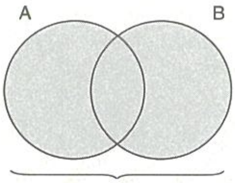
★ $(A')' = A$

★ $A \subset B \Leftrightarrow B' \subset A'$

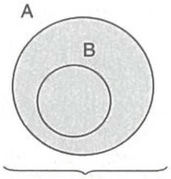
KÜMELERDE BİRLEŞİM / UNION in SETS

- A ve B kümeleri için ortak elemanları bir kez kullanılarak A nın ve B nin tüm elemanlarından oluşturulan yeni kümeye ***A ile B nin birleşim kümesi*** denir. **A ∪ B** ile gösterilir.
- A ∪ B is the union of sets A and B and is the set made up of all elements which are in A or B written once in a new set.

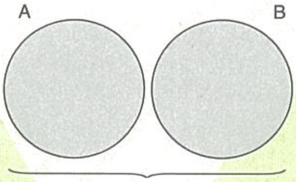
$A \cup B = \{x \mid x \in A$ veya/or $x \in B\}$



A∪B



$B \subset A \Rightarrow A \cup B = A$

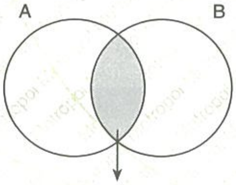


A∪B

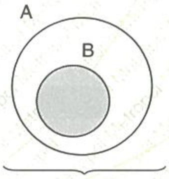
KÜMELERDE KESİŞİM / INTERSECTION in SETS

- A ve B kümeleri için bu iki kümenin ortak olan elemanlarının oluşturduğu kümeye ***A ile B nin kesişim kümesi*** denir. **A ∩ B** ile gösterilir.
- A ∩ B is the **intersection of sets A and B** and is the set made up of all elements which are in both A and B.

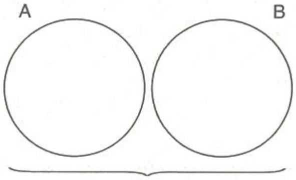
$A \cap B = \{x \mid x \in A$ ve/and $x \in B\}$



A∩B



$B \subset A \Rightarrow A \cap B = B$



$A \cap B = \varnothing$

Özellik / Property

Birleşim ve Kesişimin Özellikleri / Properties of Union and Intersection

★ $A \cup B = B \cup A$

$A \cap B = B \cap A$

★ $A \cup (B \cup C) = (A \cup B) \cup C$

$A \cap (B \cap C) = (A \cap B) \cap C$

★ $A \cup A = A$ ve / and $A \cap A = A$

★ $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

★ $(A \cup B)' = A' \cap B'$ ve / and $(A \cap B)' = A' \cup B'$ [De Morgan kuralları / De Morgan's laws]

★ $A \cup \varnothing = A$ ve / and $A \cap \varnothing = \varnothing$

★ $A \cup E = E$ ve / and $A \cap E = A$

★ $A \cup A' = E$ ve / and $A \cap A' = \varnothing$

★ $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

★ $n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(A \cap C) - n(B \cap C) + n(A \cap B \cap C)$

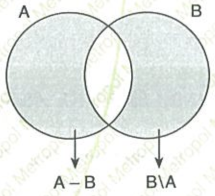
★ $n(A) + n(A') = n(E)$

KÜMELERDE FARK İŞLEMİ / OPERATION of SUBTRACTION in SETS

▼ A ve B kümeleri için A'nın elemanı olup B'nin elemanı olmayan elemanların oluşturduğu kümeye ***A fark B*** denir. **A–B** ya da **A\B** biçiminde gösterilir.

▼ For the sets A and B, a set composed of elements that exist in A but do not exist in B is called ***A minus B***. It is represented by A–B or A \ B.

$A - B = A \setminus B = \{x \mid x \in A$ ve/and $x \notin B\}$



Önemli / Important

- Ayrık kümeler, hiç ortak elemanı olmayan kümelerdir. Bu kümeler, kesişmeyen çemberler ile gösterilir.
- Disjoint(mutually exclusive) sets are sets which do not have common elements. These sets are represented by non-overlapping circles.

Özellik / Property

Fark İşleminin Özellikleri / Properties of the operation of Subtraction

★ $A - B = A \cap B'$

★ $A \subset B \Rightarrow A - B = \varnothing$

★ $A - B = A - (A \cap B)$

★ $E - A = A'$

★ $A - \varnothing = A$

★ $(A - B)' = (A \cap B')' = A' \cup B$

★ $(A - B) \cup B = A \cup B$

1. **$(A \cup B) \cap (A \cup E') = ?$**

A) E B) $\varnothing$ C) A D) B E) $A \cap B$

2. $\left.\begin{array}{l} A = \{1,2,3,4,5,6\} \\ B = \{3,4,5,6,7\} \end{array}\right\} \Rightarrow$ **$n[A-(A-B')] = ?$**

A) 0 B) 1 C) 2 D) 4 E) 5

3. $A \subset B \Rightarrow$ **$(A \cup B) \setminus (A' \cap B') = ?$**

A) A B) B C) A′ D) E E) $\varnothing$

4. $n(A \cup B) = 9,\ n(B \setminus A) = 4$

$\Rightarrow$ **$n(A) = ?$**

A) 2 B) 3 C) 4 D) 5 E) 6

5. $\left.\begin{array}{l} A = \{x \mid x \in R, -7 < x \le 14\} \\ B = \{x \mid x \in R, -5 \le x < 21\} \end{array}\right\} \Rightarrow$ **$A \cup B = ?$**

A) [–7, 21] B) (–7, 21) C) [– 5, 14] D) (–5, 14) E) [–7, 20]

6. $\left.\begin{array}{l} A' = \{1,2,3,4\} \\ B' = \{3,4,5,6,7\} \end{array}\right\} \Rightarrow$ **$(A \cup B)' = ?$**

A) {2,3} B) {1,2,3,4,5,6,7} C) {3,4} D) {1,2} E) {1,2,5,6,7}

7. $n(A \setminus B) = 4,\ n(B \setminus A) = 5,\ n(A \cup B) = 15 \Rightarrow$ **$n(B) = ?$**

A) 6 B) 8 C) 9 D) 10 E) 11

8. $n(A \cup B) = 12,\ n(A) = 5,\ n(B) = 9$

$\Rightarrow$ **$n[(A \setminus B) \cup (B \setminus A)] = ?$**

A) 5 B) 7 C) 8 D) 9 E) 10

9. $n(A) = 5,\ n(A \cap B) = 2,\ n(A \cup B) = 9 \Rightarrow$ **$n(B)=?$**

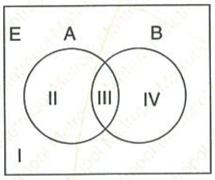
A) 4 B) 5 C) 6 D) 7 E) 8

10. $n(A \cap B) = 2 \cdot n(B \setminus A),\ n(A) = 7$

$n(A \cup B) = 8 \Rightarrow$ **$n(A \setminus B) = ?$**

A) 1 B) 2 C) 3 D) 4 E) 5

11.



$\Rightarrow$ **$(A \cap B') \cap B = ?$**

A) $\varnothing$ B) I C) II D) III E) IV

12. $C \subset B \subset A$

$n(C) = 2,\ n(B) = 4,\ n(A) = 7$

$\Rightarrow$ **$n(B \setminus C) + n(A \setminus C) = ?$**

A) 2 B) 3 C) 4 D) 5 E) 7

13. $A = \{1, 2, 3, 4, 5\}$,

$A \cup B = \{1, 2, 3, 4, 5, 6\}$,

$A \setminus B = \{1, 2, 3\} \Rightarrow$ **$B = ?$**

A) {2,3,4} B) {4,5,6} C) {3,4,5,6}

D) {5,6} E) {6}

14. $\left.\begin{array}{l} n(E) = 10 \\ n(A \cap B) = 4 \\ n(B) = 5 \\ n(A \cup B) = 7 \end{array}\right\} \Rightarrow$ **$n(A') = ?$**

A) 4 B) 5 C) 6 D) 7 E) 8

15. $\left.\begin{array}{l} n(A \setminus B) = 7 \\ n(A \cup B) = 16 \\ n(B \setminus A) = 5 \end{array}\right\} \Rightarrow$ **$n(A \cap B) = ?$**

A) 1 B) 2 C) 3 D) 4 E) 5

16. $\left.\begin{array}{l} n(A \cup B) = 7 \\ n(A \cup B)' = 2 \end{array}\right\} \Rightarrow$ **$n(E) = ?$**

A) 5 B) 6 C) 7 D) 8 E) 9

17. $\left.\begin{array}{l} n(A) = 2n(B) \\ n(A - B) = 8 \\ n(A \cap B) = 4 \end{array}\right\} \Rightarrow$ **$n(A \cup B) = ?$**

A) 12 B) 14 C) 15 D) 16 E) 18

YANITLAR / ANSWERS

1	2	3	4	5	6	7	8	9
C	C	B	D	B	C	E	E	C
10	11	12	13	14	15	16	17	
E	A	E	B	A	D	E	B	

1. $\left.\begin{array}{l} A=\{x \mid -4<x\le 1,\ x\in Z\} \\ B=\{x \mid -3<x<2,\ x\in Z\} \end{array}\right\} \Rightarrow \mathbf{A\cap B=?}$

A) {0,1} B) {–1,0,1} C) {–2, –1,0,1}
D) {–2,–1,0} E) {0,1,2}

2. $\left.\begin{array}{l} A\cup B=\{1,2,3,4\} \\ A\cup C=\{1,2,4,5,6\} \end{array}\right\} \Rightarrow \mathbf{A\cup(B\cap C)=?}$

A) {1,2,4} B) {2,3,4} C) {4,5,6}
D) {2,4} E) {5,6}

3. $\left.\begin{array}{l} n(A-B)=3 \\ n(B)=11 \\ n(A\cap B)=2 \end{array}\right\} \Rightarrow \mathbf{n(A\cup B)=?}$

A) 11 B) 12 C) 13 D) 14 E) 15

4. $\left.\begin{array}{l} A-B=\{a,b,c\} \\ A\cap B=\{d,e\} \end{array}\right\} \Rightarrow \mathbf{n(A)=?}$

A) 5 B) 4 C) 3 D) 2 E) 1

5. $\left.\begin{array}{l} n(A-B)=6 \\ n(A\cup B)=10 \\ n(A\cap B)=2 \end{array}\right\} \Rightarrow \mathbf{n(B-A)=?}$

A) 1 B) 2 C) 3 D) 4 E) 6

6. $\left.\begin{array}{l} n(A-B)=7 \\ n(B\setminus A)=3 \\ n(A)=9 \end{array}\right\} \Rightarrow \mathbf{n(A\cup B)=?}$

A) 6 B) 8 C) 9 D) 10 E) 12

7. $\left.\begin{array}{l} n(A)=18 \\ n(B)=14 \\ n(A\cup B)=28 \end{array}\right\} \Rightarrow \mathbf{n(A-B)=?}$

A) 14 B) 12 C) 11 D) 10 E) 8

8. $\left.\begin{array}{l} n(A\cup B)=15 \\ n(A)=7 \\ n(A-B)=4 \end{array}\right\} \Rightarrow \mathbf{n(B)=?}$

A) 8 B) 10 C) 11 D) 12 E) 13

9. $\mathbf{(2,5]\cup[-2,3)=?}$

A) (3, 5] B) (–2, 5] C) [–2, 3]
D) [3, 5] E) [–2, 5]

10. $\left.\begin{array}{l} n(A\cap B)=m-3 \\ n[A-(A\cap B)]=m+1 \\ n[B-(A\cap B)]=2m \\ n(A\cup B)=50 \end{array}\right\} \Rightarrow \mathbf{n(A)=?}$

A) 20 B) 22 C) 24 D) 26 E) 30

11. $A \subset B \subset C$

$n(A) = 3,\ n(B) = 6,\ n(C) = 10$

$\Rightarrow$ **$n(A \cap C) + n(A \cap B) = ?$**

A) 10 B) 9 C) 8 D) 7 E) 6

12. $A = \{x \mid -1 < x \leq 7,\ \ x \in Z\}$,

$B = \{y \mid y = 2k,\ 0 \leq k < 5,\ k \in Z\}$

$\Rightarrow$ **$n(A \cap B) = ?$**

A) 2 B) 3 C) 4 D) 5 E) 6

13. $A = \{x \mid \ 3 \leq x \leq 40\ x \in Z\}$,

$B = \{y \mid \ 3 \leq y^2 \leq 40\ y \in Z\}$

$\Rightarrow$ **$n(A \cap B) = ?$**

A) 5 B) 4 C) 3 D) 2 E) 1

14. $A, B \in E$

$n(A \cap B)' = 40,\ n(A \cup B)' = 15$

$n(A) + n(B) = 35 \quad \Rightarrow$ **$n(E) = ?$**

A) 45 B) 50 C) 55 D) 58 E) 60

15. $P \subset Q \subset R$,

$n(P \cap Q) = 3$

$n(R \cap Q) = 5,\ n(R - P) = 4 \quad \Rightarrow$ **$n(R) = ?$**

A) 12 B) 10 C) 8 D) 7 E) 6

16. $n(A \cup B) = 24,\ n(A \cap B) = 4$

$n(A) = 2 \cdot n(B - A) \quad \Rightarrow$ **$n(A - B) = ?$**

A) 14 B) 12 C) 10 D) 9 E) 8

17. $n(A) = 2 \cdot n(B),\ n(B - A) = 8,\ n(A \cap B) = 6$

$\Rightarrow$ **$n(A \cup B) = ?$**

A) 28 B) 30 C) 32 D) 34 E) 36

18. $n(A) + n(B') = 10,\ n(A') + n(B) = 8 \quad \Rightarrow$ **$n(E) = ?$**

A) 6 B) 7 C) 8 D) 9 E) 18

19. $A \subset B,\ \ n(B - A) = 16,\ n(B) = 3 \cdot n(A) \quad \Rightarrow$ **$n(A) = ?$**

A) 8 B) 7 C) 6 D) 5 E) 4

YANITLAR / ANSWERS

1	2	3	4	5	6	7	8	9	10
C	A	D	A	B	E	A	C	E	C
11	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	
E	C	B	A	D	B	E	D	A	

1. $A = \{ x \mid |x - 3| \leq 3, x \in Z \}$ ve / and

$B = \{ x \mid |x| \leq 1, x \in Z \} \Rightarrow$ **$B \cap A = ?$**

A) $\{-1\}$ B) $\{0, 1\}$
C) $\{-1, 0, 1\}$ D) $\{-1, 0, 1, 2, 3\}$
E) $\{2, 3, 4, 5\}$

2. $n(A \cup B) = 8$,
$n(A \cap B) = 3$,
$n(A - B) = 4 \Rightarrow$ **$n(B - A) = ?$**

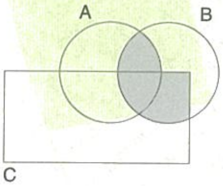
A) 1 B) 2 C) 3 D) 4 E) 5

3. $n(A) = 2 \cdot n(B)$, $n(A - B) = 10$ ve / and
$n(A \cap B) = 6 \Rightarrow$ **$n(A \cup B) = ?$**

A) 16 B) 17 C) 18 D) 19 E) 20

4. **Taralı bölge = ?**

The shaded region = ?



A) $(A \cap B) \cap C$ B) $(A \cup B) \cap C$
C) $(A \cup C) \cap B$ D) $(A \cap B) \cup C$
E) $(B \cap C) \cup A$

5. $A = \{ x \mid 10 \leq x \leq 100, x \in Z \}$
$B = \{ x \mid 20 < x \leq 30, x \in Z \}$
$\Rightarrow$ **$n(A - B) = ?$**

A) 80 B) 81 C) 82 D) 83 E) 84

6. $A = \{ x \mid (x^2 - 9) \cdot (x - 4) = 0, x \in R \} \Rightarrow$ **$n(A) = ?$**

A) $\varnothing$ B) 1 C) 2 D) 3 E) 4

7. $A = \{1, 2, 3, 4\}$, $B \cup C = \{3, 4, 5, 6\}$
$\Rightarrow$ **$(A \cap B) \cup (A \cap C) = ?$**

A) $\{3, 4, 5, 6\}$ B) $\{1, 2, 3, 4\}$ C) $\{5, 6\}$
D) $\{1, 2\}$ E) $\{3, 4\}$

8. **$(A - B) \cap B = ?$**

A) A B) B C) A′ D) B′ E) $\varnothing$

9. **$(-3, 6) \cup [6, 7) = ?$**

A) $(-3, 6)$ B) $(-3, 6]$ C) $(0, 6]$
D) $(-3, 0]$ E) $(-3, 7)$

10. $A = \{ x \mid -5 < x < 15, x \in Z \}$
$B = \{ y \mid y = 2m, 3 < m < 14, m \in Z \}$
$\Rightarrow$ **$n(A \cap B) = ?$**

A) 6 B) 5 C) 4 D) 3 E) 2

11. $A = \{0, 1, 2, 3, 4, 5\}$

$B = \{x \mid x = 2 \cdot k,\ 1 \le k < 13,\ k \in Z\}$

$\Rightarrow$ **$n(A \cup B) = ?$**

A) 10 B) 12 C) 14 D) 15 E) 16

12. **$[-10, 11] \cap [-1, 11) = ?$**

A) $[-10, 11]$ B) $[-10, -1]$ C) $[-1, 10]$ D) $[-1, 11)$ E) $(-10, 11]$

13. $A = \{-2, -1, 1, 2\}$, $B = \{0, 1, 4, 6\}$

$C = \{x \mid 1 \le x < 9,\ x \in Z\}$

$\Rightarrow$ **$n(A \cup B \cup C) = ?$**

A) 8 B) 9 C) 10 D) 11 E) 12

14. $A = \left(-\frac{5}{2}, 1\right]$, $B = [-1, 3)$, $C = \left(\frac{1}{2}, 5\right]$

$\Rightarrow$ **$(A \cap B) \cup C = ?$**

A) $[-1, 5]$ B) $(0,5]$ C) $[-1,3)$ D) $\left(-\frac{5}{2}, 5\right)$ E) $\left(-\frac{5}{2}, \frac{1}{2}\right)$

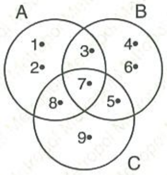
15. $n(A \cup B) = 15$,

$n(A - B) = 3$

$\Rightarrow$ **$n(B) = ?$**

A) 9 B) 10 C) 12 D) 14 E) 16

16. **$(A \cup B) \cap C = ?$**



A) $\{3, 5, 7\}$ B) $\{5, 7, 8\}$ C) $\{4, 5, 6, 7, 8\}$ D) $\{5, 7, 8, 9\}$ E) $\{3, 7, 8\}$

17. $n(A) = 10$, $n(B) = 7$

$n(A \cap B) = 2 \Rightarrow$ **$n(A \cup B) = ?$**

A) 15 B) 16 C) 17 D) 18 E) 19

18. $A \cap B = \varnothing$, $n(A) = 6$, $n(B) = 10$

$\Rightarrow$ **$n(A \cup B) = ?$**

A) 16 B) 12 C) 10 D) 8 E) 6

19. $n(A - B) = 3$, $n(A \cap B) = 2$, $n(B - A) = 5$

$\Rightarrow$ **$n(A \cup B) = ?$**

A) 9 B) 10 C) 11 D) 12 E) 13

YANITLAR / ANSWERS

1	2	3	4	5	6	7	8	9	10
B	A	C	C	B	D	E	E	E	C
11	12	13	14	15	16	17	18	19	
E	D	D	A	C	B	A	A	B	

1. $A \cap B \neq \emptyset$, $A \not\subset B$, $B \not\subset A$,

$n[(A \cup B) - (A \cap B)] = 15$

$n(A) = 6 \Rightarrow$ **$\min(n(B)) = ?$**

A) 8 B) 9 C) 10 D) 11 E) 12

2. $n(A \cup B) = 44$,

$n(B \setminus A) = 4$,

$n(A) = 4 \cdot n(A \cap B) \Rightarrow$ **$n(A \setminus B) = ?$**

A) 20 B) 26 C) 30 D) 32 E) 40

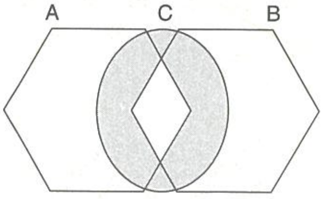
3. $n(A) = 17$,

$n(B) = 11$,

$n(A') = 13 \Rightarrow$ **$n(B') = ?$**

A) 15 B) 16 C) 17 D) 18 E) 19

4.



Taralı bölge / The shaded region **= ?**

A) $C \setminus (A \cap B)$ B) $C \setminus (A \cup B)$ C) $C \cup (A \cup B)$
D) $(A \cap B) \setminus C$ E) $(C \setminus B) \cup (C \setminus A)$

5. $A = \{x \mid -1 \leq x \leq 8,\ x \in Z\}$,

$B = \{x \mid |x - 2| \leq 4,\ x \in Z\}$

$\Rightarrow$ **$n(A \cap B) = ?$**

A) 6 B) 7 C) 8 D) 9 E) 10

6. $A = \{x \mid x = 3k,\ 26 < x \leq 203,\ k \in N^+\}$,

$B = \{x \mid x = 4k,\ 23 < x < 264,\ k \in N^+\}$

$\Rightarrow$ **$n(A \cup B) = ?$**

A) 102 B) 105 C) 108 D) 112 E) 116

7. $n(A \cap B') = 8$,

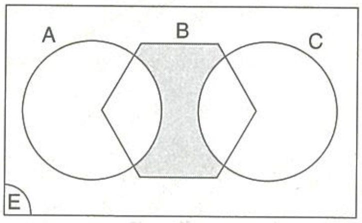
$n(B \cap A') = 7$,

$n(A \cap B) = 3$

$\Rightarrow$ **$n(A) + n(B) = ?$**

A) 15 B) 16 C) 17 D) 18 E) 21

8.



Taralı bölge / The shaded region **= ?**

A) $B \cap (A \cup C)'$ B) $B \cup (A \cup C)'$ C) $A' \cap (B \cup C)$
D) $E \setminus (A \cap C)$ E) $(A \cup C)' \setminus B$

9. $3 \cdot n(A) + 4 = 5 \cdot n(B)$

$n(B) = 14,$

$n(A \cap B) = 5 \Rightarrow \mathbf{n(A \cup B) = ?}$

A) 22 B) 24 C) 26 D) 31 E) 32

10. $\mathbf{(A \cap B) \cup [A' \cup (A \setminus B)] = ?}$

A) A B) B C) A′ D) B′ E) E

11. $n(A) = 3 \cdot n(B),$

$n(A \cup B) = 5 \cdot n(A \cap B),$

$n(B \setminus A) = 2 \Rightarrow \mathbf{n(A \setminus B) = ?}$

A) 10 B) 12 C) 14 D) 16 E) 18

12. $A = \{x \mid 141 \le x \le 239,\ x = 2k,\ k \in Z\},$

$B = \{y \mid 102 \le y < 191,\ y = 3k,\ k \in Z\}$

$\Rightarrow \mathbf{n(A' \cap B) = ?}$

A) 21 B) 22 C) 23 D) 24 E) 25

13. $n(A) + n(A') = 38,$

$n(A' \cap B) = 12,$

$n(A' \cap B') = 8 \Rightarrow \mathbf{n(A) = ?}$

A) 14 B) 15 C) 16 D) 17 E) 18

14. $n(A) = 2n + 7,$

$n(B) = 3n + 4,$

$n(A \cup B) = n + 44$

$\Rightarrow \mathbf{\min(n(A \cap B)) = ?}$

A) 2 B) 3 C) 4 D) 5 E) 6

15. $n(A' \cap B) = 3 \cdot n(A \cap B') = 4 \cdot n(A' \cap B'),$

$n(A \cap B)' = 38 \Rightarrow \mathbf{n(A \cup B)' = ?}$

A) 2 B) 3 C) 4 D) 6 E) 8

16. $A = \{x \mid 16 < x < 52,\ x = 3k,\ k \in N\},$

$B = \{y \mid 44 < y \le 120,\ y = 5k,\ k \in N\}$

$\Rightarrow \mathbf{n(A \cup B) = ?}$

A) 25 B) 26 C) 27 D) 28 E) 29

YANITLAR / ANSWERS

1	2	3	4	5	6	7	8
D	C	E	A	C	B	E	A
9	**10**	**11**	**12**	**13**	**14**	**15**	**16**
D	E	C	B	E	B	D	C

1. $A \cup B = E$, $n(A) = 15$, $n(A \cap B) = 7 \Rightarrow$ **$n(A \cap B') = ?$**

A) 5 B) 6 C) 7 D) 8 E) 10

2. $A \subset B \subset C \Rightarrow$ **$(A \cap B) \cap (A \cup C) = ?$**

A) $\varnothing$ B) A C) B D) C E) $B \setminus C$

3. $A = \{x | -3 \le x < 4, x \in Z\}$,
$B = \{x | |x - 3| < 3, x \in Z\} \Rightarrow$ **$A \cap B = ?$**

A) $\{-1, 0, 1, 2\}$ B) $\{0, 1, 2\}$ C) $\{-2, -1, 0, 1\}$
D) $\{1, 2, 3\}$ E) $\{2, 3, 4\}$

4. **$[(B \cap A)' \cap (B' \cup A)]' = ?$**

A) A B) B C) $A \cap B$ D) A' E) B'

5. **$(A \cap \varnothing)' \cap [A \cap (A \cup A')] = ?$**

A) $\varnothing$ B) A C) A' D) B E) E

6. **$(A \cup B)' \cap (A - B) = ?$**

A) B' B) A C) B D) A' E) $\varnothing$

7. $n(A \cap B)' = 35$, $n(A \cup B)' = 9$,
$n(A) + n(B) = 30 \Rightarrow$ **$n(E) = ?$**

A) 35 B) 36 C) 37 D) 38 E) 39

8. **$(B \setminus A') \setminus B = ?$**

A) $\varnothing$ B) A C) B D) A' E) B'

9. $A \cup B = A$, $n(A \cap C) = 4$, $n(B \cap C) = 2$, $n(A \setminus B) = 6$,
$n(B \setminus C) = 2$, $n(A \cup C) = 12 \Rightarrow$ **$n(C \setminus A) = ?$**

A) 10 B) 8 C) 6 D) 5 E) 2

10. $n(A \setminus B) = 0$, $n(A \cup B) = 12$, $n(B \setminus A) = 7 \Rightarrow$ **$n(A) = ?$**

A) 9 B) 8 C) 7 D) 6 E) 5

11. $A \cap B = A$, $n(A) = 4$, $n(B \setminus A) = 6 \Rightarrow$ **n(B) = ?**

A) 11 B) 10 C) 9 D) 8 E) 7

12. Bir kümenin alt ve özalt kümelerinin sayıları toplamı 63 ise bu kümenin 2 elemanlı alt kümelerinin sayısı kaçtır?

If the sum of the subsets and proper subsets of a set is 63, what is the number of the subsets that contains 2 members of this set?

A) 5 B) 8 C) 10 D) 12 E) 16

13. A = {1, 2, 3, 4, 5, 6} kümesinin 3 elemanlı alt kümelerinin kaç tanesinde 6 bulunur 5 bulunmaz?

How many of the subsets of the set A = {1, 2, 3, 4, 5, 6} that contains 3 members includes 6 but not 5?

A) 2 B) 3 C) 4 D) 6 E) 8

14. En çok 2 elemanlı alt küme sayısı 16 olan bir kümenin 3 elemanlı alt küme sayısı kaçtır?

What is the number of three – element subsets of a set which the number of its subsets with at most two elements is 16?

A) 5 B) 6 C) 7 D) 9 E) 10

15. A ve B kümeleri için A\B, A∩B, B\A kümelerinin alt küme sayıları sıra ile 64, 8, 16 dır. Buna göre, A∪B kümesi kaç elemanlıdır?

For the sets A and B, the number of the sets A\B, A∩B, and B\A are 64, 8, 16, respectively. According to that, how many members does the set A∪B have?

A) 10 B) 13 C) 15 D) 18 E) 21

16. n elemanlı bir kümenin alt küme sayısı 8^{2n-5} tir. Buna göre, bu kümenin 2 elemanlı alt küme sayısı kaçtır?

The number of the subsets of a set that contains n members is 8^{2n-5}. According to that, what is the number of the subsets of this set that contains 2 members?

A) 3 B) 5 C) 8 D) 10 E) 12

17. Bir kümenin 2 elemanlı alt kümelerinin sayısı 21'dir. Bu kümenin 3 elemanlı alt küme sayısı kaçtır?

The number of the subsets of a set that contains 2 members is 21. What is the number of the subsets of this set that contains 3 members?

A) 14 B) 18 C) 21 D) 28 E) 35

18. A = {a, b, c, d, e, f} kümesinin 3 elemanlı alt kümelerinin kaç tanesinde a ve b bulunur?

How many of the subsets of the set A = {a, b, c, d, e, f}. that has 3 members contains a and b?

A) 4 B) 6 C) 9 D) 10 E) 12

19. A = {1, 2, {a,b}, Δ, ∅, c} ve B = {a, b, Δ, 5} olduğuna göre, A\B kümesinin 2 elemanlı alt küme sayısı kaçtır?

If A = {1, 2, {a,b}, Δ , ∅, c} and B = {a, b, Δ, 5}, what is the number of the subsets of the set A\B that contains 2 members?

A) 6 B) 8 C) 10 D) 12 E) 15

YANITLAR / ANSWERS

1	2	3	4	5	6	7	8	9	10
D	B	D	B	B	E	C	A	E	E
11	12	13	14	15	16	17	18	19	
B	C	D	E	B	A	E	A	C	

BÖLÜM
CHAPTER
12

TARİHSEL NOT / HISTORICAL NOTE

Atiyah, Sir Michael Francis [1929 –]

Topoloji, Geometri, Analiz, cebirsel değişimlerin transdantal teorisi, diferansiyel, işlemler ve kuantum alan teorisine önemli katkıları olan ingiliz matematikçidir. 1966 da Field Medal, 2004 da Abel ödülü ile ödüllendirilmiştir.

British mathematician who made important contributions in topology, geometry, analysis, the transcenolantal theory of algebraic varieties, differential, operations and quantum field theory. he was awarded the Fields Madel in 1966 and the Abel Prize in 2004.

BAĞINTI
RELATION

Bu bölüm, 15 test sorusu içermektedir.
This chapter includes 15 test questions.

BÖLÜM / CHAPTER 12

BAĞINTI / RELATION

- Bağıntı / Relation .... 217 – 222

BÖLÜM

12

CHAPTER

BAĞINTI
RELATİON

BAĞINTI / RELATION

SIRALI İKİLİ / ORDERED PAIR

▼ x ve y gibi iki elemanın bir öncelik sırasına göre (x, y) biçiminde tek bir eleman olarak yazılmasına sıralı ikili veya sadece ikili denir.

▼ If two elements like x and y are written as (x, y), with the order of them being important, the result is called an ordered pair.

▼ "x" e bu ikilinin birinci bileşeni, "y"ye bu ikilinin ikinci bileşeni denir. (Asal sayı, Tek sayı) ikililerinden bazıları (2,1), (3,5), (5,–9), (7,–15) gibi

▼ Examples of (prime number, odd number) ordered pairs: (2,5), (3,5), (5,5), (7,15)

Özellik / Property

★ $(a,b) = (c,d) \Leftrightarrow a = c$ ve / and $b = d$

KARTEZYEN ÇARPIM / CARTESIAN PRODUCT

▼ A ve b boş olmayan iki küme olsun. Birinci bileşeni A kümesinden ikinci bileşeni B kümesinden alınarak oluşturulan tüm sıralı ikililerin kümesine A ile B nin kartezyen çarpımı denir.

$AxB = \{(x,y) \mid x \in A$ ve / and $y \in B\}$ şeklinde gösterilir.

▼ Let A and B be two sets, set of all the ordered pairs in which their fist and second components are taken from first and second sets respectively is called A cartesian product B and is shown by $AxB = \{(x,y) \mid x \in A \wedge y \in B\}$.

Özellik / Property

- ★ $A \neq B \Rightarrow A \times B \neq B \times A$
- ★ $n(A \times B) = n(B \times A) = n(A) \times n(B)$
- ★ $A \times (B \times C) = (A \times B) \times C$
- ★ $A \times (B \cup C) = (A \times B) \cup (A \times C)$
- ★ $A \times (B \cap C) = (A \times B) \cap (A \times C)$

KARTEZYEN ÇARPININ GRAFİĞİ / GRAPH of CARTESİAN PRODUCT

▼ AxB nin grafiği oluştururken, A nın elemanları yatay(x) ekseni üzerinde, B nin elemanları düşey (y) ekseni üzerinde alınır.

▼ When drowing the graph of AxB, the elements of set A make the horizontal(x) axis and the elements of set B make the vertical(y) axis.

Önemli / Important

A x B nin grafinde / In the graph of AxB

- A nın elemanları sayılamaz, B nin elemanları sayılabilir çoklukta ise grafik **x eksenine paralel doğrular**dır.
- If the elements of A are infinite and the elements of B are finite numbers, the graph consists of lines parallel to x axis.
- A nın elemanları sayılabilir, B nin elemanları sayılamaz çoklukta ise grafik **y eksenine paralel doğrular**dır.
- If the elements of A are finite numbers and the elements of B are infinite, the graph consists of lines parallel to y axis.
- AxB veya BxA çarpımında her iki kümenin elemanları da sayılamaz çoklukta ise grafikleri bir **alan** oluşturur.
- If in the AxB or BxA, the elements of A and the elements of B are both infinite, the graphs create a region.

BAĞINTI / RELATION

Tanım: A ve B kümeleri için, AxB nin her bir alt kümesine A dan B ye bağıntı denir. A dan B ye bağıntı sayısı $2^{n(A) \times n(B)}$ dir.

Definition: From set A to set B, each subset of AxB is called a relation from A to B. The number of relations from A to B are $2^{n(A) \times n(B)}$.

1. $(x^3 + 3,\ 18 - y) = (11,\ 2^x + y)$

$\Rightarrow$ **$x + y = ?$**

A) 11 B) 10 C) 9 D) 8 E) 7

2. $(a^2 + 3,\ 6 - b) = (28,\ 8)$

$\Rightarrow$ **$\sum a + b = ?$**

A) – 6 B) – 4 C) – 2 D) 0 E) 2

3. $(x + y,\ 2^{x-1}) = (5,\ 4^{x+1})$

$\Rightarrow$ **$x - y = ?$**

A) – 8 B) – 9 C) – 10 D) – 11 E) – 12

4. $A = \{-3, -1, 2, 4, 6\}$

$B = \{-9, -6, -3, 1, 2, 5, 6\}$

$\Rightarrow$ **$n(AxB) = ?$**

A) 25 B) 28 C) 32 D) 35 E) 42

5. $A = \{-1, 0, 1, 2, 3, 4\}$

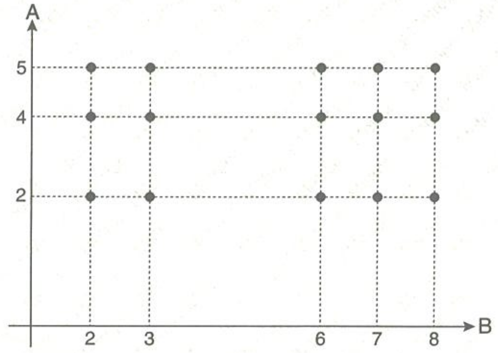
$B = \{-2, -1, 2, 4, 5, 6\}$

$C = \{-3, -1, 0, 1, 2, 3\}$

$\Rightarrow$ **$n[Ax(B \cap C)] = ?$**

A) 12 B) 14 C) 16 D) 18 E) 20

6.



Yukarıda BxA nın grafiği verilmiştir. Buna göre, A∪B kümesinin eleman sayısı kaçtır?

The graph of BxA is given in the figure above. Accordingly, what is the number of elements of the set A∪B ?

A) 4 B) 5 C) 6 D) 7 E) 8

7. $A = \{x \mid -4 < x \leq 1,\ x \in Z\}$

$B = \{x \mid -6 \leq x \leq 5,\ x \in N\}$

$\Rightarrow$ **$n(A x B) = ?$**

A) 25 B) 28 C) 30 D) 32 E) 36

8. $A = \{1, 2, 3\}$,

$B \cap C = \{6, 7, 8, 9\}$

$\Rightarrow$ **$n[(A x B) \cap (A x C)] = ?$**

A) 7 B) 9 C) 12 D) 36 E) 64

9. **$A = \{1, 2, 3, 4\}$**

$B = \{a, b, c\}$

ise A x B nin 3 elemanlı alt küme sayısı kaçtır?

What is the number of three-element subsets of AxB?

A) 210 B) 220 C) 230 D) 240 E) 250

10. $A = \{1, 2, 3\}$,
$B = \{a, b, c\}$,
$C = \{x, y, z, t\}$
$\Rightarrow$ **$n[(A \times B) \times C] = ?$**

A) 11 B) 24 C) 28 D) 36 E) 48

11. A, B, C ayrık kümeler / disjoint sets
$n(A \times B) = 30$, $n(B \times C) = 20$
$\Rightarrow$ **$\min[n(A \times (B \cup C))] = ?$**

A) 36 B) 72 C) 80 D) 120 E) 156

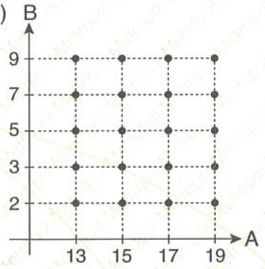
12. $A \neq \varnothing$, $B \neq \varnothing$
$n(A) + n(B) = 8$
$n(A \times B) = 12$
$\Rightarrow$ **$\max(n[A \times (A \cup B)] = ?$**

A) 12 B) 16 C) 24 D) 32 E) 48

13. $n(A \times B) = 24$
$n(B \times C) = 36$
$n(A \times C) = 54$
$\Rightarrow$ **$\max[n(A \cup B \cup C)] = ?$**

A) 16 B) 17 C) 18 D) 19 E) 20

14. A = {21 den küçük iki basamaklı asal sayılar}
B = {Asal olmayan rakamlar}
ise BxA nın grafiği aşağıdakilerden hangisidir?
A = {two-digit prime numbers smaller than 21}
B = {non-prime digits}
Which of the following is the graph of BxA?

A)


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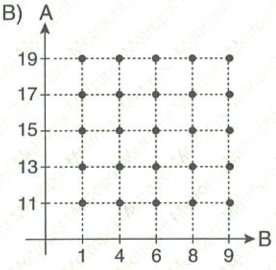
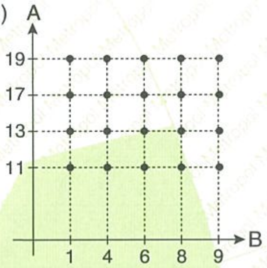
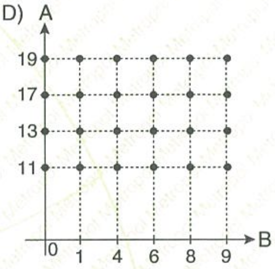
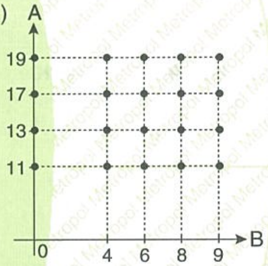
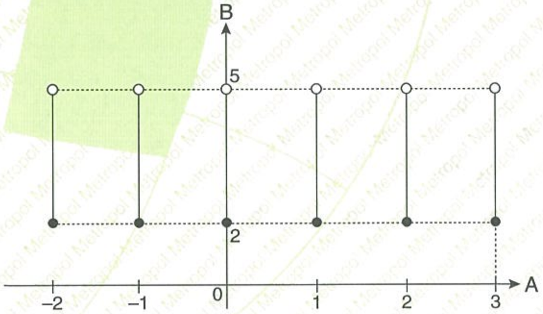


C)


D)


E)


15.



Yukarıda AxB nin grafiği çizilmiştir. A'∩B kümesinin tam sayı elemanlarının toplamı kaçtır?

The graph of AxB is given above. What is the sum of integer elements of the set A'∩B ?

A) – 2 B) – 1 C) 0 D) 4 E) 6

YANITLAR / ANSWERS

1	2	3	4	5	6	7	8
C	B	D	D	A	D	C	C
9	10	11	12	13	14	15	16
B	D	A	E	D	D	D	

BÖLÜM
CHAPTER

13

TARİHSEL NOT / HISTORICAL NOTE

Bessel Friedrich Wilhelm. [1784 – 1846]

Alman gök bilimci ve matematikçi. Bessel fonksiyonlarını ortaya çıkarmakla matematiğe büyük katkıda bulunmuştur. Bessel fonksiyonları, diferansiyel denklemlerin elde edilmesinde ve muhtemelen temel fonksiyonlardan sonra mühendislik ve fizik alanında en sık kullanılan fonksiyonlardır.

German astronomer and mathematician who made a major contribution to mathematics in the development of what are now called Bessel functions. These functions satisfy certain differential equations, and probably are the most commonly occurring functions in physics and engineering after the elementary functions.

FONKSİYONLAR
FUNCTIONS

Bu bölüm, 155 test sorusu içermektedir.
This chapter includes 155 test questions.

BÖLÜM / CHAPTER 13

FONKSİYONLAR / FUNCTIONS

- Fonksiyonlar / Functions .... 223 – 250

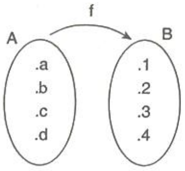
BÖLÜM 13 CHAPTER

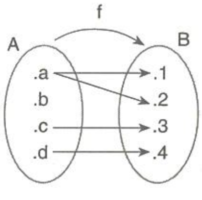
FONKSİYONLAR
FUNCTIONS

FONKSİYONLAR / FUNCTIONS

Tanım: A dan B ye bir f bağıntısı A nın her elemanını B de bir ve yalnız bir elemanla eşliyorsa ve A da eşlenmeyen eleman kalmıyorsa **f, A dan B ye bir fonksiyon**dur denir ve f: A → B ile gösterilir. A kümesine ***tanım kümesi***, B kümesine ***değer kümesi*** denir.

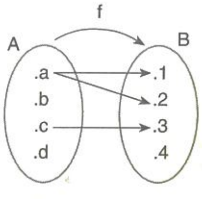
Definition: If a relation f from the set A to set B makes an assignment to each element in the set A and assigns one element of A to one and only one element in the set B then f is called *a function*. Set A is called the *domain* and set B is called the *range* of the function.





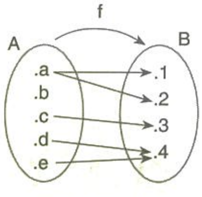
Fonksiyon değil

Not a function



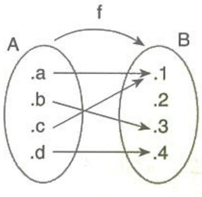
Fonksiyon değil

Not a function



Fonksiyon değil

Not a function



Fonksiyon

A function

Önemli / Important

- Bir fonksiyonun tanım kümesi, fonksiyonun tanımlı olduğu aralıktaki değerleridir. Fonksiyonun görüntü kümesi, fonksiyon uygulandığında elde edilen sonuçlardır.
- The domain of a function is the set of all the values for which the function is defined. The range of a function is the set of all values that are the outputs, or results, of applying the function.

Önemli / Important

- $\sqrt[n]{g(x)}$ biçimindeki fonksiyonların tanım kümesi,
  - **a.** n tek ise, R'dir.
  - **b.** n çift ise, $g(x) \geq 0$ dır.

- $\frac{a}{b}$ biçimindeki fonksiyonların tanım kümesi bulunurken, önce payda sıfıra eşitlenir ve paydayı sıfır yapan değer bulunur. Tanım kümesi, R'den paydayı sıfır yapan değer çıkarılarak bulunur.
- Polinom şeklindeki fonksiyonların tanım kümesi R'dir.

- The domain set of functions in $\sqrt[n]{g(x)}$ form;
  - **a.** If n is odd, it is R.
  - **b.** If n is even, then $g(x) \geq 0$.

- The domain set of functions in $\frac{a}{b}$ form; you should first find the values of which make denominator equal to zero and the function undefined. The domain of the function is R minus these vales.
- The domain set of polynomial functions is R.

FONKSİYONLARDA İŞLEMLER / OPERATIONS in FUNCTIONS

▼ $f : A \rightarrow R,\ g : A \rightarrow R$

$(f \pm g)(x) = f(x) \pm g(x)$

ÇARPMA ve BÖLME İŞLEMİ / MULTIPLICATION and DIVISION OPERATION

$f: A \rightarrow R,\ g: A \rightarrow R$

▼ $(f \cdot g)(x) = f(x) \cdot g(x)$

▼ $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)},\ g(x) \neq 0$

FONKSİYON ÇEŞİTLERİ / FUNCTION TYPES

▼ Birim (Etkisiz) Fonksiyon	(Identity Function)
▼ Sabit Fonksiyon	(Constant Function)
▼ Ters Fonksiyon	(Inverse Function)
▼ Bileşke Fonksiyon	(Composite Function)
▼ Permütasyon Fonksiyonu	(Permutation Function)
▼ Doğrusal Fonksiyon	(Linear Function)

BİRİM (ETKİSİZ) FONKSİYON / IDENTITY FUNCTION

▼ $f(x) = x \Rightarrow f$ birim fonksiyondur. $f(x) = I(x) = x$ ile gösterilir.

▼ $f(x) = x \Rightarrow f$ is identity function. It is denoted by $f(x) = Ix = x$.

SABİT FONKSİYON / CONSTANT FUNCTION

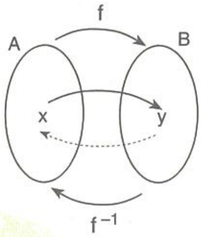
▼ $f(x) = c,\ c \in R \Rightarrow$ f sabit fonksiyondur.

▼ $f(x) = c,\ c \in R \Rightarrow$ f is a constant function.

TERS FONKSİYON / INVERSE FUNCTION

▼ $f : A \rightarrow B$;

$f^{-1} : B \rightarrow A$



$$f(x) = y \Rightarrow f^{-1}(y) = x$$

Önemli / Important

Bir fonksiyonun tersini bulmak için,

- x yalnız bırakılır.
- y yerine x, x yerine $f^{-1}(x)$ yazılır.

To the find the inverse of a function,

- isolate x.
- write x instead of y and $f^{-1}(x)$ instead of x.

Önemli / Important

- f^{-1} fonksiyonunun grafiği, f fonksiyonunun $y = x$ doğrusuna göre simetriğidir.
- The graph of f^{-1} is a reflection of the graph of f in the line $y = x$.

Özellik / Property

★ $(f^{-1})^{-1}(x) = f(x)$

★ $f(x) = I(x) \Rightarrow \mathbf{f^{-1}(x) = f(x)}$

Önemli / Important

- Bir fonksiyonun tersi kendisine eşit ise bu fonksiyon birim fonksiyon olmak zorunda değildir.
- If a function's reverse equels the function itself, the function doesnt need to be identity function.

BİR FONKSİYONUN TERSİNİ BULMADA PRATİK FORMÜLLER
PRACTICAL FORMULAS to FIND the INVERSE of a FUNCTION

▼ $f(x) = ax + b \Rightarrow f^{-1}(x) = \frac{x-b}{a}$

▼ $f(x) = \frac{ax+b}{cx+d} \Rightarrow f^{-1}(x) = \frac{-dx+b}{cx-a}$

▼ $f(x) = a^x \Rightarrow f^{-1}(x) = \log_a x$

Önemli / Important

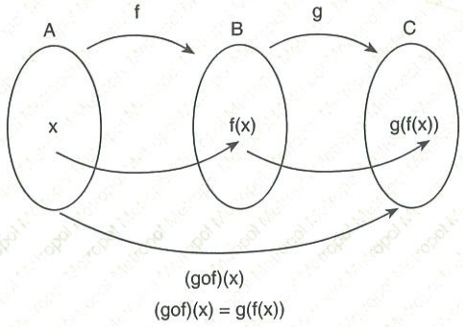
- İkinci dereceden bir fonksiyonun tersi alınacaksa fonksiyon önce tam kare ifadeye çevrilir.
- When inversing second degree function, we need to convert the function to a compete square first.

Önemli / Important

- Üçüncü dereceden bir fonksiyonun tersi alınacaksa fonksiyonunun önce tam küp ifadeye çevrilir.
- To inverse a third degree function, we need to convert the function to a complete cube first.

BİLEŞKE FONKSİYON / COMPOSITE FUNCTION

▼



(gof)(x) = g(f(x))

Önemli / Important

- $(fog)(x)$: f fonksiyonundaki x'lerin yerine $g(x)$ konulur.
- $(gof)(x)$: g fonksiyonundaki x'lerin yerine $f(x)$ konulur.
- $fog(x)$: Insert $g(x)$ instead of x in $f(x)$.
- $gof(x)$: Insert $f(x)$ instead of x in $g(x)$.

Özellik / Property

★ $fog \neq gof$

★ $(fog)oh = fo(goh)$

★ $(fog)^{-1}(x) = \mathbf{(g^{-1}of^{-1})(x)}$

★ $(f^{-1}of)(x) = (fof^{-1})(x) = I(x)$

★ $(foI)(x) = (Iof)(x) = f(x)$

★ $(fog)(x) = h(x) \Rightarrow g(x) = f^{-1}(h(x))$

★ $(fog)(x) = h(x) \Rightarrow f(x) = h(g^{-1}(x))$

PERMÜTASYON FONKSİYON / PERMUTATION FUNCTION

▼ Tanım ve değer kümeleri aynı olan fonksiyondur.

▼ A function that its range set is same as its domain set.

$$f = \begin{pmatrix} a & b & c & d \\ d & c & b & a \end{pmatrix} \Rightarrow \begin{cases} \mathbf{f(a) = d,} & \mathbf{f^{-1}(d) = a} \\ \mathbf{f(b) = c,} & \mathbf{f^{-1}(c) = b} \\ \mathbf{f(c) = b,} & \mathbf{f^{-1}(b) = c} \\ \mathbf{f(d) = a,} & \mathbf{f^{-1}(a) = d} \end{cases}$$

Önemli / Important

•

$$\begin{array}{cccc} \underline{g} & \underline{f} & & fog(x) \\ a \to c & c \to d & \Rightarrow & a \to d \\ b \to b & b \to c & \Rightarrow & b \to c \\ c \to d & d \to a & \Rightarrow & c \to a \\ d \to a & a \to b & \Rightarrow & d \to b \end{array}$$

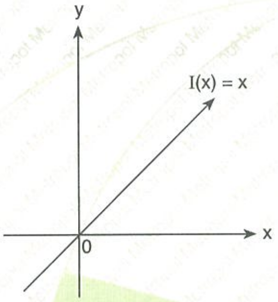
DOĞRUSAL FONKSİYON / LINEAR FUNCTION

▼ a ve b sabit sayılar olmak üzere, $y = ax + b$ şeklindeki fonksiyonlardır.

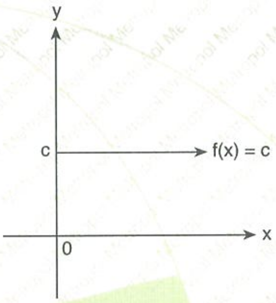
▼ Provided that a and b are constants, $y = ax + b$ is a linear function.

FONKSİYON GRAFİKLERİ / GRAPHS of FUNCTIONS

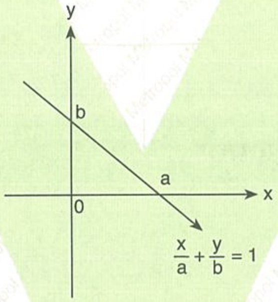
- Bir fonksiyonun tanım kümesinin elemanlarının x ekseninden görüntü kümesinin elemanlarının y ekseninden, yazılarak oluşturulan noktalarının dik izdüşümlerinin kesişim noktaları birleştirilmesiyle oluşan şekle **fonksiyon grafiği** denir.
- The figure formed by combining cross points of orthographic projeetions of the element of domain of a function written in x axis, with the elements of range of the same function written in y axis is called the graph of the function.



Birim fonksiyon grafiği / graph of Identity function



Sabit fonksiyon grafiği / graph of constant function



Doğrusal fonksiyon grafiği / graph of linear function

1. $f(x) = 3x^2 + 2x - 1$

$\Rightarrow$ **$f(2) + f(-1) + f(0) = ?$**

A) 13 B) 14 C) 15 D) 16 E) 17

2. $f(x) = 3x - 2$

$\Rightarrow$ **$f(2x + 1) = ?$**

A) $6x - 1$ B) $6x$ C) $6x + 1$ D) $6x + 3$ E) $6x + 4$

3. $f(3x + 2) = 4x^2 - 3$

$\Rightarrow$ **$f(8) = ?$**

A) 11 B) 13 C) 14 D) 15 E) 16

4. $f(3^x - 2) = 2^x + 4x - 1$

$\Rightarrow$ **$f(25) = ?$**

A) 19 B) 20 C) 21 D) 23 E) 25

5. $f(x) = x^2 - 2x + 3$

$\Rightarrow$ **$f(x + 1) = ?$**

A) $x^2 - 5x + 3$ B) $x^2 + 3x - 4$ C) $x^2 - 1$ D) $x^2 - 3x + 4$ E) $x^2 + 2$

6. $f(x) = 3|x - 2| + 4x$

$\Rightarrow$ **$f(5) + f(0) = ?$**

A) 33 B) 34 C) 35 D) 36 E) 37

7. $f(x) = 3x - 6 \Rightarrow$ **$f^{-1}(x) = ?$**

A) $\frac{x-6}{3}$ B) $\frac{3x-1}{6}$ C) $\frac{x+3}{6}$ D) $\frac{x+6}{3}$ E) $3x+6$

8. $f(x) = \frac{3x-4}{2x+5} \Rightarrow$ **$f^{-1}(1) = ?$**

A) $\frac{7}{2}$ B) 4 C) $\frac{9}{2}$ D) $\frac{17}{2}$ E) 9

9. $f(2x+3) = 5^x - 2$

$\Rightarrow$ **$f^{-1}(123) = ?$**

A) 3 B) 7 C) 9 D) 11 E) 13

10. $f(x) = \sqrt[3]{2x+1} \Rightarrow$ **$f^{-1}(3) = ?$**

A) 12 B) 13 C) 14 D) 15 E) 16

11. $f(2x+3) = 4x - 1 \Rightarrow$ **$f(x) = ?$**

A) $2x - 3$ B) $2x + 3$ C) $2x - 5$ D) $4x - 7$ E) $2x - 7$

12. $f(3x+4) = 6x - 8 \Rightarrow$ **$f^{-1}(10) = ?$**

A) 4 B) 7 C) 10 D) 13 E) 16

13. $f(g(x)) = 3g(x) + 4$

$\Rightarrow$ **$f(2) = ?$**

A) 4 B) 7 C) 10 D) 12 E) 14

14. $f(3^x) = 4 \cdot 3^{2x} + 5 \cdot 3^x + 3$

$\Rightarrow$ **$f(x) = ?$**

A) $9x + 3$ B) $4x^2 + 3x + 3$ C) $4x^2 + 5x + 3$ D) $3x^2 + 5x + 3$ E) $-x^2 + 3$

15. $f(x) = \dfrac{4x-6}{x+2a}$

$f(x) = f^{-1}(x) \Rightarrow$ **$a = ?$**

A) -2 B) -1 C) 0 D) 1 E) 2

16. $f(x) = x + 3 \Rightarrow$ **$f(2x+1) = ?$**

A) $f(x) - 2$ B) $2f(x)$ C) $2f(x) - 1$ D) $2f(x) - 2$ E) $2f(x) + 1$

17. $f(3x+1) = 9x^2 + 3x - 1$

$\Rightarrow$ **$f(x) = ?$**

A) $x^2 - x - 1$ B) $\dfrac{3x^2 - 5x + 1}{2}$ C) $\dfrac{3x^2 + 5x - 1}{2}$ D) $x^2 + x + 1$ E) $\dfrac{3x^2 - 5x + 2}{2}$

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1	2	3	4	5	6	7	8	9
B	C	B	A	E	C	D	E	C
10	**11**	**12**	**13**	**14**	**15**	**16**	**17**	
B	E	D	C	C	A	D	A	

1. $f(3x-1) = x^2 + x + m$

$f(5) = 10 \Rightarrow$ **$m = ?$**

A) 0 B) 1 C) 2 D) 3 E) 4

2. $f: R \rightarrow R$,

$f(x) = x^2 + x - 1 \Rightarrow$ **$f(3) = ?$**

A) 9 B) 10 C) 11 D) 12 E) 13

3. $f(x^3 + 1) = 2x + 8 \Rightarrow$ **$f(65) = ?$**

A) 12 B) 13 C) 15 D) 16 E) 19

4. $f: R \rightarrow R$,

$x \rightarrow y = f(2x+1) = 2x - 1$

$\Rightarrow$ **$f(x) = ?$**

A) $x-2$ B) $x-1$ C) x D) $x+1$ E) $x+2$

5. $f: R^+ \rightarrow R$

$f(x^2 + 5x + 1) = 2x + 3$

$\Rightarrow$ **$f(7) = ?$**

A) 5 B) 9 C) 10 D) 11 E) 12

6. $f: R - \{m\} \rightarrow R - \{n\}$,

$f(x) = \frac{2x-7}{x+3} \Rightarrow$ **$m \cdot n = ?$**

A) –2 B) –3 C) –6 D) –8 E) –10

7. $f(x) + 6x = 3f(x+1)$

$f(0) = 54 \Rightarrow$ **$f(2) = ?$**

A) 4 B) 6 C) 8 D) 9 E) 12

8. $3f(x) = x \cdot f(x+1)$,

$f(5) = \frac{9}{16} \Rightarrow$ **$f(3) = ?$**

A) $\frac{1}{4}$ B) $\frac{1}{3}$ C) $\frac{1}{2}$ D) $\frac{3}{4}$ E) $\frac{3}{2}$

9. $f: R - \{2\} \rightarrow R - \{-3\}$

$f(x) = \frac{ax+4}{1-bx} \Rightarrow$ **$a + b = ?$**

A) $\frac{1}{2}$ B) $\frac{3}{4}$ C) 1 D) $\frac{3}{2}$ E) 2

10. $f(x) = f(x+1) + 2x$

$f(10) = 10 \Rightarrow$ **$f(1) = ?$**

A) 90 B) 95 C) 100 D) 110 E) 120

11. $f(x) = \dfrac{4f(x+1)-3}{4}$, $f(0) = 17 \Rightarrow$ **$4 \cdot f(26) = ?$**

A) 130 B) 146 C) 152 D) 160 E)180

12. $f : R \rightarrow R$,

$f(x-1) + f(x+1) = 8x - 18 \Rightarrow$ **$f(3) = ?$**

A) 6 B) −3 C) 0 D) 3 E) 7

13. $f : R \rightarrow (0, \infty)$

$f(x) = x^2 + 4x + 4 \Rightarrow$ **$f^{-1}(x) = ?$**

A) $\sqrt{x} - 2$ B) $\sqrt{x} + 2$ C) $\sqrt{x-2}$

D) $\sqrt{x+2}$ E) $2\sqrt{x}-1$

14. $f : R \rightarrow R^+$

$f(x) = x^2 - 6x + 11$

$\Rightarrow$ **$f^{-1}(x) = ?$**

A) $\sqrt{x} - 3$ B) $\sqrt{x-3}$ C) $\sqrt{x-3} + 2$

D) $\sqrt{x-2} + 3$ E) $\sqrt{x-2} - 3$

15. $f : R \rightarrow R$

$f(x) = \sqrt[3]{x+2} \Rightarrow$ **$f^{-1}(x) = ?$**

A) $\sqrt[3]{x}-2$ B) $x^3 + 2$ C) $x^3 - 2$

D) $\sqrt[3]{x-2}$ E) $x + 2$

16. $f(x) = 2x + 3 \Rightarrow$ **$f(3x+2) = ?$**

A) $\dfrac{3}{2}f(x)$ B) $2\,f(x) + 3$ C) $2\,f(x) - 3$

D) $3\,f(x) + 2$ E) $3\,f(x) - 2$

17. $f(2x) - f(3x-1) = f(x^2) + 1 \Rightarrow$ **$f(5) = ?$**

A) −2 B) −1 C) 0 D) 1 E) 2

18. $f(x+4) + f(x-2) = 6x + 3 \Rightarrow$ **$f(6) - f(-6) = ?$**

A) 32 B) 36 C) 42 D) 48 E) 54

YANITLAR / ANSWERS

1	2	3	4	5	6	7	8	9
E	C	D	A	A	C	C	D	E
10	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**
C	B	D	A	D	C	E	B	B

1. $f(x) = 3x - 4 \Rightarrow \mathbf{f^{-1}(x) = ?}$

A) $3x + 4$ B) $\frac{1}{3x-4}$ C) $4x - 3$

D) $\frac{x-4}{3}$ E) $\frac{x+4}{3}$

2. $f\left(\frac{3x+7}{2x-1}\right) = 2x + 6 \Rightarrow \mathbf{f^{-1}(6) = ?}$

A) –7 B) –1 C) 0 D) 1 E) 7

3. $f\left(\frac{2x+1}{x-2}\right) = x - 1 \Rightarrow \mathbf{f^{-1}(2) = ?}$

A) 3 B) 4 C) 7 D) 11 E) 13

4. $f(x) = x^3 - 2ax - 9$, $f^{-1}(-2) = -1 \Rightarrow \mathbf{a = ?}$

A) 4 B) 5 C) 6 D) 8 E) 9

5. $f(3x - 1) = 2mx + 5$

$f^{-1}(10) = 2 \Rightarrow \mathbf{m = ?}$

A) 2 B) $\frac{5}{2}$ C) 3 D) $\frac{7}{2}$ E) 4

6. $f : R \rightarrow R$, $f(x) = 2x - a + 1$

$f^{-1}(-2) = 4 \Rightarrow \mathbf{a = ?}$

A) 2 B) 3 C) 5 D) 9 E) 11

7. $f(3x - 2) = 5x + 3$

$f^{-1}(a - 2) = 1 \Rightarrow \mathbf{a = ?}$

A) 5 B) 6 C) 10 D) 12 E) 18

8. $f(2^x + 3^x) = 4^x - 19 \Rightarrow \mathbf{f^{-1}(45) = ?}$

A) 3 B) 18 C) 27 D) 35 E) 64

9. $\left.\begin{array}{l} f(x)=4x-3 \\ g(x)=x^2-1 \end{array}\right\} \Rightarrow \mathbf{(fog)(2)=?}$

A) 9 B) 10 C) 11 D) 12 E) 13

10. $\left.\begin{array}{l} f(x)=6x+3 \\ (fog)(x)=12x+27 \end{array}\right\} \Rightarrow \mathbf{g(x)=?}$

A) $2x+1$ B) $x+4$ C) $2x+4$ D) $3x-1$ E) $3x+1$

11. $f(3x+4)=6x-1 \Rightarrow \mathbf{f(x)=?}$

A) $2x-7$ B) $2x-8$ C) $2x-9$ D) $2x+7$ E) $2x+8$

12. $\left.\begin{array}{l} f(x)=3x-1 \\ (gof)(x)=12x-7 \end{array}\right\} \Rightarrow \mathbf{g(x)=?}$

A) $4x-3$ B) $4x+3$ C) $4x-7$ D) $4x+7$ E) $3x+12$

13. $\left.\begin{array}{l} f(x)=2x+1 \\ g(x)=x-1 \end{array}\right\} \Rightarrow (fog)^{-1}(1)=?$

A) 1 B) 2 C) 3 D) 4 E) 5

14. $\left.\begin{array}{l} f(x)=2x-5 \\ (fog)(x)=4x-1 \end{array}\right\} \Rightarrow \mathbf{g^{-1}(0)=?}$

A) –3 B) –1 C) 0 D) 1 E) 3

15. $f(x-2)=2x+3$

$g(x)=x+2$

$\Rightarrow \mathbf{(fog^{-1})(5)=?}$

A) 8 B) 10 C) 12 D) 13 E) 15

16. $f: R \rightarrow R,\ f(x)=4x-5,$

$g: R \rightarrow R,\ g(x)=\frac{1}{3}x-2$

$\Rightarrow \mathbf{(fog^{-1})(x)=?}$

A) $12x+19$ B) $12x+29$ C) $4x+5$ D) $6x+7$ E) $8x-7$

17. $(fog)(x)=x$

$(g^{-1}of^{-1}oh)(x)=3x-2$

$\Rightarrow \mathbf{h(0)=?}$

A) –2 B) 1 C) 4 D) 7 E) 9

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1	2	3	4	5	6	7	8	9
E	A	C	A	B	E	C	D	A
10	**11**	**12**	**13**	**14**	**15**	**16**	**17**	
C	C	A	A	B	D	A	A	

1. $f: R - \{1\} \rightarrow R$,

$f(x) = \frac{1}{x-1} \cdot f(x+1)$,

$f(2) = 6 \Rightarrow$ **$f(5) = ?$**

A) 18 B) 24 C) 36 D) 48 E) 54

2. $f^2(x+1) - 2f(2x) = -1$,

$\Rightarrow$ **$f(2) = ?$**

A) -1 B) $-\frac{1}{2}$ C) 0 D) 1 E) $\frac{1}{2}$

3. $f: R - \{7\} \rightarrow R - \{5\}$,

$f(x) = \frac{mx-4}{3x+n} \Rightarrow$ **$m + n = ?$**

A) -7 B) -6 C) -4 D) -3 E) -1

4. $f(x) = f(x+1) + 10$,

$f(1) = 10 \Rightarrow$ **$f(20) = ?$**

A) -180 B) -160 C) -120 D) 120 E) 180

5. $f(x-1) = x^3 - 3x^2 + 3x - 3$,

$\Rightarrow$ **$f(x) = ?$**

A) x^3 B) $x^3 - 1$ C) $x^3 - 2$

D) $x^3 + 1$ E) $x^3 + 2$

6. $f(x) = \frac{x}{x-1} \Rightarrow$ **$f(2x) = ?$**

A) $\frac{2f(x)}{f(x)-2}$ B) $\frac{2f(x)}{f(x)-1}$ C) $\frac{2f(x)}{2f(x)-1}$

D) $\frac{2f(x)}{f(x)+1}$ E) $\frac{2f(x)}{2f(x)+1}$

7. $f^{-1}(x-1) = 2x + 3$

$\Rightarrow$ **$f(x+1) = ?$**

A) $\frac{x}{2}$ B) $\frac{x-1}{2}$ C) $\frac{x-2}{2}$

D) $\frac{x-3}{2}$ E) $\frac{x-4}{2}$

8. $(f^{-1}og)(x) = 3x - 11$,

$f(x) = \frac{1}{x-1} \Rightarrow$ **$g(2) = ?$**

A) $-\frac{1}{2}$ B) $-\frac{1}{4}$ C) $-\frac{1}{6}$ D) $\frac{1}{4}$ E) $\frac{1}{2}$

9. $f\left(\dfrac{x+2}{x-3}\right)=\dfrac{x-3}{x+2}+1 \Rightarrow$ **$f(x)$ = ?**

A) $\dfrac{x+1}{x}$　B) $\dfrac{x+2}{x}$　C) $\dfrac{x+3}{x}$　D) $\dfrac{x+1}{x-1}$　E) $\dfrac{x-1}{x+1}$

10. $f(x)=\dfrac{3x}{x+1} \Rightarrow$ **$f(2x)$ = ?**

A) $\dfrac{f(x)}{2f(x)+1}$　B) $\dfrac{3f(x)}{2f(x)+1}$　C) $\dfrac{3f(x)}{f(x)+3}$　D) $\dfrac{6f(x)}{f(x)+1}$　E) $\dfrac{6f(x)}{f(x)+3}$

11. $f(2x-7)=3x^2+1$,

$f^{-1}(3m+7)=5 \Rightarrow$ **m = ?**

A) 36　B) 34　C) 32　D) 27　E) 24

12. $f(x)=x^2+x$,

$gof(x)=2x^2+2x+4 \Rightarrow$ **$g^{-1}(x)$ = ?**

A) $\dfrac{x-1}{2}$　B) $\dfrac{x-2}{2}$　C) $\dfrac{x-3}{2}$　D) $\dfrac{x-4}{2}$　E) $\dfrac{x+1}{2}$

13. $f(x)=\begin{cases} x^2-4x+5, & x\geq 2 \\ 2x-3, & x<2 \end{cases}$

$g(x)=2x+1 \Rightarrow$ **$(g^{-1}of)(-1)$ = ?**

A) –4　B) –3　C) –2　D) 2　E) 3

14. $f(x+1)=x-3$,

$fog(x)=x^2+2x-3 \Rightarrow$ **$g(x)$ = ?**

A) x^2+x　B) x^2+x+1　C) x^2+2x　D) x^2-2x+1　E) x^2+2x+1

15. $f(x)=x^2-2x+1$

$g[f(x+1)]=x^2+4 \Rightarrow$ **$g(4)$ = ?**

A) 2　B) 4　C) 8　D) 16　E) 24

16. $f: R-\{4\} \rightarrow R-\{2m+7\}$,

$f(x)=\dfrac{5x+6}{x-n} \Rightarrow$ **m + n = ?**

A) –3　B) –2　C) 0　D) 2　E) 3

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1	2	3	4	5	6	7	8
C	D	B	A	C	D	E	C
9	**10**	**11**	**12**	**13**	**14**	**15**	**16**
A	E	B	D	B	E	C	E

1. $f(x) = ax + b$,

$(fof)(x) = 3f(x) + 5 \Rightarrow$ **$f(2) = ?$**

A) 11 B) 12 C) 13 D) 14 E) 15

2. $(gof)(x) = \dfrac{2x-1}{x-1}$, $g(x) = 2 + \dfrac{1}{x} \Rightarrow$ **$f(4) = ?$**

A) 1 B) 2 C) 3 D) 4 E) 5

3. $f(x) = 2^x + 5^x$

$g(x) = \dfrac{f(x+1)}{f(x-1)} \Rightarrow$ **$g(0) = ?$**

A) $\frac{5}{2}$ B) $\frac{7}{2}$ C) $\frac{11}{2}$ D) 10 E) 14

4. $f(x) = x + 11$,

$g(x) = \dfrac{3x+1}{4}$

$(gof)(a) = 1 \Rightarrow$ **$a = ?$**

A) −10 B) − 8 C) − 5 D) − 3 E) −1

5. $f(x) = \begin{cases} x^2-3x+1, & x \le 0 \\ 2-x, & x < 0 \end{cases}$

$\Rightarrow$ **$(fofof)(-2) = ?$**

A) 101 B) 103 C) 107 D) 109 E) 113

6. $g(x) = 2x + 5$

$(fog)(x) = \dfrac{x+3}{x+2} \Rightarrow$ **$f(x) = ?$**

A) $\dfrac{x+1}{x-1}$ B) $\dfrac{x^2-1}{x^2+1}$ C) $\dfrac{x^2-1}{(x-1)^2}$

D) $\dfrac{x+1}{x^2-1}$ E) x^2-1

7. $f(x) = \begin{cases} x+3, & x \equiv 1 \pmod 2 \\ 3x+1, & x \equiv 0 \pmod 2 \end{cases}$

$\Rightarrow$ **$(fof)(2) + f(3) = ?$**

A) 13 B) 16 C) 18 D) 22 E) 25

8. $f(x) = \dfrac{x+1}{x} \Rightarrow$ **$f(2x) = ?$**

A) $\dfrac{f(x)+1}{f(x)}$ B) $\dfrac{f(x)-1}{f(x)}$ C) $\dfrac{f(x)+1}{2}$

D) $\dfrac{2}{1-f(x)}$ E) $\dfrac{f(x)}{f(x)-1}$

9. $f(x+1) = x \cdot f(x)$

$f(1) = 10 \Rightarrow \mathbf{f(10) = ?}$

A) 11! B) 10·10! C) 10! D) 9! E) 45

10. $f(x) = 2^{x+1} \Rightarrow \mathbf{f(x) + f(x-1) = ?}$

A) $2f(x)$ B) $\frac{f(x)}{3}$ C) $f(x)^2$

D) $\frac{3f(x)}{2}$ E) $\frac{f(x)^2}{2}$

11. $f(x) = -2x + 5 \Rightarrow \mathbf{f(3x) = ?}$

A) $3f(x) + 5$ B) $3f(x) + 7$ C) $3f(x) - 20$

D) $3f(x) - 10$ E) $3f(x) + 10$

12. $A = \{1,2,3,4\}$,

f: A → A, g: A → A

$f = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 1 & 4 & 2 \end{pmatrix}$

$g = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 4 & 2 & 1 & 3 \end{pmatrix} \Rightarrow \mathbf{(fog) = ?}$

A) $\begin{pmatrix} 1 & 2 & 3 & 4 \\ 4 & 1 & 3 & 2 \end{pmatrix}$ B) $\begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 1 & 2 & 4 \end{pmatrix}$

C) $\begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 1 & 3 & 4 \end{pmatrix}$ D) $\begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 1 & 4 & 3 \end{pmatrix}$

E) $\begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & 3 & 2 & 4 \end{pmatrix}$

13. $A = \{a,b,c,d\}$,

f: A → A,

$f = \begin{pmatrix} a & b & c & d \\ d & b & a & c \end{pmatrix} \Rightarrow \mathbf{(fof) = ?}$

A) $\begin{pmatrix} a & b & c & d \\ b & c & d & a \end{pmatrix}$ B) $\begin{pmatrix} a & b & c & d \\ c & b & d & a \end{pmatrix}$

C) $\begin{pmatrix} a & b & c & d \\ c & d & b & a \end{pmatrix}$ D) $\begin{pmatrix} a & b & c & d \\ d & b & c & a \end{pmatrix}$

E) $\begin{pmatrix} a & b & c & d \\ a & b & d & c \end{pmatrix}$

14. $A = \{1,2,3,4\}$,

f : A → A, g : A → A

$f = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 1 & 4 & 2 \end{pmatrix}$

$(fog)^{-1} = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & 2 & 3 & 4 \end{pmatrix} \Rightarrow \mathbf{g = ?}$

A) $\begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 4 & 3 & 1 \end{pmatrix}$ B) $\begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 4 & 1 & 3 \end{pmatrix}$

C) $\begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 1 & 3 & 4 \end{pmatrix}$ D) $\begin{pmatrix} 1 & 2 & 3 & 4 \\ 4 & 2 & 1 & 3 \end{pmatrix}$

E) $\begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 1 & 4 & 2 \end{pmatrix}$

15. $A = \{1, 2, 3, 4\}$

f: A → A, g: A → A

$f = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 4 & 1 & 2 \end{pmatrix}$

$fog = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 4 & 1 & 3 & 2 \end{pmatrix} \Rightarrow \mathbf{g = ?}$

A) $\begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 1 & 4 & 2 \end{pmatrix}$ B) $\begin{pmatrix} 1 & 2 & 3 & 4 \\ 4 & 3 & 1 & 2 \end{pmatrix}$

C) $\begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 1 & 4 \end{pmatrix}$ D) $\begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 1 & 4 & 3 \end{pmatrix}$

E) $\begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & 1 \end{pmatrix}$

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1	2	3	4	5	6	7	8
A	C	D	A	D	A	B	C
9	**10**	**11**	**12**	**13**	**14**	**15**	**16**
C	D	D	C	B	B	C	C

1. $(fog)(x) = 3(gof)(x) + x^2$

$g(x) = 3x + 2 \Rightarrow$ **$f(-1) = ?$**

A) -1 B) $-\frac{7}{8}$ C) $-\frac{3}{4}$ D) $-\frac{5}{7}$ E) $-\frac{3}{5}$

2. $(f(x))^3 - 3x \cdot f(x) + 4 = 0$

$\Rightarrow$ **$f^{-1}(2) = ?$**

A) $\frac{1}{3}$ B) $\frac{1}{2}$ C) 1 D) 2 E) 3

3. $f(g(x^4 + 1) - 2) = 4x^8 + 5x^4 + 6$

$g(-2) = 5 \Rightarrow$ **$f(3) = ?$**

A) 24 B) 25 C) 26 D) 27 E) 28

4. $(fog)(x) = g(x) + 2a$

$f^{-1}(11) = 3 \Rightarrow$ **$a = ?$**

A) 3 B) 4 C) 5 D) 6 E) 7

5. $(x - 2)f(x) = x^3 + 3f(x + 1)$

$\Rightarrow$ **$f(4) = ?$**

A) $-\frac{89}{9}$ B) $-\frac{67}{9}$ C) $-\frac{44}{9}$ D) $-\frac{31}{9}$ E) $-\frac{23}{9}$

6. $f\left(\frac{4}{x}\right) + 2f\left(\frac{x}{4}\right) = 2x$

$\Rightarrow$ **$f(2) = ?$**

A) 9 B) $\frac{28}{3}$ C) $\frac{29}{3}$ D) 10 E) $\frac{31}{3}$

7. $f(x) = 3^x - 4 \Rightarrow$ **$f(2x) = ?$**

A) $f^2(x) + 4$ B) $f^2(x) + 16$ C) $f^2(x) + 8f(x) + 16$ D) $f^2(x) + 8f(x) + 20$ E) $f^2(x) + 8f(x) + 12$

8. $f(x^2 + 4x + 2) = x^4 + 8x^3 + 16x^2 + 12$

$\Rightarrow$ **$f(x) = ?$**

A) $x^2 - 4x + 4$ B) $x^2 + 4x + 4$ C) $x^2 - 4x + 12$ D) $x^2 - 4x + 16$ E) $x^2 + 4x + 12$

9. $f(x^2+4)=3x^2+15$

$\Rightarrow$ **$f(x)=?$**

A) $3x-1$ B) $3x+1$ C) $3x+3$ D) $2x+3$ E) $2x+1$

10. $(fogoh)(x)=4x+3$

$h(x)=2x+1$

$g(5)=7$

$\Rightarrow$ **$f^{-1}(11)=?$**

A) $\frac{7}{3}$ B) 4 C) $\frac{11}{2}$ D) $\frac{13}{3}$ E) 7

11. $f\left(x-\frac{2}{x}\right)=x^3-6x+\frac{12}{x}-\frac{8}{x^3}+4$

$\Rightarrow$ **$f(-2)=?$**

A) -8 B) -6 C) -4 D) 4 E) 8

12. $f(x)=\frac{3x-1}{2x+a}$

$(fofofofof)(x)=f(x)$

$\Rightarrow$ **$a=?$**

A) 3 B) 2 C) 1 D) -2 E) -3

13. $f: R\rightarrow R,\quad g: R\rightarrow R$

$(fog)(x+4)=f(2x+7)$,

$(gof)(x-3)=g(4x+1)\ \Rightarrow$ **$(f+g)(1)=?$**

A) 15 B) 16 C) 17 D) 18 E) 19

14. $f(x+y)=f(x)+f(y)$

$f(2)=6\ \Rightarrow$ **$f(6)=?$**

A) 12 B) 18 C) 24 D) 30 E) 36

15. $f(x\cdot y)=f(x)\cdot f(y)$

$f(2)=3\ \Rightarrow$ **$f(8)=?$**

A) 8 B) 9 C) 12 D) 18 E) 27

16. $5f\left(\frac{x}{5}\right)=f\left(\frac{5}{x}\right)+x^2+a$

$f(1)=7\Rightarrow$ **$a=?$**

A) 1 B) 2 C) 3 D) 4 E) 5

YANITLAR / ANSWERS

1	2	3	4	5	6	7	8
B	D	D	B	A	B	D	D
9	**10**	**11**	**12**	**13**	**14**	**15**	**16**
C	E	C	E	D	B	E	C

1. $f(x)=\dfrac{f^2(x)-4f(x)+3x}{f(x)-x} \Rightarrow$ $\mathbf{f^{-1}(x)=\,?}$

A) $\dfrac{4x}{x+3}$ B) $\dfrac{-4x}{x+3}$ C) $\dfrac{3x}{x+4}$

D) $\dfrac{-3x}{x+4}$ E) $\dfrac{-3x}{x-4}$

2. $fog(x)=12x+1$

$hog(x)=3x-2 \Rightarrow$ $\mathbf{(foh^{-1})(x)=\,?}$

A) $4x-9$ B) $4x+9$ C) $4x-7$

D) $4x+7$ E) $4x-1$

3. $A=\{1, 2, 3, 4\}$,

$f=\begin{pmatrix}1 & 2 & 3 & 4\\ 2 & 4 & 3 & 1\end{pmatrix}$, $gof=\begin{pmatrix}1 & 2 & 3 & 4\\ 3 & 2 & 1 & 4\end{pmatrix}$

$\Rightarrow$ $\mathbf{g=\,?}$

A) $\begin{pmatrix}1 & 2 & 3 & 4\\ 4 & 1 & 2 & 3\end{pmatrix}$ B) $\begin{pmatrix}1 & 2 & 3 & 4\\ 2 & 1 & 3 & 4\end{pmatrix}$

C) $\begin{pmatrix}1 & 2 & 3 & 4\\ 2 & 1 & 4 & 3\end{pmatrix}$ D) $\begin{pmatrix}1 & 2 & 3 & 4\\ 4 & 3 & 1 & 2\end{pmatrix}$

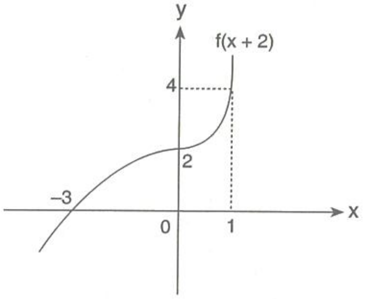
E) $\begin{pmatrix}1 & 2 & 3 & 4\\ 3 & 4 & 1 & 2\end{pmatrix}$

4. $f(x^2-4x+5)=x+1 \Rightarrow$ $\mathbf{f^{-1}(x)=\,?}$

A) x^2-4x+6 B) x^2-4x+8 C) $x^2-4x+10$

D) x^2-6x+9 E) $x^2-6x+10$

5.



$\Rightarrow$ $\mathbf{\dfrac{f(3)+f(-1)}{f^{-1}(2)}=\,?}$

A) 2 B) 3 C) 4 D) 5 E) 6

6. $g(2x^2+3x-5)=6x^2+9x+5$,

$f\left(\dfrac{x-4}{x+1}\right)=2\cdot\left(\dfrac{x+1}{x-4}\right)-3 \Rightarrow$ $\mathbf{(fog)(-6)=\,?}$

A) –4 B) –3 C) –2 D) 2 E) 4

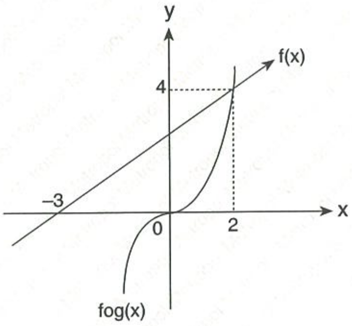
7. $(gof^{-1})(2x+4)=g(x+1) \Rightarrow$ $\mathbf{f(2)=\,?}$

A) 3 B) 4 C) 5 D) 6 E) 8

8. $f\left(\dfrac{x}{3}\right)=x\cdot f\left(\dfrac{3}{x}\right)+5 \Rightarrow$ $\mathbf{f(3)=\,?}$

A) $\dfrac{-29}{4}$ B) $\dfrac{-27}{4}$ C) $\dfrac{-25}{4}$ D) $\dfrac{25}{4}$ E) $\dfrac{27}{4}$

9.



$\Rightarrow$ **fog(2) + g(2) = ?**

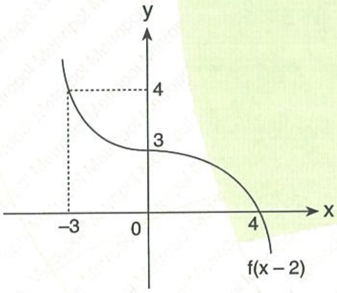
A) 2 B) 4 C) 6 D) 8 E) 10

10. $(fog)^{-1}(3) = -5$,

$g(-5) = 7 \Rightarrow$ **f(7) = ?**

A) –5 B) –3 C) 3 D) 5 E) 7

11.



$$\Rightarrow \frac{f(-2)+f^{-1}(0)}{f^{-1}(4)} = ?$$

A) –2 B) –1 C) 0 D) 1 E) 2

12. $f(x) = x - 1$,

$g(x) = \dfrac{3x+1}{x-5}$,

$(g^{-1}of)(x) = 4 \Rightarrow$ **x = ?**

A) –12 B) –8 C) –6 D) –4 E) –2

13. $f^{-1}(x) = 2x - 3$, $fog^{-1}(x) = \dfrac{x+4}{3} \Rightarrow$ **g(x) = ?**

A) $\dfrac{3x-1}{4}$ B) $\dfrac{3x+1}{2}$ C) $\dfrac{3x-1}{2}$

D) $\dfrac{3x+1}{4}$ E) $\dfrac{3x-2}{4}$

14. f : {(1, 2), (2, 3), (3, 4), (–1, 5), (–2, 6)}

g : {(0, –2), (3, –4), (–2, 1), (5, 7)}

$\Rightarrow$ **2f + g = ?**

A) {(3, 4), (–2, 13)}

B) {(3, 0), (–2, 10)}

C) {(1, 3), (3, 4), (–2, 10)}

D) {(6, 0), (6, 5)}

E) {(5, 4), (3, 4), (–2, 11)}

15. $f(6x + 2) = 2x - 1$,

$f^{-1}og^{-1}(x) = 3x \Rightarrow$ **g(2) = ?**

A) $\dfrac{11}{4}$ B) $\dfrac{11}{3}$ C) $\dfrac{11}{2}$ D) 11 E) 22

16. $f(x) = \dfrac{x+1}{x-2} \Rightarrow$ **f(x + 2) = ?**

A) $\dfrac{5f(x)+2}{f(x)-1}$ B) $\dfrac{5f(x)-2}{2f(x)-1}$ C) $\dfrac{5f(x)-2}{2f(x)+1}$

D) $\dfrac{f(x)-2}{5f(x)+1}$ E) $\dfrac{f(x)+2}{5f(x)-1}$

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1	2	3	4	5	6	7	8
A	B	D	E	A	C	D	C
9	**10**	**11**	**12**	**13**	**14**	**15**	**16**
C	C	B	A	B	A	B	C

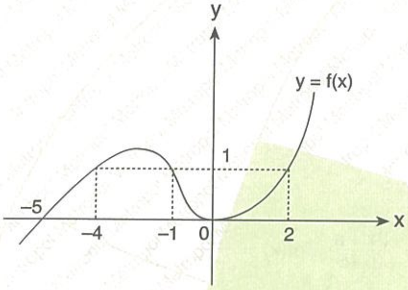
1. $f(x) = \dfrac{2x+1}{x+a}$,

$g(x) = 3x - 5$,

$(g^{-1}of)^{-1}(2) = 3 \Rightarrow$ **a = ?**

A) –4 B) –3 C) –1 D) 3 E) 4

2.



$f(x + 3) = 1 \Rightarrow \sum x = ?$

A) –12 B) –7 C) –5 D) –3 E) –1

3. $f(x^2 + x) = 3x^2 + 3x + 2$

$\Rightarrow$ $\mathbf{f^{-1}(-2) = ?}$

A) $-\frac{5}{3}$ B) $-\frac{4}{3}$ C) –1 D) $-\frac{2}{3}$ E) $-\frac{1}{3}$

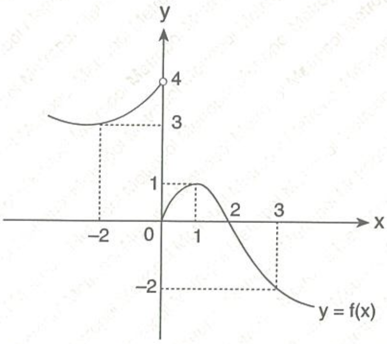
4. $f(x) = x - 3$,

$gof(x) = x^2 - 6x + 7 \Rightarrow$ **g(x) = ?**

A) $x^2 - 3x + 1$ B) $x^2 - 2x + 3$ C) $x^2 + 2$

D) $x^2 - 2$ E) $x^2 - 3$

5.

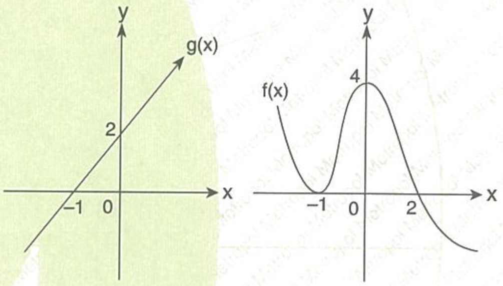


$f[f(x_1)] = 3$

$f[f(x_2)] = -2 \Rightarrow$ $\mathbf{x_1 + x_2 = ?}$

A) –2 B) –1 C) 0 D) 1 E) 2

6.



$\Rightarrow$ $\mathbf{(g^{-1}ofof)(2) = ?}$

A) – 1 B) $-\frac{1}{2}$ C) $\frac{1}{2}$ D) 1 E) $\frac{3}{2}$

7. $f(x^3) = x^6 + 2x^3 + 1 \Rightarrow$ **f(3) = ?**

A) 5 B) 6 C) 7 D) 9 E) 16

8. $f(x) = x + 3$,

$g^{-1}(x) = \dfrac{1-x}{x+1} \Rightarrow$ $\mathbf{(f^{-1}og)(1) = ?}$

A) –3 B) –2 C) 0 D) 2 E) 3

9. $f(x) = I(x) = x,$

$f(ax + b) = (12 - 3a)x + 30 - 2b \Rightarrow$ **$a + b = ?$**

A) 12 B) 13 C) 15 D) 18 E) 21

10. $f(x) = c,$

$f(x) = (m - 4)x^2 + (3m - n + 3)x + m \cdot n - 5$

$\Rightarrow$ **$f(2012) = ?$**

A) –65 B) –55 C) 0 D) 55 E) 65

11. $f(x) = \begin{cases} x^2 - 3, & x < -3 \\ 2x + 7, & -3 \leq x < 2 \\ 2 - x^2, & 2 < x \end{cases}$

$\Rightarrow$ **$f(-4) + f(-2) + f(3) = ?$**

A) 6 B) 7 C) 9 D) 10 E) 12

12. $f(x) = 3^{2x-1} \Rightarrow$ **$f(2x + 1) = ?$**

A) $\frac{f^2(x)}{27}$ B) $\frac{f^2(x)}{9}$ C) $\frac{f^2(x)}{3}$

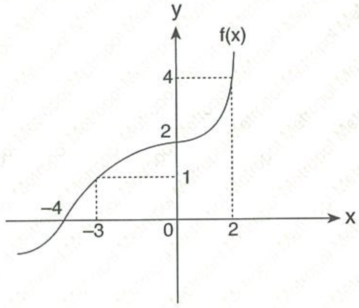
D) $9f^2(x)$ E) $27f^2(x)$

13. $f(x) - 2 = \frac{5f(x) + 3}{x + 1} \Rightarrow$ **$f^{-1}(x) = ?$**

A) $\frac{3x + 2}{x - 1}$ B) $\frac{2x + 3}{x - 3}$ C) $\frac{4x + 5}{x - 2}$

D) $\frac{x - 3}{2x - 1}$ E) $\frac{x - 2}{3x + 1}$

14.



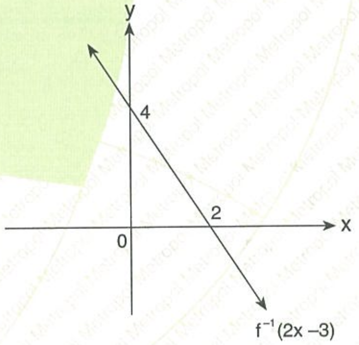
$\Rightarrow$ **$\frac{f(-3) + f^{-1}(0)}{fof(0)} = ?$**

A) $-\frac{3}{4}$ B) $-\frac{1}{2}$ C) –1 D) 1 E) $\frac{3}{4}$

15. $f(x) = \frac{3x + a}{2x - 5}$, $fof(x) = \frac{23x - 14}{-4x + 39} \Rightarrow$ **$a = ?$**

A) 4 B) 5 C) 6 D) 7 E) 8

16.



$\Rightarrow$ **$f(x) = ?$**

A) $-1 - x$ B) $1 - x$ C) $2 - x$ D) $x + 1$ E) $x + 2$

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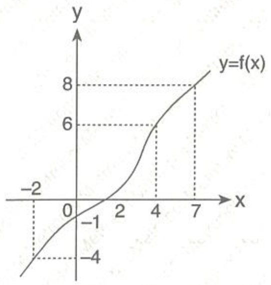
1	2	3	4	5	6	7	8
E	A	B	D	D	D	E	A
9	**10**	**11**	**12**	**13**	**14**	**15**	**16**
B	D	C	E	C	A	D	B

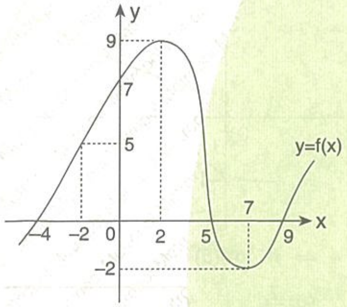
1.



$\Rightarrow$ $f(0) + f^{-1}(6) + f(7) = ?$

A) 6 B) 8 C) 9 D) 11 E) 12

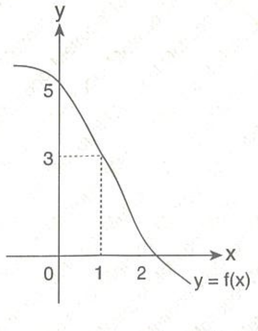
2.

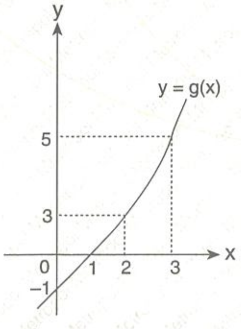


$\Rightarrow$ $f(f(f(5))) = ?$

A) 9 B) 7 C) 5 D) 0 E) –2

3.

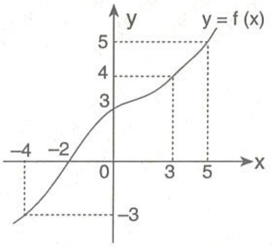




$\Rightarrow$ $(fog)(1) + (gof^{-1})(5) = ?$

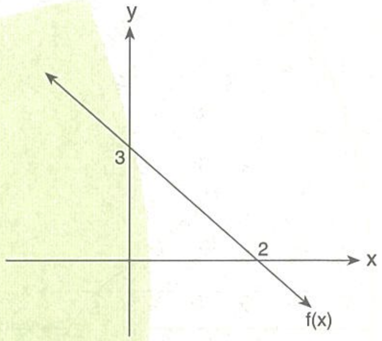
A) 1 B) 2 C) 3 D) 4 E) 5

4. $\dfrac{f^{-1}(5) + f(0)}{f^{-1}(-3) - f^{-1}(4)} = ?$



A) $-\frac{7}{8}$ B) $-\frac{5}{6}$ C) $-\frac{7}{5}$ D) $-\frac{8}{7}$ E) $-\frac{9}{7}$

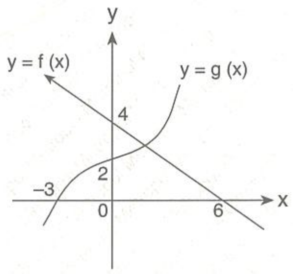
5.



$\Rightarrow$ $f(4) + f^{-1}(6) = ?$

A) –5 B) –8 C) –9 D) –10 E) –11

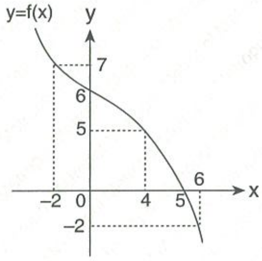
6.



$\Rightarrow$ $(fog)(-3) + (f^{-1}og)(0) = ?$

A) 3 B) 4 C) 5 D) 6 E) 7

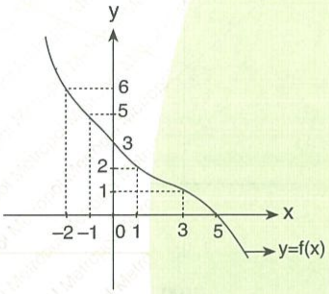
7.



$f^{-1}(f^{-1}(x+5)) = 5 \Rightarrow \mathbf{x = ?}$

A) 1 B) 2 C) 3 D) 4 E) 5

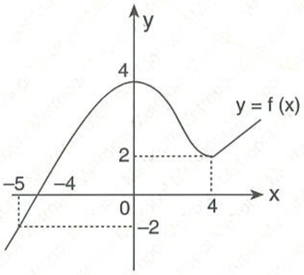
8.



$(f \circ f \circ f)(x) = 3 \Rightarrow \mathbf{x = ?}$

A) –2 B) –1 C) 0 D) 1 E) 5

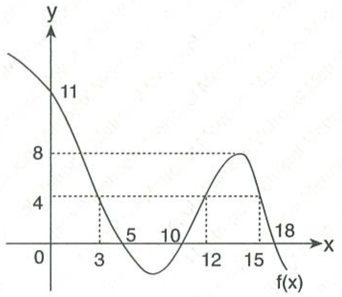
9.



$(f \circ f \circ f)(a-1) = 2 \Rightarrow \mathbf{a = ?}$

A) –6 B) –5 C) –4 D) –3 E) –2

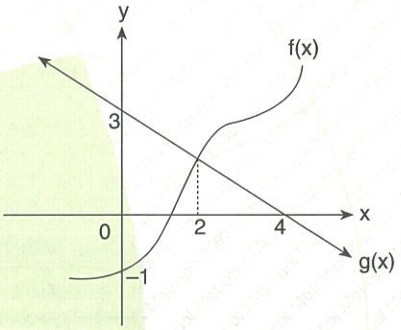
10.



$f(x-2) = 4 \Rightarrow \sum \mathbf{x = ?}$

A) 24 B) 30 C) 36 D) 42 E) 48

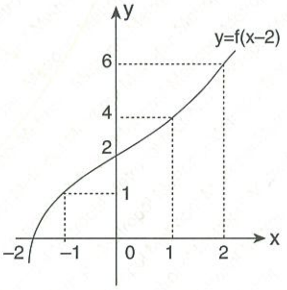
11.



$\Rightarrow \mathbf{(f^{-1} \circ f)(5) + g(8) + f(2) = ?}$

A) 3 B) $\frac{7}{2}$ C) 4 D) $\frac{9}{2}$ E) 5

12. $\dfrac{f(0)+f^{-1}(6)}{f(-1)+f^{-1}(1)} = ?$



A) 6 B) 3 C) 2 D) $\frac{4}{3}$ E) $\frac{2}{3}$

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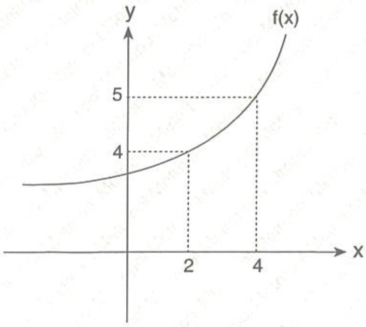
1	2	3	4	5	6	7	8	9	10	11	12
D	E	D	D	A	E	A	B	D	C	B	A

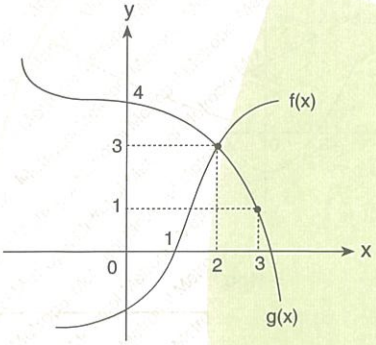
1.



$f^{-1}(5) = f(a+1) \Rightarrow$ **a = ?**

A) 0 B) 1 C) 2 D) 3 E) 4

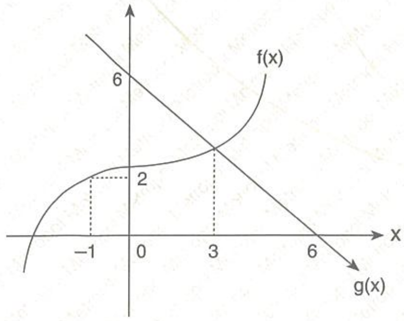
2.


y
4
f(x)
3
1
1
0
2
3
x
g(x)

$h(x) = x^2 \cdot (gof)(x) \Rightarrow$ **h(2) = ?**

A) 4 B) 8 C) 12 D) 16 E) 20

3.

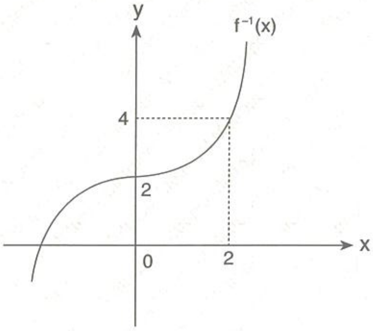

y
f(x)
6
2
–1
0
3
6
x
g(x)

$h(x) = f(x) \cdot g(x)$

$\Rightarrow$ **h(3) + h(–1) = ?**

A) 20 B) 21 C) 22 D) 23 E) 24

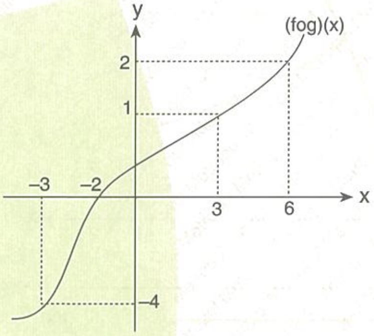
4.



$f(2x+6) = 2 \Rightarrow$ **x = ?**

A) –1 B) –2 C) –3 D) –4 E) –5

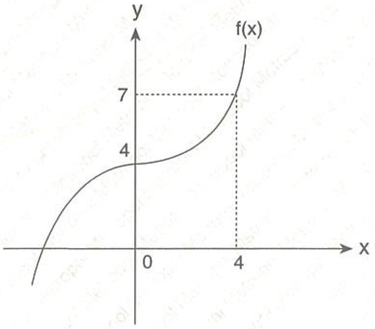
5.


y
(fog)(x)
2
1
–3
–2
3
6
x
–4

$f^{-1}(2) = 4 \Rightarrow$ **g(6) = ?**

A) 2 B) 3 C) 4 D) 5 E) 6

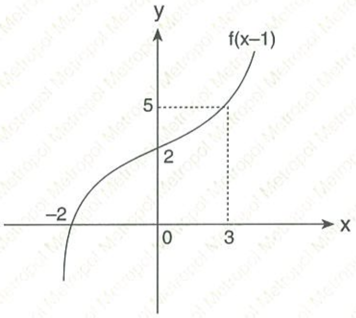
6.



$(fof)(3a-6) = 7 \Rightarrow$ **a = ?**

A) 7 B) 6 C) 4 D) 2 E) 1

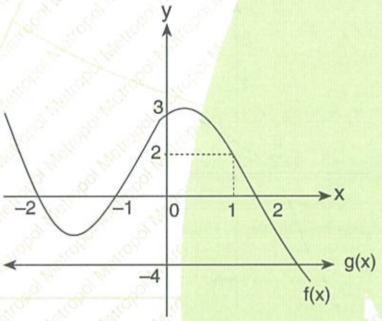
7.



$\Rightarrow (fof)(-1) = ?$

A) 1 B) 2 C) 3 D) 4 E) 5

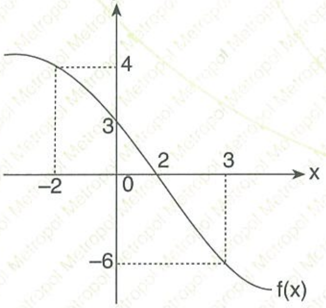
8.



$\Rightarrow (gof^{-1})(2) = ?$

A) 2 B) 1 C) 0 D) –2 E) –4

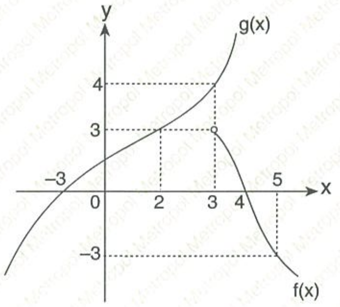
9.



$$\Rightarrow \frac{f^{-1}(4)+(fof^{-1})(-6)}{f(2)+f^{-1}(0)} = ?$$

A) –4 B) –3 C)–2 D) –1 E) 0

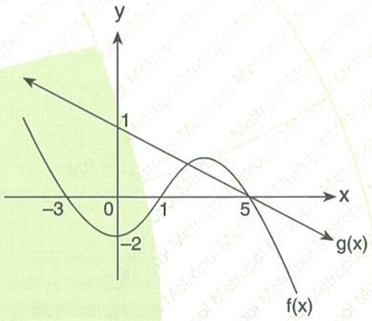
10.



$$\Rightarrow \frac{g(2)+(fog)(3)}{f(5)} = ?$$

A) –1 B) –2 C) 2 D) 3 E) 4

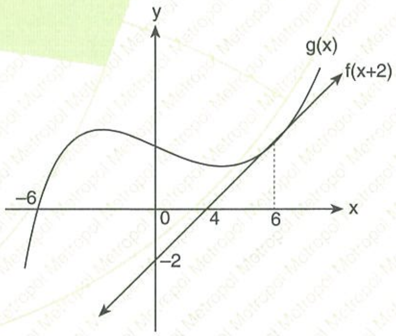
11.



$\Rightarrow (fog^{-1}of)(1) = ?$

A) 5 B) 1 C) 0 D) –2 E) –3

12.



$\Rightarrow (g^{-1}of)(6) + f^{-1}(8) = ?$

A) 8 B) 16 C) 22 D) 28 E) 32

YANITLAR / ANSWERS

1	2	3	4	5	6	7	8	9	10	11	12
B	A	D	A	C	D	E	E	A	A	C	B

BÖLÜM
CHAPTER
14

TARİHSEL NOT / HISTORICAL NOTE

Borel Felix Edouard Justin Emile [1871 - 1956]

Borel Felix Edouard Justin Emile Fransız bir matematikçi ve politikacıydı. Bir matematikçi olarak, ölçü teorisi ve olasılık alanlarında kurucu çalışmaları ile tanınıyordu.

Borel Felix Edouard Justin Emile was a French mathematician and politician. As a mathematician, he was known for his founding work in the areas of measure theory and probability.

İŞLEM ve MODÜLER ARİTMETİK
OPERATION and MODULAR SYSTEMS

Bu bölüm, 65 test sorusu içermektedir.
This chapter includes 65 test questions.

BÖLÜM / CHAPTER 14

İŞLEM ve MODÜLER ARİTMETİK / OPERATION and MODULAR SYSTEMS

- İşlem ve Modüler Aritmetik / Operation and Modular Systems .... 251 – 264

Metropol

BÖLÜM

14

CHAPTER

İŞLEM VE MODÜLER ARİTMETİK
OPERATION AND MODULAR SYSTEMS

İŞLEM / OPERATION

Tanım: $A \neq \varnothing$ olmak üzere A x A kümesinden A ya tanımlanan her fonksiyona, A da ***ikili işlem*** ya da kısaca ***işlem*** denir.

İşlemler ⊕, ●, ✳, □ , Δ , ... gibi simgelerle gösterilir.

Definition: Provided that $A \neq \varnothing$, each function defined from A x A to A is called as a *binary operation* or simply *operation*.

Operations are shown by symbols like ⊕,●, ✳, ■ ,Δ.

Özellik / Property

★ **Kapalılık Özelliği:**

A kümesi üzerinde tanımlanan bir Δ işlemi için, $\forall x, y \in A$ için $x \, \Delta \, y \in A$ ise, A kümesi Δ işlemine göre **kapalıdır** denir.

★ **Closure Property**

an operation Δ defined on set A is **closed** if

$\forall x, y \in A$, then $x \, \Delta \, y \in A$.

★ **Değişme Özelliği :**

A kümesi üzerinde tanımlanan bir Δ işlemi için, $\forall x,y \in A$ için $x \, \Delta \, y = y \, \Delta \, x$ ise Δ işlemi **değişme özelliğine** sahiptir denir.

★ **Commutative Property**

An operation Δ, defined on set A is **commutative** if for

$\forall x,y \in A, x \, \Delta \, y = y \, \Delta \, x$.

★ **Birleşme Özelliği:**

A kümesi üzerinde tanımlanan bir Δ işlemi için, $\forall x, y, z \in A$ için $(x \Delta y) \Delta z = x \Delta (y \Delta z)$ ise Δ işlemi **birleşme özelliğine** sahiptir denir.

★ **Associative Property**

An operation Δ, defined on set A is **associative** if

for $\forall x, y, z \in A$, $(x \, \Delta \, y) \, \Delta \, z = x \, \Delta \, (y \, \Delta \, z)$.

★ **Dağılma Özelliği :**

A kümesi üzerinde tanımlanan Δ ve $\square$ işlemlerinde

$\forall x,y \in A$ için $x \Delta (y \square z) = (x \Delta y) \square (x \Delta z)$ ise, Δ işleminin $\square$ işlemi üzerinde **dağılma özelliği vardır** denir.

★ **Distributive Property:**

Let Δ and $\square$ be are two operations defined on set A, if

for $\forall x,y \in A$, $x \Delta (y \square z) = (x \Delta y) \square (x \Delta z)$, Δ is distributive over $\square$.

★ **Birim (Etkisiz) Eleman :**

A kümesi üzerinde tanımlanan bir Δ işleminde, $\forall x \in A$ için $x \Delta e = e \Delta x = x$ olacak biçimde $e \in A$ varsa $e \in A$ ya Δ işlemine göre birim **(etkisiz) elemandır** denir.

★ **Identity Element:**

Let Δ be an operation on set A, if for $\forall x \in A$, $x \Delta e = e \Delta x = x$, $e \in A$ is identity element of the operation Δ.

Önemli / Important

- Bir işlemin birim elemanı varsa bir tanedir.
- The indentity element of an operation is unique, if it exists.

★ **Ters Eleman :**

A kümesi üzerinde tanımlanan Δ işleminde, A'nın Δ işlemine göre birim elemanı e ise, $\forall x \in A$ için $x \Delta y = y \Delta x = e$ koşulunu sağlayan $y \in A$ varsa bu y elemanına Δ işlemine göre **x'in tersi** denir ve $x^{-1} = y$ biçiminde gösterilir.

★ **Inverse Element:**

For an operation Δ defined on set A, if identity element for the set A under the operation Δ is e and for $\forall x \in A$ there exist $y \in A$ satisfying $x \Delta y = y \Delta x = e$, y is said to be the inverse of x under Δ and denoted as $x^{-1} = y$.

★ **Yutan Eleman :**

A kümesi üzerinde bir Δ işlemi verildiğinde, $\forall x \in A$ için $x \Delta y = y \Delta x = y$ ise y elemanına Δ işleminin **yutan elemanı** denir.

★ **Null Element:**

Let Δ be an operation defined on set A. If for $\forall x,y \in A$, $x \Delta y = y$ then y is said to be the null element.

MODÜLER ARİTMETİK / MODULER SYSTEMS

▼ $a, b, m \in Z$, $m > 1$ olmak üzere

$\beta = \{(a,b) : m,\ (a-b)$ yi tam böler$\}$ bağıntısı bir denklik bağıntısıdır.

Her $(a,b) \in \beta$ için

$a \equiv b \pmod m$ biçiminde yazılır ve **a sayısı modül m de b ye denktir** denir.

▼ Provided that $a, b, m \in Z$, and $m > 1$

The relation $\beta = \{(a,b) : m$ divides $(a-b)\}$ is an equation relation.

Each $(a,b) \in \beta$ is written as $a \equiv b \pmod m$ and read as a is equivalent to b in mod m.

$$\begin{array}{r|l} a & m \\ \cline{2-2} - & k \\ \hline b & \end{array} \quad \Rightarrow a \equiv b \pmod m$$

$$a \equiv m \cdot k + b, \quad k \in Z$$

Önemli / Important

- a sayısı m sayısına bölünürse 0, 1, 2, 3, ..., $(m-1)$ kalanlarını verir.
- If the number a is divided to m, the remainders will be 0, 1, 2, 3, ..., $(m-1)$.

DENKLİK SINIFLARI / EQUIVALENCE CLASSES

▼ Her tamsayı m ile bölündüğünde hangi kalanı veriyor ise o kalana denktir. m ye bölündüğünde aynı kalanı veren sayıların kümesine **kalan sınıfların kümesi** denir.

$Z_{/m} = \{\overline{0}, \overline{1}, \overline{2}, \overline{3}, \ldots, \overline{m-1}\}$ şeklinde gösterilir.

▼ When dividing an integer to m, the integer is called equivalent to that remainder. Set of numbers with same remainder when divided to m, is called set of remainders class.

it is shown by $Z_{/m} = \{\overline{0}, \overline{1}, \overline{2}, \overline{3}, \ldots, \overline{m-1}\}$.

BİR SAYININ KUVVETİNİN BİRLER BASAMAĞI / ONES PLACE of an EXPONENT of a NUMBER

▼ Bir sayının birler basamağını bulma işlemi (mod10)'a göre kalanın bulunması ile eş anlamlıdır.

▼ To find ones place of an exponential is same as to find the remainder when the number is divided by 10 (mod 10).

Özellik / Property

$x \equiv y \pmod{m}$

$u \equiv v \pmod{m}$, olmak üzere / provided that

★ $x + u \equiv (y + v) \pmod{m}$

★ $x - u \equiv (y - v) \pmod{m}$

★ $x \cdot u \equiv y \cdot v \pmod{m}$

★ $x^n \equiv y^n \pmod{m}$

★ $ax \equiv ay \pmod{m}$, $a \neq 0$

1. $a \Delta b = \frac{1}{a} + \frac{2}{b} \Rightarrow \left(\frac{1}{2} \Delta \frac{1}{3}\right) \Delta \frac{1}{4} = ?$

A) $\frac{65}{8}$ B) $\frac{45}{8}$ C) $\frac{35}{8}$ D) $\frac{25}{8}$ E) $\frac{15}{8}$

2. $\frac{x-y}{x \Delta y} = \frac{1}{x+y} \Rightarrow$ **$3\Delta5 = ?$**

A) 4 B) 8 C) −12 D) −14 E) −16

3. $\forall a, b \in R$

$a \Delta b = \begin{cases} a+2b, & a > b \\ 2a+b, & a < b \end{cases} \Rightarrow$ **$2\Delta(4\Delta2) = ?$**

A) 4 B) 6 C) 8 D) 10 E) 12

4. $a \Delta b = a + b + 1$

$(2 \Delta a) \Delta a = 6$

$\Rightarrow$ **$a \Delta a = ?$**

A) 3 B) 2 C) 1 D) 0 E) −1

5. $\forall x, y \in R$

$x \Delta y = 3x + 2y - 4$

$x \star y = x + 2y + 3$

$(2 \Delta x) \star (x \Delta 1) = 0 \Rightarrow x = ?$

A) −1 B) $-\frac{1}{2}$ C) $-\frac{1}{4}$ D) $-\frac{1}{8}$ E) 2

6. $a \blacksquare b = a - 4b$ ve / and $a\Delta b = \begin{cases} 2a, & a+b > 0 \\ a+2, & a+b \leq 0 \end{cases}$

$\Rightarrow$ **$(2 \blacksquare 3) \Delta (-2 \Delta 4) = ?$**

A) −8 B) −6 C) −4 D) 2 E) 4

7. $a, b \in R$

$a \star b = , \sqrt{|a.b+1|}$

$a \Delta b = ab - 2a + 1$

$\Rightarrow$ **$(3 \star 5) \Delta 2 = ?$**

A) −1 B) 1 C) 2 D) 3 E) 4

8. $\forall a, b \in R$

$(3a + b) \Delta (a - 3b) = a^2 + b^2$

$23 \Delta 1 = x \Rightarrow$ **$x = ?$**

A) 10 B) 30 C) 50 D) 53 E) 96

9. $a \blacksquare b = 3 \cdot a \cdot b + a + b$,

$a \blacksquare \frac{1}{a} = 5 \Rightarrow$ **$a = ?$**

A) 1 B) 2 C) 3 D) 4 E) 5

10. $A = \{a, b, 1, 2, 3\}$,

$\forall x, y \in A$

$f_x(y) = x \Delta y$

$g_y(x) = (y \Delta 3) \Delta x$

$\Rightarrow$ **$(f_b o\, g_2)(1) = ?$**

Δ	a	b	1	2	3
a	a	b	1	2	3
b	b	1	2	3	a
1	1	2	3	a	b
2	2	3	a	b	1
3	3	a	b	1	2

A) a B) b C) 1 D) 2 E) 3

11. $x \Delta y = \frac{x}{x^y + 1} + \frac{x}{x^{-y} + 1}$

$2 \Delta x = 2x - 6 \Rightarrow$ **$x = ?$**

A) 2 B) 4 C) 6 D) 8 E) 9

12. $x, y \in IR$

$x \Delta y = x^2 + y^2 - 2xy$ ve / and

$x \bigstar y = 3^{x-y}$,

$(8888 \Delta 8885) \bigstar x = 81$

$\Rightarrow$ **$x = ?$**

A) 3 B) 4 C) 5 D) 6 E) 13

13. $x, y \in R$

$x \Delta y = x^2 + y^2 + 2(y \Delta x)$

$\Rightarrow$ **$3 \Delta 4 = ?$**

A) –4 B) –5 C) –9 D) –16 E) –25

14. $x, y \in Z$

$$x \text{ o } y = \begin{cases} x^2 - y, & x < y \\ x + y^2, & x \geq y \end{cases}$$

$\Rightarrow$ **$(2 \text{ o } 3) \text{ o } 0 = ?$**

A) 0 B) 1 C) 2 D) 3 E) 4

15. $a, b \in R - \{0\}$

$a \square b = a^b - a$,

$3 \square x = 24$

$\Rightarrow$ **$x = ?$**

A) 1 B) 2 C) 3 D) 4 E) 5

16. $x \Delta y = \frac{3}{x} + \frac{3}{y} \Rightarrow$ **$(2\Delta3)\Delta4 = ?$**

A) 1 B) $\frac{27}{20}$ C) $\frac{33}{20}$ D) $\frac{39}{20}$ E) 2

17. $\frac{a \Delta b}{a + b} = \frac{3(b \Delta a) + 1}{a.b} \Rightarrow$ **$3\Delta1 = ?$**

A) $\frac{2}{3}$ B) $\frac{1}{3}$ C) 6 D) $-\frac{2}{9}$ E) $-\frac{4}{9}$

YANITLAR / ANSWERS

1	2	3	4	5	6	7	8	9
A	E	E	A	D	A	B	D	A
10	**11**	**12**	**13**	**14**	**15**	**16**	**17**	
A	B	C	E	B	C	D	E	

1. $x \,\Delta\, y = x^2 - 2xy + y^2 + \frac{x}{y}$

$\Rightarrow$ **$(2 \,\Delta\, 3) \,\Delta\, 1 = ?$**

A) $\frac{19}{9}$ B) $\frac{4}{3}$ C) 1 D) $\frac{2}{3}$ E) $\frac{1}{3}$

2. $x \,\Delta\, y = x^2 + y^2$

$x \,\square\, y = (x \,\Delta\, y) \,\Delta\, 2 \;\Rightarrow\;$ **$2 \,\square\, 3 = ?$**

A) 148 B) 154 C) 162 D) 168 E) 173

3. $x \,\Delta\, y = 3x - y + 1$

$5 \,\Delta\, (1 \,\Delta\, a) = 13 \;\Rightarrow\;$ **$a = ?$**

A) –2 B) –1 C) 0 D) 1 E) 2

4. $\frac{1}{x} \,\square\, \frac{1}{y} = \frac{3}{x} + \frac{4}{y} - \frac{x}{y}$

$\Rightarrow$ **$4 \,\square\, 3 = ?$**

A) $\frac{87}{4}$ B) $\frac{89}{4}$ C) $\frac{93}{4}$ D) $\frac{97}{4}$ E) $\frac{99}{4}$

5. $x \,\Delta\, e = e \,\Delta\, x = x$

$x \,\Delta\, y = 5x + 4xy + 5y + 5$

$\Rightarrow$ **$e = ?$**

A) –1 B) $-\frac{1}{2}$ C) $\frac{1}{2}$ D) 1 E) $\frac{3}{2}$

6. $x \,\Delta\, y = \min(4x + y,\, x^3 - y)$

$x \,\square\, y = \max(2^x - 3^y,\, 2x + 3y)$

$\Rightarrow$ **$(2 \,\Delta\, 3) \,\Delta\, (3 \,\square\, 2) = ?$**

A) 29 B) 30 C) 31 D) 32 E) 33

7. $x \,\Delta\, e = e \,\Delta\, x = x$

$x \,\Delta\, x^{-1} = x^{-1} \,\Delta\, x = e$

$x \,\Delta\, y = -3x - 3y + 4xy + 3 \;\Rightarrow\;$ **$2^{-1} = ?$**

A) $\frac{1}{2}$ B) $\frac{2}{3}$ C) $\frac{3}{4}$ D) $\frac{4}{5}$ E) $\frac{5}{6}$

8. $x \,\square\, y = \begin{cases} x.y + x, & x > y \\ x + y + 2xy, & x \le y \end{cases}$

$\Rightarrow$ **$(2 \,\square\, 2) \,\square\, 10 = ?$**

A) 120 B) 132 C) 136 D) 144 E) 148

9. $\dfrac{3}{x \square y} = \dfrac{5}{x} - \dfrac{2}{y}$

$4 \square a = 12 \Rightarrow$ **$a = ?$**

A) −2 B) −1 C) 0 D) 1 E) 2

10. $x \Delta e = x$

$x \square y = x + y - 4$

$x \Delta y = (x \square y) + 5$

$x \Delta x = e \Rightarrow$ **$x = ?$**

A) −3 B) −2 C) −1 D) 1 E) 2

11. $(x, y) \Delta (a, b) = (2x + a, 3y - b)$

$(1, 2) \Delta (m, 4) = (5, n) \Rightarrow$ **$m + n = ?$**

A) 2 B) 3 C) 4 D) 5 E) 6

12. $x \Delta y$ = OBEB / GCF(x, y)

$a \square b$ = OKEK / LCM(a, b)

$\Rightarrow$ **$(90 \Delta 150) \square (60 \Delta 36) = ?$**

A) 36 B) 48 C) 60 D) 72 E) 90

13.

Δ	C	A	N	E	R
C	N	E	R	C	A
A	E	R	C	A	N
N	R	C	A	N	E
E	C	A	N	E	R
R	A	N	E	R	C

$x^{-1} \Delta x = e$

$\Rightarrow$ **$(C \Delta N^{-1}) \Delta (C^{-1} \Delta N) = ?$**

A) C B) A C) N D) E E) R

14. $x \Delta y = x + y + 2a - 3$

$x \Delta e = x,$

$5 \Delta 7 = e \Rightarrow$ **$a = ?$**

A) $-\frac{3}{2}$ B) −1 C) $-\frac{1}{2}$ D) $\frac{1}{2}$ E) 1

15.

❑	1	2	3	4
1	1	2	3	4
2	2	1	4	3
3	3	2	1	4
4	4	2	3	1

$x \square e = x,\ x \square x^{-1} = e$

$(4 \square y)^{-1} = 3,$

$3 \square (x \square 2)^{-1} = 3^{-1}$

$\Rightarrow$ **$\dfrac{2}{y \square x} = ?$**

A) $\frac{1}{3}$ B) $-\frac{1}{2}$ C) 1 D) 2 E) 3

16. $x \Delta y = 4x + 4y + 12xy + 1$

$x \Delta e = x \Rightarrow$ **$e = ?$**

A) $-\frac{1}{6}$ B) $-\frac{1}{4}$ C) $-\frac{1}{2}$ D) $\frac{1}{2}$ E) $\frac{1}{4}$

YANITLAR / ANSWERS

1	2	3	4	5	6	7	8
A	E	D	C	A	D	D	B
9	10	11	12	13	14	15	16
E	C	D	C	D	A	C	B

1. $74 < x < 81$

$x \equiv 3 \pmod 7 \Rightarrow \mathbf{x = ?}$

A) 75 B) 76 C) 77 D) 78 E) 80

2. $0 < x < 8$

$73 \equiv x \pmod 8 \Rightarrow \mathbf{x = ?}$

A) 1 B) 2 C) 3 D) 5 E) 7

3. $x \in N$

$x \equiv 5 \pmod 7 \equiv 3 \pmod 5 \equiv 2 \pmod 4$

$\Rightarrow$ **min(x) = ?**

A) 136 B) 138 C) 140 D) 142 E) 144

4. $f: Z_{/5} \rightarrow Z_{/5}$,

$f(x) = x^2 + \overline{1}$

$f(a) = \overline{0}$

$\Rightarrow$ **ÇK(SS) = ?**

A) $\{\overline{1}, \overline{4}\}$ B) $\{\overline{2}\}$ C) $\{\overline{1}, \overline{3}\}$

D) $\{\overline{2}, \overline{3}\}$ E) $\{\overline{1}, \overline{2}, \overline{3}\}$

5. $3^{79} \equiv x \pmod 5 \Rightarrow \mathbf{x = ?}$

A) 0 B) 1 C) 2 D) 3 E) 4

6. $(41)^{1983} \equiv x \pmod 6 \Rightarrow \mathbf{x = ?}$

A) 0 B) 1 C) 2 D) 3 E) 5

7. $(-16)^{92} \equiv x \pmod 7 \Rightarrow \mathbf{x = ?}$

A) 1 B) 2 C) 3 D) 4 E) 5

8. $A = (73)^{98} \Rightarrow$

$$\begin{array}{r|l} A & 10 \\ \cline{2-2} - & \\ \hline \mathbf{?} & \end{array}$$

A) 1 B) 3 C) 4 D) 8 E) 9

9. $f(x) = \begin{cases} 2x-8 & x \equiv 0 \pmod 3 \\ x^2+1 & x \equiv 1 \pmod 3 \\ 3x-2 & x \equiv 2 \pmod 3 \end{cases} \Rightarrow$ **$(fofof)(6) = ?$**

A) 49 B) 50 C) 51 D) 52 E) 53

10. $4^{3n+17} \equiv x \pmod 7 \Rightarrow$ **$x = ?$**

A) 1 B) 2 C) 4 D) 5 E) 6

11. $x > 1$,

$\left.\begin{array}{l} 152 \equiv 2 \pmod x \\ 214 \equiv 4 \pmod x \end{array}\right\} \Rightarrow$ **$\max(x) = ?$**

A) 24 B) 27 C) 30 D) 35 E) 60

12. $(296)^9 + (313)^5 \equiv x \pmod{10} \Rightarrow$ **$x = ?$**

A) 6 B) 7 C) 8 D) 9 E) 10

13. $24! + 25! \equiv x \pmod 8 \Rightarrow$ **$x = ?$**

A) 0 B) 2 C) 3 D) 5 E) 7

14. $f : Z_{/5} \to Z_{/5}$,

$f(x) = \bar{3}x + \bar{4}$

$\Rightarrow$ **$f(\bar{3}) = ?$**

A) 0 B) 1 C) 2 D) 3 E) 4

15. $f : Z_{/7} \to Z_{/7}$,

$g : Z_{/7} \to Z_{/7}$,

$\left.\begin{array}{l} f(x) = \bar{2}x + \bar{1} \\ g(x) = \bar{5}x + \bar{3} \end{array}\right\} \Rightarrow$ **$(fog)(x) = ?$**

A) $\bar{3}x + \bar{1}$ B) $\bar{4}x + \bar{1}$ C) $\bar{3}x + \bar{5}$

D) $\bar{3}x + \bar{3}$ E) $\bar{3}x$

16. $f : Z_{/5} \to Z_{/5}$,

$f(x) = \bar{3}x + \bar{4}$

$\Rightarrow$ **$f^{-1}(x) = ?$**

A) $\bar{2}x + \bar{1}$ B) $\bar{2}x + \bar{2}$ C) $\bar{2}x + \bar{3}$

D) $\bar{3}x + \bar{5}$ E) $\bar{3}x + \bar{2}$

YANITLAR / ANSWERS

1	2	3	4	5	6	7	8
E	A	B	D	C	E	D	E
9	10	11	12	13	14	15	16
A	B	C	D	A	D	E	B

1. $1955^{1955} \equiv x \pmod 6$

$\Rightarrow$ **x = ?**

A) 1 B) 2 C) 3 D) 4 E) 5

2. $2008^{2012} \equiv \ldots A \Rightarrow$ **A = ?**

A) 0 B) 2 C) 4 D) 6 E) 8

3. $f: Z_{/9} \to Z_{/9}$,

$f(x) = \overline{5}x + \overline{7} \Rightarrow$ $\mathbf{f^{-1}(x) = ?}$

A) $\overline{2}x + \overline{3}$ B) $\overline{2}x + \overline{4}$ C) $\overline{2}x + \overline{5}$ D) $\overline{4}x + \overline{3}$ E) $\overline{4}x + \overline{5}$

4. $3x + 5 \equiv x + 1 \pmod 7 \Rightarrow$ **x = ?**

A) 2 B) 3 C) 4 D) 5 E) 6

5. $f: Z_{/7} \to Z_{/7}$,

$g: Z_{/7} \to Z_{/7}$,

$fog(x) = \overline{2}x + \overline{5}$,

$g(x) = \overline{3}x + \overline{4} \Rightarrow$ **f(x) = ?**

A) $\overline{3}x$ B) $\overline{3}x + 1$ C) $\overline{3}x + 2$ D) $\overline{3}x + 4$ E) $\overline{3}x + 5$

6. $\left(\frac{4}{5}\right)^{50} \equiv x \pmod 7 \Rightarrow$ **x = ?**

A) 1 B) 2 C) 4 D) 5 E) 6

7. $f: Z_{/5} \to Z_{/5}$

$g: Z_{/5} \to Z_{/5}$

$f(x) = \overline{3}x + \overline{3}$,

$g(x) = x^2 + \overline{4}x + \overline{5}$

$\Rightarrow$ $\mathbf{f^{-1}og(\overline{2}) = ?}$

A) 0 B) 1 C) 2 D) 3 E) 4

8. $3x^2 + 1 \equiv 2 \pmod 5$

$\Rightarrow$ **ÇK(SS) = ?**

A) ∅ B) {0} C) {1} D) {2} E) {3}

9. $x^2 - 2x - 1 \equiv 0 \pmod{7}$

$\Rightarrow$ **ÇK(SS) = ?**

A) {2, 3} B) {3, 4} C) {4, 5}

D) {5, 6} E) ∅

10. $x > 5$,

$3x + 17 \equiv 2x - 1 \; [\bmod\, (x - 4)]$

$\Rightarrow \sum x = ?$

A) 35 B) 37 C) 42 D) 45 E) 47

11. $999^{2013} \equiv x \pmod{7}$

$\Rightarrow$ **x = ?**

A) 1 B) 2 C) 3 D) 5 E) 6

12. $3^{-5} + \left(-\frac{1}{2}\right)^3 \cdot 13 \equiv A \pmod{5}$

$\Rightarrow$ **A = ?**

A) 0 B) 1 C) 2 D) 3 E) 4

13. $\left(\frac{5}{2}\right)^{-2008} \equiv x \pmod{9}$

$\Rightarrow$ **x = ?**

A) 2 B) 3 C) 4 D) 6 E) 7

14. $f: Z_{/7} \to Z_{/7}$,

$g: Z_{/7} \to Z_{/7}$,

$f(x) = \overline{2}x + \overline{6}$,

$gof(x) = \overline{3}x + \overline{5} \Rightarrow$ **g(x) = ?**

A) $\overline{5}x + \overline{2}$ B) $\overline{5}x + \overline{3}$ C) $\overline{5}x + \overline{4}$

D) $\overline{3}x + \overline{2}$ E) $\overline{3}x + \overline{5}$

15. $x^2 + x + 10 \equiv 0 \pmod{11}$

$\Rightarrow$ **ÇK(SS) = ?**

A) {3, 7} B) {3, 6} C) {3, 5}

D) {3, 4} E) {3, 2}

16. $7^{20} + 3 \cdot 8^{17} + 10! \equiv A \pmod{5}$

$\Rightarrow$ **A = ?**

A) 0 B) 1 C) 2 D) 3 E) 4

YANITLAR / ANSWERS

1	2	3	4	5	6	7	8
E	D	B	D	A	C	D	A
9	**10**	**11**	**12**	**13**	**14**	**15**	**16**
C	E	E	B	C	B	A	A

BÖLÜM CHAPTER 15

TARİHSEL NOT / HISTORICAL NOTE

Sir David Roxbee Cox [1924 -]

Matematiksel istatik, olasılık ve yöneylem araştırması alanlarında çalışmış İngiliz istatistikçi. Deney tasarımı, ikili dataların analizi, stokastik süreçler, kalite kontolünde ve yöneylem araştırmasında yeni yöntemlerin geliştirilmesi, kuyruk ve yenilenme teorisine olan katkıları ile tanınır.

British statistician distinguished for his work in mathematical statistics and applied probability, especially in relation to industrial and operational research. His varied contributions include the design and analysis of statistical experiments, analysis of binary data and of point stochastic process and the development of new methods in quality control and operational research particularity in relation to queuing and renewal theory.

SAYMA İLKELERİ (PERMÜTASYON, KOMBİNASYON), BİNOM AÇILIMI ve OLASILIK
COUNTING PRINCIPLES (PERMUTATIONS, COMBINATIONS), BINOMIAL EXPANSION and PROBABILTY

Bu bölüm, 89 test sorusu içermektedir.

This chapter includes 89 test questions.

BÖLÜM / CHAPTER 15

SAYMA İLKELERİ (PERMÜTASYON, KOMBİNASYON), BİNOM AÇILIMI ve OLASILIK
COUNTING PRINCIPLES (PERMUTATIONS, COMBINATIONS), BINOMIAL EXPANSION and PROBABILTY

- Sayma İlkeleri (Permütasyon, Kombinasyon), Binom Açılımı ve Olasılık
  Counting Principles (Permutations, Combinations), Binomial Expansion and Probabilty .... 265 – 286

BÖLÜM 15 CHAPTER

SAYMA İLKELERİ (PERMÜTASYON, KOMBİNASYON), BİNOM ve OLASILIK
COUNTING PRINCIPLES (PERMUTATIONS, COMBINATIONS), BINOMIAL EXPANSION and PROBABILITY

SAYMA ve SAYMANIN İLKELERİ
COUNTING and PRINCIPLES of COUNTING

Tanım: Sayma, nesnelerin ve olayların sayılarla birebir eşlenmesidir. Saymanın temel ilkeleri, **toplama ve çarpma**dır.

Definition: Counting is matching the objects and the events with the numbers. The fundamental principles of counting are the principle of **addition** and the principle of **multiplication.**

TOPLAMA İLKESİ / PRINCIPLE of ADDITION

▼ Birincisi n_1 yolla, ikincisi n_2 yolla yapılabilen iki işlem göz önüne alındığında, iki işlemden ancak birinden biri yapılabiliyorsa, bu işlemlerden biri **veya** diğeri $(n_1 + n_2)$ yolla yapılabilir. Bu ilke, sonlu sayıda işlemi içine alan durumlar için genelleştirilebilir.

▼ When two different events which the first one can happen in n_1 ways and the second one can happen in n_2 ways are taken into account for an operation, if only one way of one of two events can happen in a single run, the operation itself can happen **either** by the first way **or** the second way in $n_1 + n_2$ ways. This principle can be generalized for finite number of operations.

ÇARPMA İLKESİ / THE PRINCIPLE of MULTIPLICATION

▼ n_1 farklı yolla ortaya çıkan bir olay göz önüne alındığında, her n_1 yolu için, bu yolu izleyen ikinci olay n_2 farklı yolla ortaya çıkarsa, tüm olayın farklı biçimde ortaya çıkma sayısı $(n_1 \cdot n_2)$ olur. Eğer n_2 olayının her biri için bunu izleyen üçüncü olay n_3 yolla ortaya çıkarsa, tüm olayın ortaya çıkma sayısı $(n_1 \cdot n_2 \cdot n_3)$ olur. Bu şekilde devam edilirse,

$n_1 \cdot n_2 \cdot n_3 \cdot \ldots \cdot n_k$ yazılabilir. Buna, **çarpma ilkesi** denir.

▼ When an event which can happen in n_1 different ways is taken into account, if for all of the n_1 ways of this event, there is a follow up event which can happen in n_2 different ways, the total number of different ways in which the whole event can happen is $(n_1 \cdot n_2)$. if for all of the n_2 ways of this event, there is a follow up event which can happen in n3 different ways, the total number of different ways in which the whole event can happen is $(n_1 \cdot n_2 \cdot n_3)$. If we continue this way, $n_1 \cdot n_2 \cdot n_3 \cdot \ldots \cdot n_k$ can be written. This principle is called the **principle of multiplication**.

n_1	n_2	n_3	n_4		n_{r-1}	n_r

$$n_1 \cdot n_2 \cdot n_3 \cdot n_4 \cdot \ldots \cdot n_{r-1} \cdot n_r$$

Örneğin; Bir okuldaki 2 matematik, 3 Türkçe, 4 tarih öğretmeni arasından 1 matematik, 1 Türkçe ve 1 tarih öğretmeni, $2 \cdot 3 \cdot 4 = 24$ farklı yolla seçilebilir.

For example; From 2 Mathematics, 3 Literature and 4 History teachers in one school, 1 Mathematics, 1 Literature and 1 History teacher can be chosen in $2 \cdot 3 \cdot 4 = 24$ different ways.

Önemli / Important

- Toplama yoluyla sayma yapılırken **"veya"** ile **"ya da"**, çarpma yoluyla sayma yapılırken ise **"ve"** bağlacının kullanıldığına dikkat edilmelidir.
- You need to take into consideration that when counting by the principle addition, **"either"** and **"or"** is used; and when counting by the principle multiplication, **"and"** is used.

PERMÜTASYON / PERMUTATIONS

Tanım: n farklı elemanın bir elemanı birinci, diğeri ikinci bir diğeri de üçüncü olacak şekilde sıralanarak devam etmesine **permütasyon** denir.

▼ n farklı nesnenin (her biri bir kez kullanılması koşuluyla) r tanesi ($r \le n$) sıralanırsa, elde edilecek farklı sıralanışların sayısı P(n, r) ile gösterilirse,

Definition: A permutation of n different elements is an ordering of the elements such that one element is first, one is second, one is third and so on.

▼ The number of permutations (arrangements, orders) of n elements taken r at a time. It is denoted by P(n, r).

$P(n,r)=\dfrac{n!}{(n-r)!}$ yazılır.

Burada, P (n, r), n elemanlı bir kümenin r li permütasyonlarının sayısını göstermektedir.

Here, P(n,r) denotes the number of permutations of set of n elements, taken r at a time.

Önemli / Important

- $P(n, n) = n!$
- $P(n, 1) = n$

Önemli / Important

- Permütasyon tanımına göre birbirinden farklı diziliş olayları permütasyonla çözülebilir. Her problem çarpma kuralı ile de çözülebilir.
- According to the definition of permutation, it can be used in determining the number of ways that elements can be arranged (in order). Each problem can also be solved by the principle of multiplication.

TEKRARLI (YİNELEMELİ) PERMÜTASYON
REPEATING PERMUTATIONS

▼ n tane nesnenin n_1 tanesi bir türden, n_2 tanesi ikinci türden, ..., n_k tanesi de k ninci türden olmak üzere n nesnenin n tanesi sıralandığında, $\left(\dfrac{n!}{n_1!\cdot n_2!\cdot \ldots \cdot n_k!}\right)$ tane farklı düzen elde edilebilir.

▼ The number of arrangements of n things taken all at a time, of which n_1 are from first type, n_2 are from second type, ..., and n_k are from type k, can be happen in $\dfrac{n!}{n_1!\cdot n_2!\ldots n_k!}$ different ways.

DAİRESEL PERMÜTASYON / CIRCULAR PERMUTATIONS

▼ n tane farklı nesnenin bir daire etrafında farklı sıralanmasında nesnelerden birisi bir yere yerleştirilir ve diğer (n–1) nesnenin sıralanışı, $(n-1)\cdot(n-2)\cdot \ldots \cdot 2\cdot 1=(n-1)!$ olur. Bu da, n nesnenin dairesel sıralanış sayısını verir.

▼ The number of ways of arranging n different objects around a circular object is $(n-1)!$.
in other words, one of the objects get placed, and the remaining (n – 1) objects get placed in $(n-1)\cdot(n-2)\cdot \ldots \cdot 2\cdot 1=(n-1)!$ ways.

KOMBİNASYON / COMBINATIONS

Tanım: n tane farklı nesne verilmiş ise, bu nesnelerden herhangi r ≤ n (r pozitif bir tamsayı) tanesi seçilip alınır ve bu r nesne herhangi bir sırada yazılırsa, buna **r ninci mertebeden kombinasyon** denir.

Definition: From n different objects which are given, if we take r ≤ n objects (r is a positive integer) and write them in any order, it is called the combination of n objects, taken r at a time.

▼ Permütasyon, sıralama problemlerinin çözümünü içerir ve sıralama önemlidir. Kümenin elemanlarından alt küme seçme problemleri kombinasyon problemi olup sıralama önemli değildir.

▼ Permutations solves problems concerning different ways of ordering, and therefore the order of objects is important. On the other hand, the problems concerning subsets of a set are solved using combination, and therefore the order of objects is not important.

A = {a, b, c} kümesinin 2 li permütasyonları ve kombinasyonları aşağıda gösterilmiştir.

The number of permutations and combinations of the set A = {a, b, c}, taken 2 at a time is as follows.

Farklı üç nesnenin 6 tane 2 li permütasyonu varken, 2 li kombinasyonları 3 tanedir.

3 different objects have 6 permutations with 2 elements while they have 3 combinations with 2 elements.

Permütasyonlar Permutations	**Kombinasyonlar** Combinations
ab	{a, b}
ba	{a, c}
ac	{b, c}
ca	
bc	
cb	

▼ n elemanlı bir kümenin r elemanlı (n ≥ r) kombinasyonlarının sayısı:

▼ The number of combinations of n different objects taken r at a time (n ≥ r) is:

$$C(n,r)=\binom{n}{r}=\frac{n!}{r!\cdot(n-r)!}=\frac{P(n,r)}{r!}$$

Önemli / Important

- n elemanlı bir kümenin r elemanlı (n ≥ r) alt kümelerinden her birine n'in r'li kombinasyonu denir.
- Each one of r element subsets of a set with n elements (n ≥ r) is called a combination of r elements from n elements.

Özellik / Property

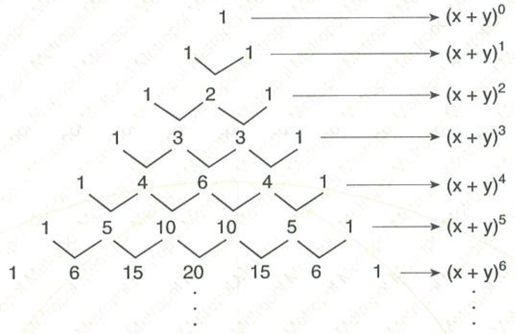
★ $C(n,0)=\binom{n}{0}=1$

★ $C(n,1)=\binom{n}{1}=\binom{n}{n-1}=n$

★ $C(n,0)+C(n,1)+C(n,2)+\ldots+C(n,n)=2^n$

★ $C(n,r)=C(n,k)\Rightarrow$ **i.** $n=r+k$ **ii.** $r=k$

BİNOM AÇILIMI ve PASCAL ÜÇGENİ / BINOMIAL EXPANSION and PASCAL'S TRIANGLE



$$(x+y)^n = \binom{n}{0}x^n + \binom{n}{1}x^{n-1}y + \binom{n}{2}x^{n-2}y^2 + \ldots + \binom{n}{n}y^n$$

Önemli / Important

- Pascal üçgeninin her bir satırındaki ilk ve son sayı 1'dir. Her satırdaki diğer sayılar, üzerinde yer alan iki sayının toplamından elde edilir.
- The first and last number in each row of Pascal's triangle is 1. Every other number in each row is formed by adding the two numbers immediately above the number.

Önemli / Important

- Her bir açılımda, x ve y simetriktir. x'in kuvveti 1 azalırken, y'nin kuvveti 1 artar.
- In each expansion, x and y have symmetrical. The powers of x decrease by 1 in successive terms, whereas the powers of y increase by 1.

Özellik / Property

★ $(x + y)^n$ nin açılımında, n + 1 tane terim vardır.
In the expansion of $(x + y)^n$, (n + 1) terms exist.

★ $(x + y)^n$ açılımında, her terimdeki x ve y nin üslerinin toplamı n'ye eşittir.
The sum of the power of x and y is equal to n in each terms of the expansion of $(x + y)^n$.

★ Baştan (r + 1). terim, $\binom{n}{r} \cdot x^{n-r} \cdot y^r$ dir.
The $(r + 1)^{th}$ term from beginning is $\binom{n}{r}\ x^{n-r} \cdot y^r$.

Özellik / Property

★ $p, q, r \in \mathbb{N}$,
$p + q + r = n \Rightarrow (ax + by + cz)^n = (ax)^p \cdot (by)^q \cdot (cz)^r \cdot \dfrac{n!}{p! \cdot q! \cdot r!}$

OLASILIK / PROBABILITY

Tanım: Olabilecek sonuçların belli fakat hangisinin olacağı kesin olmayan olayların matematiksel sayılarla ifade edilmesine **olasılık** denir.

Definition: The numeric expression of the results which are definite but it is not certain which one will happen is called probability.

- **Deney:** Sonucun gözlenmesi için yapılan her işe **olasılıkta deney** denir.
- **Experiment:** Any happening to observe the result is called a probability **experiment**

- **Çıktı :** Yapılan deneyin sonuçlarından her birine **çıktı** denir.
- **Output:** Every result of the experiment is called **output**.

- **Örnek Uzay:** Tüm çıktıların oluşturduğu kümeye **örnek uzay** denir ve E ile gösterilir.
- **Sample Space:** The set that consists of all the outputs is called **sample space**, and it is denoted by E.

- **Olay:** Bir örnek uzayın her alt kümesine **olay** denir.
- **Event:** Every subset of a sample space is called an event.

- **Olanaksız (İmkansız), Kesin Olaylar:** E örnek uzayının bir alt kümesi A olsun. Eğer $A = \varnothing$ ise olanaksız, $A = E$ ise kesin olaydır.

  Örneğin;

  a. Bir zarın atılması deneyinde gelen sayının 6 dan büyük olması olayı olanaksız olaydır.

  b. Bir zarın atılması deneyinde gelen sayının 6 dan küçük veya eşit gelmesi olayı kesin olaydır.

- **Impossible Event, Certain Event:** A is a subset of E. If $A = \varnothing$, then A is an impossible event. If $A = E$, then A is a certain event.

  For example;

  a. The event of getting a number greater than 6 on the experiment of rolling a dice is an impossible event.

  b. The event of getting a number equal to or less than 6 on the experiment of rolling a dice is a certain event.

- **Ayrık Olaylar:** A ve B kümeleri, E örnek uzayın birer alt kümesi olmak üzere, $A \cap B = \varnothing$ ise A ve B olaylarına **ayrık olaylar** denir. Örneğin; bir zarın atılması deneyinde gelen sayının 2 den küçük gelmesi olayı ile 4 ten büyük gelmesi olayları ayrık olaylardır.
- **Mutually Exclusive Events:** A and B are the subsets of E. If $A \cap B = \varnothing$, then the event A and the event B are called **mutually exclusive events**. For instance the event of getting a number less than 2 and the event of getting a number more than 4 are mutually exclusive events when a dice is rolled.

Önemli / Important

- Bir madeni paranın havaya atılması olayında, paranın üst yüzünün yazı veya tura geleceği bilinir, fakat hangi oranda yazı, hangi oranda tura geleceği olasılık hesaplamalarıyla bulunabilir.
- When a coin is tossed, we know the surface of the coin is either heads or tails but we use probability calculations to find out what ratio of the event is heads and what ratio of the event is tails.

Önemli / Important

- Bir zarın atılması deneyinde çıktılar 1, 2, 3, 4, 5, 6; bir paranın iki kere havaya atılması deneyinde çıktılar YY, YT, TY, TT dır (T: Tura, Y: Yazı).
- The outputs of the experiment of rolling a dice are 1, 2, 3, 4, 5, 6. The outputs of the experiment of tossing a coin for two times are YY, YT, TY, TT. (Y = tail, T: head)

Önemli / Important

- İki madeni paranın atılması olayında örnek uzay, E = {(YY), (YT), (TY), (TT)} dır.
- The sample space of the event of tossing two coins is E = {(YY), (YT), (TY), (TT)} .

OLASILIK FONKSİYONU / THE PROBABILITY FUNCTION

Tanım: E örnek uzayının tüm alt kümelerinin oluşturduğu kümeye S denirse,

P: S → [0, 1] şeklinde tanımlanan fonksiyona **olasılık fonksiyonu** denir ve $A \in S$ olmak üzere, P(A) sayısına da **A olayının olasılığı** denir.

Olasılıkla ilgili, aşağıdaki aksiyomlar verilebilir:

i. Olasılık fonksiyonunda görüntü kümesi [0, 1] olduğundan,

$A \in S$ için $0 \leq P(A) \leq 1$ dir.

Definition: If S is a set that consists of all the subsets of the sample space E, the function P that defined from S to [0, 1] is called the **probability function**. If $A \in S$, P(A) is **the probability of the event A**.

The following axioms are given below about the probability:

i. Since the range of the probability function is [0, 1],

for $A \in S$, $0 \leq P(A) \leq 1$.

Önemli / Important

- E örnek uzayında A ve B kümeleri için küme işlemleri geçerlidir.

  $A \cap B$, A ve B olayı

  $A \cup B$, A veya B olayı

  A′ , A olayının tümleyenidir.

  $A \cup A' = E$
- The operations of sets are obtained for the sets A and B in the sample space E.

  The event of A and B $A \cap B$,

  The event of A or B is $A \cup B$,

  The complement of A′ is A.

  $A \cup A' = E$

Özellik / Property

★ $A \cap B \neq \varnothing$, $P(A \cup B) = P(A) + P(B) - P(A \cap B)$.

★ $P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C)$

★ A olayının tümleyeni A′ olmak üzere / If the complement of A is A′, then,
$P(A) = 1 - P(A')$.

★ $A \subset B$ ise / if $P(A) \leq P(B)$.

Önemli / Important

- Bir olayın olma olasılığı 0 dan küçük, 1 den büyük olamaz.

i. $P(A) + P(A') = 1$

ii. Kesin olay için **$P(E) = 1$**

İmkansız olay için **$P(\varnothing) = 0$** dır.

iii. $A, B \in S$ için $A \cap B = \varnothing$ (A ve B ayrık olaylar) ise **$P(A \cup B) = P(A) + P(B)$** dir.

- The probability of an event can **not be less than 0 or greater** than **1**.

i. $P(A) + P(A') = 1$

ii. For a certain event; **$P(E) = 1$**

For an impossible event; **$P(\varnothing) = 0$**

iii. For $A, B \in S$, if $A \cap B = \varnothing$ (A an **B are independent events), then**

$P(A \cup B) = P(A) + P(B)$.

EŞ OLUMLU ÖRNEK UZAYDA OLASILIK HESAPLARI
CALCULATING PROBABILITY in EQUALLY LIKELY SAMPLE SPACES

Tanım: Bir deneyde bütün sonuçların olabilme olasılığı eşit ise bu şekildeki örnek uzaylara eş olumlu örnek uzay denir. Örneğin; bir zar atılması olayında 6 sayının da gelme olasılığı aynıdır. O halde, zar atma deneyinde örnek uzay bir eş olumlu örnek uzaydır.

Definition: If the happening probabilities of all the events are equal, then this kind of space is called **equally likely sample space**. For instance the probability of each number, from 1 to 6 inclusively, is equal, when a dice is tossed. Because of that, for the experiment of rolling a dice the sample is an equally likely sample space.

Eş olumlu bir örnek uzayında, $A \subset E$ olmak üzere,

In an equally likely sample space, if $A \subset E$,

$$P(A) = \frac{s(A)}{s(E)}$$

$s(A) \rightarrow$ İstenilen sonuç sayısı / Number of desired results

$s(E) \rightarrow$ Tüm olası sonuç sayısı / Number of all results

Örneğin; bir madeni paranın atılması deneyinde,

E = {Y, T} olup tura gelme olasılığı,

$P(T)=\frac{s(T)}{s(E)}=\frac{1}{2}$ dir.

Aynı şekilde bir zar atma deneyinde 2 gelme olasılığı $\frac{1}{6}$ dır.

For instance the sample space of the experiment of tossing a coin is E = {Y, T}.

The probability of getting a head is $P(T)=\frac{s(T)}{s(E)}=\frac{1}{2}$

With the same logic, the probability of getting 2 when a dice is rolled is $\frac{1}{6}$.

Önemli / Important

- $E = (A_1, A_2, \ldots, A_n\}$ örnek uzayında $P(A_1) = P(A_2) = \ldots = P(A_n)$ ise E eş olumlu bir örnek uzaydır.
- If $P(A_1) = P(A_2) = \ldots = P(A_n)$, then $E = (A_1, A_2, \ldots, A_n\}$ is an equally likely sample space.

KOŞULLU OLASILIK (BAĞIMLI OLAYLAR)
CONDITIONAL PROBABILITY (DEPENDENT EVENTS)

Tanım: Bir olayın gerçekleşmesi başka bir olayın gerçekleşmesine bağlı ise bu iki olaya **bağımlı olaylar** denir.

Definition: Two events are called **dependent events** if the occurrance of one is dependent on the occurrance of the other.

▼ Bir örnek uzayının iki olayı A ve B olup A nın gerçekleşmesi, B olayının gerçekleşmesine bağlı ise **A olayının, B olayına bağlı koşullu olasılığı P(A\B)** ile gösterilir.

▼ A and B are two events of a sample space and if the occurrance of A is depended on occurrance of B, then **the conditional probability of A conditioned to B is shown as P(A\B)**.

$$P(A \setminus B)=\frac{P(A\cap B)}{P(B)}$$

BAĞIMSIZ OLAYLAR / INDEPENDENT EVENTS

Tanım: A ve B, E örnek uzayının iki olayı olmak üzere bu iki olaydan birinin olması diğerinin olmasını etkilemiyorsa A ve B olaylarına **bağımsız olaylar** denir.

Definition: Two events A and B, two events of the sample space, are called as **independent events** if the occurrence of one has no effect on the occurrence of the other.

A ve B bağımsız olaylar / A and B are independent events $\Rightarrow P(A\cap B) = P(A) \cdot P(B) \quad [P(A) \neq 0,\ P(B) \neq 0]$

1. $P(4, 3) + P(6, 2) - P(5, 1) = ?$

A) 42 B) 46 C) 48 D) 49 E) 50

2. $P(n, 0) + P(n, 1) = 8 \Rightarrow n = ?$

A) 7 B) 6 C) 5 D) 4 E) 3

3. $P(n, 2) = 42 \Rightarrow n = ?$

A) 6 B) 7 C) 8 D) 9 E) 10

4. $\dfrac{P(n,2)}{P(n,3)} = \dfrac{1}{2} \Rightarrow n = ?$

A) 8 B) 7 C) 6 D) 5 E) 4

5. $P(n, 5) = 12 \cdot P(n, 4) \Rightarrow n = ?$

A) 15 B) 16 C) 18 D) 21 E) 24

6. $\dfrac{P(n+1, 2)}{P(n+1, 4)} = \dfrac{1}{2} \Rightarrow n = ?$

A) 3 B) 4 C) 5 D) 6 E) 7

7. $P(n, 2) - 28 = P(2n, 1) \Rightarrow n = ?$

A) 6 B) 7 C) 8 D) 9 E) 10

8. $2 \cdot P(n, 2) + P(n, 2) = 270 \Rightarrow n = ?$

A) 8 B) 10 C) 16 D) 18 E) 20

9. A = {0, 1, 2, 3, 4, 5} kümesinin elemanları kullanılarak rakamları farklı 3 basamaklı kaç tane doğal sayı yazılabilir?

By using the elements of the set A = {0, 1, 2, 3, 4, 5}, how many different three-digit natural numbers can be written if repetition of digits is not allowed?

A) 50 B) 100 C) 125 D) 150 E) 175

10. B = {1, 2, 3, 4, 5} kümesinin elemanları kullanılarak rakamları farklı üç basamaklı 200 den büyük kaç tane doğal sayı yazılabilir?

By using the elements of the set B = {1, 2, 3, 4, 5}, how many different three-digit natural numbers bigger than 200 can be written if repetition of digits is not allowed?

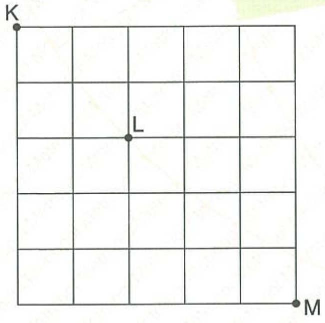
A) 48 B) 56 C) 96 D) 100 E) 128

11. 3344455 sayısının rakamlarının yerleri değiştirilerek 7 basamaklı kaç farklı sayı yazılabilir?

By changing the place of digits of the number 3344555, how many different seven-digit natural numbers can be written?

A) 220 B) 210 C) 214 D) 212 E) 210

12.



K dan başlanarak, L ye uğramak koşuluyla M ye en kısa yoldan kaç farklı şekilde gidilebilir?

Starting from the point K and passing through the point L, in how many different ways can someone go to point M using the shortest route?

A) 140 B) 120 C) 90 D) 60 E) 48

13. 6 rakamlı telefon numarasının ilk 3 numarası ile son rakamı bilinmektedir. Kaç farklı deneme ile bu telefon numarası kesin olarak tespit edilebilir?

The first three and the last digit of a six-digit phone number are known. Accordingly, with how many different trials the whole phone number can certainly be found?

A) 60 B) 70 C) 72 D) 80 E) 100

14. 4 farklı IQ kitabı 3 öğrenciye dağıtılacaktır. Her öğrenciye en az bir IQ kitabı verilmek koşuluyla tüm kitaplar kaç farklı şekilde dağıtılabilir?

4 different IQ books are to be distributed among 3 students. If each student should get at least one IQ book, in how many different ways can these 4 books be distributed?

A) 30 B) 32 C) 34 D) 35 E) 36

15. 5 kişinin katıldığı bir sınav, kişilerin başarılı veya başarısız olması yönünden kaç farklı şekilde sonuçlanabilir?

In an exam which 5 people participated in, the results of success or failure can be expressed in how many different ways?

A) 10 B) 5^2 C) 2^5 D) 5^5 E) 5^{10}

YANITLAR / ANSWERS

1	2	3	4	5	6	7	8
D	A	B	E	B	A	B	B
9	10	11	12	13	14	15	16
B	A	B	B	E	E	C	

1. $\binom{n+2}{3}=\binom{n+2}{6} \Rightarrow$ **n = ?**

A) 9 B) 8 C) 7 D) 6 E) 5

2. C(8, 1) + C(8, 2) + C(9, 3) = C(n, 3)

$\Rightarrow$ **n = ?**

A) 6 B) 7 C) 8 D) 9 E) 10

3. $P(7,5)=\binom{7}{5}\cdot x \Rightarrow$ **x = ?**

A) 2 B) 6 C) 24 D) 120 E) 720

4. $\binom{n}{1}+P(n,1)=24 \Rightarrow$ **n = ?**

A) 10 B) 11 C) 12 D) 13 E) 14

5. $\binom{7}{0}+\binom{7}{1}+\ldots+\binom{7}{7}=$ **?**

A) 24 B) 36 C) 2^6 D) 2^7 E) 2^8

6. $\binom{10}{1}+\binom{10}{2}+\ldots+\binom{10}{9}=$ **?**

A) 2^{10} B) 2^{11} C) 2^9

D) $2^{10}-1$ E) $2^{10}-2$

7. $\binom{9}{0}+\binom{9}{1}+\ldots+\binom{9}{9}=A$

$\Rightarrow \binom{9}{1}+\binom{9}{2}+\ldots+\binom{9}{4}=$ **?**

A) A–1 B) A–2 C) $\frac{A-1}{2}$

D) $\frac{A-3}{3}$ E) $\frac{A-2}{2}$

8. $\binom{10}{0}+\binom{10}{1}+\binom{10}{2}+\ldots+\binom{10}{10}=$ **?**

A) 2^5 B) 4^5 C) 10^2 D) 10^3 E) 10^{10}

9. A kentinden B kentine 6 farklı yol, B kentinden C kentine 5 farklı yol vardır.

B kentine uğramak şartıyla A dan C ye kaç farklı yolla gidilebilir?

There are 6 different ways to go from city A to city B, and There are 5 different ways to go from city B to city C.

In how many different ways can someone go to city C from city A passing through the city B?

A) 11 B) 12 C) 18 D) 20 E) 30

10. 6 elemanlı bir kümenin 2 elemanlı alt küme sayısı kaçtır?

What is the number of the subsets of a set with 6 elements, taken 2 at a time?

A) 30 B) 28 C) 24 D) 20 E) 15

11. 8 soruluk bir sınavda 5 sorunun yanıtlanması isteniyor. İlk 5 sorudan 3 ü zorunlu olmak üzere, yanıtlanacak 5 soru kaç farklı şekilde seçilebilir?

5 of the questions of an exam with 8 questions should be answered. If 3 of the first 5 questions should be answered, in how many different ways can the 5 questions to be answered be selected?

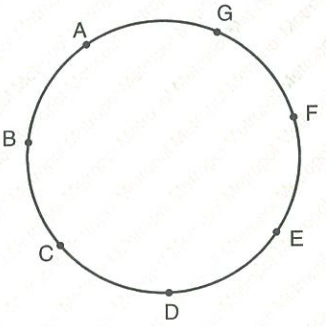
A) 54 B) 50 C) 48 D) 46 E) 40

12. YÖS sonucuyla 9 yabancı uyruklu öğrenci arasından 5 öğrenci İstanbul Üniversitesine, 4'ü Ankara Üniversitesine kaç farklı şekilde gönderilebilir?

Using the results of the YÖS exam of 9 people, in how many different ways can we send 5 of these people to the Istanbul University and other 4 to the Ankara University?

A) 170 B) 156 C) 140 D) 126 E) 112

13.



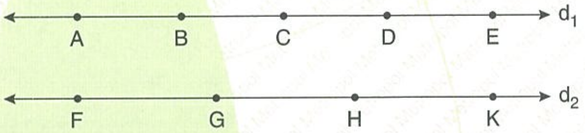
Yukarıda bir çember üzerinde A, B, C, D, E, F ve G noktaları verilmiştir.

Buna göre, köşeleri bu noktalar olan kaç farklı çokgen çizilebilir?

The points A, B, C, D, E, F, and G are given on the circle above. Accordingly, how many polygons can be drown using these points?

A) 103 B) 102 C) 101 D) 100 E) 99

14.

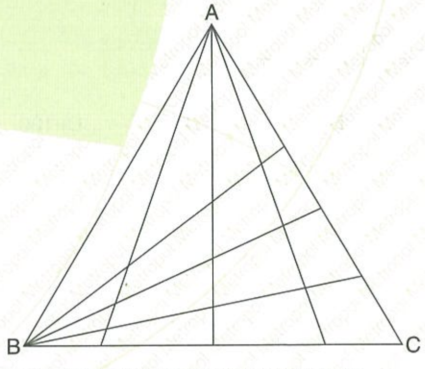


d_1 // d_2 olmak üzere, köşeleri şekilde verilen 9 noktadan herhangi dördü olan kaç farklı dörtgen çizilebilir?

If d_1 // d_2, how many different quadrangles can be drown using 4 of the 9 points on the figure?

A) 60 B) 56 C) 52 D) 50 E) 48

15.



Yukarıdaki şekilde kaç farklı üçgen vardır?

How many different triangles exist on the figure above?

A) 52 B) 54 C) 56 D) 60 E) 64

YANITLAR / ANSWERS

1	2	3	4	5	6	7	8
C	E	D	C	D	D	E	B
9	10	11	12	13	14	15	16
E	E	D	D	E	A	E	

1. $\dfrac{P(n,2)+P(n,3)}{P(n,3)-P(n,2)}=\dfrac{4}{3} \Rightarrow$ **$n = ?$**

A) 6 B) 7 C) 8 D) 9 E) 10

2. $P(n, 3) + P(n, 1) = P(7n, 1) \Rightarrow$ **$n = ?$**

A) 3 B) 4 C) 5 D) 6 E) 7

3. $\dfrac{P(n+1,n)}{n}=5\cdot 3! \Rightarrow$ **$n = ?$**

A) 4 B) 5 C) 6 D) 7 E) 8

4. $\dfrac{P(n,4)}{P(n,3)}=43 \Rightarrow$ **$n = ?$**

A) 40 B) 43 C) 46 D) 49 E) 52

5. $P(n, k) = 42 \Rightarrow$ **$\max(n) = ?$**

A) 6 B) 7 C) 20 D) 40 E) 42

6. $P(x, y) = 5! \Rightarrow$ **$\max(x + y) = ?$**

A) 5 B) 6 C) 100 D) 120 E) 121

7. $P(n, 3) = 42\cdot P(n, 1) \Rightarrow$ **$n = ?$**

A) 5 B) 6 C) 7 D) 8 E) 9

8. $P(n, 2) - 110 = 0 \Rightarrow n = ?$

A) 10 B) 11 C) 12 D) 13 E) 14

9. $\binom{n}{0}+\binom{n}{1}+\binom{n}{n}=7 \Rightarrow n = ?$

A) 2 B) 3 C) 4 D) 5 E) 6

10. **A = {1, 2, 3, 4, 5} kümesinin elemanları ile rakamları farklı 300 den küçük üç basamaklı kaç doğal sayı yazılabilir?**

By using the elements of the set A = {1, 2, 3, 4, 5}, how many different three-digit natural numbers smaller than 300 can be written if repetition of digits is not allowed?

A) 12 B) 20 C) 24 D) 36 E) 48

11. **4 mektup 5 posta kutusuna kaç farklı yolla atılabilir?**

In how many different ways can 4 letters be put in 5 letter boxes?

A) 4 B) 5 C) 20 D) 5^4 E) 4^5

12. $\binom{10}{0}-\binom{10}{1}+\binom{10}{2}-\binom{10}{3}+\ldots+\binom{10}{10}=?$

A) 0 B) 16 C) 32 D) 64 E) 128

13. **5 elemanlı bir kümenin en çok 2 elemanlı alt küme sayısı kaçtır?**

What is the number of subsets consisting of at most 2 elements, of a set consisting of 5 elements?

A) 15 B) 16 C) 20 D) 25 E) 26

14. **5 farklı IQ ve 4 farklı geometri kitabı arasından seçilecek 3 IQ ve 2 geometri kitabı bir rafa kaç farklı şekilde dizilebilir?**

In how many different ways can 3 IQ books chosen from 5 different IQ books and 2 geometry books chosen from 4 different geometry books be ordered on a single shelf?

A) 7200 B) 7080 C) 6900 D) 6520 E) 6440

YANITLAR / ANSWERS

1	2	3	4	5	6	7
D	B	A	C	E	E	D
8	**9**	**10**	**11**	**12**	**13**	**14**
B	D	C	D	A	B	A

1. $(x + 2)^5 = Ax^5 + Bx^4 + C\cdot x^3 + Dx^2 + Ex + F$

$\Rightarrow$ **C = ?**

A) 10 B) 20 C) 30 D) 40 E) 50

2. $\left(x + \frac{2}{x}\right)^5 = Ax^5 + Bx^3 + Cx + D\cdot x^{-1} + Ex^{-3} + Fx^{-5}$

$\Rightarrow$ **D = ?**

A) 20 B) 40 C) 50 D) 60 E) 80

3. $(2x - y)^4 = Ax^4 + Bx^3y + Cx^2y^2 + D\cdot xy^3 + Ey^4$

$\Rightarrow$ **D = ?**

A) –8 B) –4 C) 4 D) 8 E) 16

4. $(x + 3y)^7 = Ax^7 + B\cdot x^6y + \ldots + Mxy^6 + Ky^7$

$\Rightarrow$ **B = ?**

A) 7 B) 14 C) 21 D) 42 E) 63

5. $\left(x + \frac{1}{x}\right)^{12} = Ax^{12} + Bx^{10} + \ldots + D + \ldots + Ex^{-12}$

$\Rightarrow$ **D = ?**

A) 824 B) 860 C) 890 D) 904 E) 924

6. $(x - y)^6 = x^6 + Ax^5y + Cx^4y^2 + Dx^3y^3 + E\cdot x^2y^4 + Fxy^5 + Ky^6$

$\Rightarrow$ **E = ?**

A) –20 B) –15 C) –6 D) 15 E) 20

7. $(x^3 + y^2)^7 = Ax^{21} + Bx^{18}y^2 + C\cdot x^{15}y^4 + \ldots + Ky^{14}$

$\Rightarrow$ **C = ?**

A) 7 B) 21 C) 35 D) 42 E) 70

8. $(x^3 + x)^8 = Ax^{24} + Bx^{22} + \ldots + C\cdot x^{14} + \ldots + Kx^8$

$\Rightarrow$ **C = ?**

A) 8 B) 28 C) 56 D) 98 E) 112

9. $(x^2 + y)^{12} = x^{24} + \ldots + K \cdot x^6y^9 + \ldots y^{12}$

$\Rightarrow$ **K = ?**

A) 105 B) 120 C) 180 D) 200 E) 220

10. $\left(x - \frac{1}{x^2}\right)^9 = Ax^9 + \ldots + K + \ldots + M \cdot x^{-18} \Rightarrow$ **K = ?**

A) –84 B) –36 C) 1 D) 36 E) 84

11. $(x + y)^{18} = x^{18} + A \cdot x^{17}y + \ldots \Rightarrow$ **A = ?**

A) 2 B) 17 C) 18 D) 36 E) 72

12. $(x - 2y)^{14} = x^{14} - 28x^{13}y + A \cdot x^{12}y^2 + \ldots \Rightarrow$ **A = ?**

A) –1446 B) –182 C) –81 D) 182 E) 364

13. $\left(x - \frac{3}{x}\right)^{16} = x^{16} + Ax^{14} + \ldots + K \cdot x^{-14} + Mx^{-15} + \ldots$

$\Rightarrow$ **K = ?**

A) $-16 \cdot 3^{15}$ B) $-15 \cdot 3^{15}$ C) $-16 \cdot 3^{16}$ D) $-15 \cdot 3^{16}$ E) $16 \cdot 3^{15}$

14. $(x + y + z)^8 = \ldots + M \cdot x^4y^2z^2 + \ldots \Rightarrow$ **M = ?**

A) 420 B) 640 C) 720 D) 800 E) 840

15. $(x + y - z)^7 = \ldots + M \cdot x^3y^2z^2 + \ldots \Rightarrow$ **M = ?**

A) –210 B) –105 C) –42 D) 105 E) 210

16. $(x - 2y + 3z)^5 = \ldots + M \cdot x^3yz + \ldots \Rightarrow$ **M = ?**

A) –120 B) –40 C) –10 D) 10 E) 20

17. $(x + y^2 + z^3)^5 = \ldots + M \cdot x^2y^4z^3 + \ldots \Rightarrow$ **M = ?**

A) 15 B) 20 C) 30 D) 40 E) 60

YANITLAR / ANSWERS

1	2	3	4	5	6	7	8	9
D	E	A	C	E	D	B	C	E
10	**11**	**12**	**13**	**14**	**15**	**16**	**17**	
A	C	E	A	A	E	A	C	

1. $(x-2y)^5 = \ldots + K \cdot x^3y^2 + \ldots \Rightarrow$ **K = ?**

A) 24 B) 36 C) 40 D) 48 E) 54

2. $\left(2x-\frac{1}{y}\right)^7 = \ldots\ldots + M \cdot x^4y^n + \ldots \Rightarrow$ **M + n = ?**

A) –563 B) –560 C) –557 D) 557 E) 560

3. $\left(3x-\frac{1}{2x}\right)^6 = \ldots + C + \ldots \Rightarrow$ **C = ?**

A) –270 B) $-\frac{135}{2}$ C) $-\frac{135}{4}$

D) $\frac{135}{4}$ E) $\frac{135}{2}$

4. $\left(x-\frac{2}{x^2}\right)^8 = \ldots + Ax^5 + \ldots \Rightarrow$ **A = ?**

A) –64 B) –32 C) –16 D) 16 E) 32

5. $\left(x^3+\frac{2}{x}\right)^5 = \ldots + Mx^{11} + \ldots \Rightarrow$ **M = ?**

A) 5 B) 10 C) 15 D) 20 E) 40

6. $(3x^2-2y)^n = \ldots + K \cdot x^4y^3 + \ldots \Rightarrow$ **K + n = ?**

A) –920 B) –915 C) –820

D) –720 E) –715

7. $\left(\frac{2}{x}-x\right)^9 = \ldots + A \cdot x^5 + \ldots \Rightarrow$ **A = ?**

A) –144 B) –120 C) –80 D) –72 E) –60

8. $\left(x^2-\frac{1}{2x^3}\right)^{10} = \ldots + A + \ldots \Rightarrow$ **A = ?**

A) $\frac{55}{2}$ B) $\frac{105}{8}$ C) $\frac{75}{4}$ D) $\frac{125}{8}$ E) $\frac{65}{2}$

9. $(1+x^3)^8 = \ldots + A\cdot x^{12} + \ldots + B\cdot x^{15} + \ldots \Rightarrow \mathbf{\frac{A}{B} = ?}$

A) $\frac{3}{2}$ B) $\frac{4}{3}$ C) $\frac{5}{4}$ D) $\frac{6}{5}$ E) $\frac{7}{6}$

10. $(x - y + z)^8 = \ldots + A\cdot xy^2z^5 + \ldots$

$\Rightarrow$ **A = ?**

A) 120 B) 132 C) 144 D) 156 E) 168

11. $\left(x^2 - \frac{1}{y}\right)^6 = \ldots + A\cdot x^4y^{-4} + \ldots \Rightarrow \mathbf{A = ?}$

A) 6 B) 8 C) 12 D) 15 E) 18

12. $\left(\frac{1}{\sqrt[3]{x}} - \frac{1}{x}\right)^8 = \ldots + A\cdot x^{-6} + \ldots \Rightarrow \mathbf{A = ?}$

A) –56 B) –48 C) –32 D) –24 E) –18

13. $(2a - b - 3c)^6 = \ldots + M\cdot a^2b^3c + \ldots \Rightarrow \mathbf{M = ?}$

A) –740 B) –720 C) –360 D) 720 E) 740

14. $(3x - 2y - z)^5 = \ldots + A\cdot xyz^3 + \ldots \Rightarrow \mathbf{A = ?}$

A) –360 B) –240 C) 120 D) 240 E) 360

15. $\left(x^4 + \frac{1}{\sqrt{x}}\right)^9 = \ldots + A\cdot x^{18} \Rightarrow \mathbf{A = ?}$

A) $\binom{9}{4}$ B) $\binom{9}{3}$ C) $\binom{9}{2}$ D) $\binom{9}{1}$ E) $\binom{9}{0}$

16. $\left(\sqrt{x} + \frac{1}{\sqrt{x}}\right)^8 = \ldots + A\cdot x^0 \Rightarrow \mathbf{A = ?}$

A) –210 B) –140 C) 140 D) 70 E) 210

YANITLAR / ANSWERS

1	2	3	4	5	6	7	8
C	A	B	C	B	E	A	B
9	**10**	**11**	**12**	**13**	**14**	**15**	**16**
C	E	D	A	D	C	A	D

1. A ve B ayrık iki olay / A and B are two independent events

$P(A) = \frac{2}{3}, P(B) = \frac{1}{4} \Rightarrow$ **$P(AUB) = ?$**

A) $\frac{1}{6}$ B) $\frac{1}{4}$ C) $\frac{3}{4}$ D) $\frac{5}{6}$ E) $\frac{11}{12}$

2. $P(A') = \frac{5}{7}, P(B) = \frac{9}{14}, P(A \cap B) = \frac{5}{21} \Rightarrow$ **$P(A \cup B) = ?$**

A) $\frac{23}{42}$ B) $\frac{29}{35}$ C) $\frac{31}{35}$ D) $\frac{29}{42}$ E) $\frac{31}{42}$

3. **Bir metal para art arda 3 kez atılıyor. Bu üç atış sonunda paranın 2 kez tura, bir kez yazı gelme olasılığı kaçtır?**

A coin is tossed 3 times. What is the probability of getting 2 heads and 1 tails?

A) $\frac{1}{8}$ B) $\frac{1}{4}$ C) $\frac{3}{8}$ D) $\frac{1}{2}$ E) $\frac{5}{8}$

4. **6 evli çiftin bulunduğu bir gruptan rastgele seçilen iki kişinin evli olma olasılığı kaçtır?**

What is the probability of being a married couple when two are chosen among a group that consists of 6 married couple?

A) $\frac{1}{12}$ B) $\frac{1}{11}$ C) $\frac{1}{10}$ D) $\frac{1}{9}$ E) $\frac{1}{6}$

5. **4 hemşire 3 doktor arasından iki kişilik bir sağlık ekibi oluşturulacaktır. Seçilen iki kişinin de doktor olması olasılığı kaçtır?**

A health team of 2 persons is chosen from among 4 nurses and 3 doctors. What is the probability of both selected people being a doctor?

A) $\frac{1}{7}$ B) $\frac{2}{7}$ C) $\frac{1}{4}$ D) $\frac{3}{8}$ E) $\frac{3}{7}$

6. **Bir zar atılma deneyinde zarın tek sayı geldiği biliniyor. Buna göre, bu sayının asal sayı olması olasılığı kaçtır?**

It is known that the number which come up is odd when a dice is rolled. What is the probability of it being a prime number?

A) 1 B) $\frac{1}{2}$ C) $\frac{1}{3}$ D) $\frac{1}{4}$ E) $\frac{2}{3}$

7. **Bir zar ve bir metal para birlikte atılıyor. Zarın 4 veya 4'ten büyük bir sayı ve paranın tura gelme olasılığı kaçtır?**

A dice is rolled and a coin is tossed simultaneously. What is the probability of getting a number equal to or greater than 4 and getting a heads?

A) $\frac{1}{2}$ B) $\frac{1}{3}$ C) $\frac{1}{4}$ D) $\frac{1}{5}$ E) $\frac{1}{6}$

8. **Bir kutuda 7'si çürük 12 ceviz vardır. Kutudan rastgele iki ceviz alınıyor. Bu cevizlerden en az birinin sağlam olma olasılığı kaçtır?**

There are 12 walnuts such that 7 of them are spoiled in a box. What is the probability of picking at least one unspoiled walnuts, when two walnuts are picked randomly from the box?

A) $\frac{7}{22}$ B) $\frac{9}{22}$ C) $\frac{13}{22}$ D) $\frac{15}{22}$ E) $\frac{19}{22}$

9. M ve N herhangi iki olay / M and N are any two events events

$P(M) = \frac{1}{2}, P(N) = \frac{1}{3}, P(M \cap N) = \frac{1}{6} \Rightarrow \mathbf{P(M \cup N) = ?}$

A) $\frac{1}{3}$ B) $\frac{1}{2}$ C) $\frac{2}{3}$ D) $\frac{3}{4}$ E) $\frac{5}{6}$

10. **Bir torbada 3 beyaz, 5 mavi ve 4 kırmızı top vardır. Torbadan rastgele seçilen 3 toptan ikisinin kırmızı ve birinin beyaz olma olasılığı kaçtır?**

There are 3 white, 5 blue and 4 red balls in a pouch. What is the probability of picking 2 red and 1 white balls when 3 balls are picked randomly from the pouch?

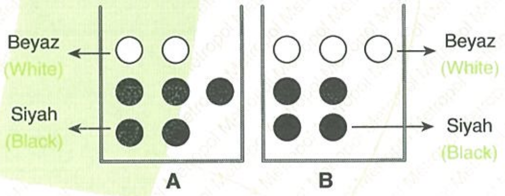
A) $\frac{7}{110}$ B) $\frac{9}{110}$ C) $\frac{19}{220}$ D) $\frac{21}{220}$ E) $\frac{17}{110}$

11. **Bir sınıfta 4'ü gözlüklü 10 kız ve 5'i gözlüklü 12 erkek öğrenci vardır. Sınıftan seçilen bir öğrencinin erkek veya gözlüklü olması olasılığı kaçtır?**

There are 10 female students such that 4 of them are spectacled and 12 male students such that 5 of them are spectacled in a class. What is the probability of choosing a male student or a spectacled student from among the class when the we select one form students randomly?

A) $\frac{9}{22}$ B) $\frac{6}{11}$ C) $\frac{8}{11}$ D) $\frac{9}{11}$ E) $\frac{16}{23}$

12.



A kutusundan bir top alınıp rengine bakılmadan B kutusuna atılıyor. Sonra B kutusundan bir top alınıyor. Alınan bu topun beyaz olması olasılığı kaçtır?

A rondom ball is picked from the Box A and then put into Box B without looking at its color. After that, a ball from Box B is picked. What is the probability of being white for the ball that is picked?

A) $\frac{1}{2}$ B) $\frac{4}{49}$ C) $\frac{1}{14}$ D) $\frac{23}{49}$ E) $\frac{23}{56}$

YANITLAR / ANSWERS

1	2	3	4	5	6
E	D	C	B	A	E
7	8	9	10	11	12
C	D	C	B	C	E

BÖLÜM CHAPTER 16

TARİHSEL NOT / HISTORICAL NOTE

CAHİT ARF [1910 – 1997]

Türk matematikçi ve bilim insanı. TÜBİTAK Bilim Kolu eski başkanı. Cebir konusundaki çalışmalarıyla dünyaca ün kazanmıştır. Cisimlerin kuadratik formlarının sınıflandırılmasında ortaya çıkan değişmezlere ilişkin ARF değişmezi ve ARF halkaları gibi literatürde adıyla anılan çalışmaların yanı sıra "Hasse-ARF Teoremi" adı ile anılan teoremi matematik bilimine kazandırmıştır. Cahit ARF, matematiği bir meslek dalı olarak değil, bir yaşam tarzı olarak görmüştür. Öğrencilerine sürekli: "Matematiği ezberlemeyin, kendiniz yapın ve anlayın." demiştir.

Turkish mathematician and scientist. Former chairman of the TUBITAK Science Branch. His work on algebra has gained worldwide fame. In addition to the studies in the literature such as the Arf invariant and Arf rings related to the invariants that emerged in the classification of quadratic forms of the objects, he introduced the theorem called "Hasse-Arf Theorem" to mathematical science. Cahit Arf saw mathematics as a lifestyle rather than a profession. He constantly told his students: "Do not memorize the math, do it yourself and understand it."

POLİNOMLAR / POLYNOMIALS

Bu bölüm, 117 test sorusu içermektedir.
This chapter includes 117 test questions.

BÖLÜM / CHAPTER 16

POLİNOMLAR / POLYNOMIALS

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BÖLÜM

16

CHAPTER

POLİNOMLAR
POLYNOMIALS

POLİNOMLAR / POLYNOMIALS

Tanım : $a_0, a_1, a_2, ..., a_n \in R$, $n \in N$ olmak üzere,

$P(x) = a_0 + a_1 \cdot x + a_2 \cdot x^2 + ... + a_n \cdot x^n$ biçiminde tanımlanan ifadelere ***polinom*** denir.

Definition: Provided that $a_0, a_1, a_2 ... a_n \in R$ and $n \in N$,

A **polynomial** is an expression of the form $P(x) = a_0 + a_1 x + a_2 x^2 + ... + a_n x^n$

Önemli / Important

$f(x) = a_n \cdot x^n + a_{n-1} \cdot x^{n-1} + ... + a_1 \cdot x + a_0$

Polinomlar, fonksiyonların $n \in N$ durumundaki özel halidir.

- $x^{1/2} + x^2$ fonksiyondur, polinom değildir.
- $x + x^2$ fonksiyondur, polinomdur.

$f(x) = a_n \cdot x^n + a_{n-1} \cdot x^{n-1} + ... + a_1 \cdot x + a_0$

Polynomial is a special case of the functions in situation $n \in N$.

- $x^{1/2} + x^2$ is a function, but not a polynomial.
- $x + x^2$ is a function and a polynomial.

Özellik / Property

★ Polinomun terimleri: a_0 , $a_1 \cdot x$, $a_2 \cdot x^2$, ... , $a_n \cdot x^n$

★ Terms of polynomial: a_0 , $a_1 x$, $a_2 x^2$, ... , $a_n x^n$

★ Polinomun katsayıları: a_0 , a_1 , a_2 , ... , a_n

★ Coefficients of polynomial: a_0 , a_1 , a_2 , ... , a_n

★ Başkatsayı: a_n

★ Leading coefficient: a_n

★ Sabit terim: a_0

★ Constant term: a_0

★ **Derece:** Polinomda yer alan terimlerden derecesi en büyük olan terimin derecesi, **polinomun derecesi**dir ve **der (P(x))** ile gösterilir.

★ **Degree:** The degree of a polynomial is the greatest of the degrees of its terms that is denoted by der (P(x)).

★ Polinomlar; P(x), Q(x), A(x), B(x,y) gibi büyük harflerle gösterilir.

★ Polynomials are denoted by capital letters like P(x), Q(x), A(x), B(x, y).

★ $C(x,y) = 4x^3y^5 - 3x^2y^2 + 6xy + 7$ biçimindeki polinomlara ***iki bilinmeyenli polinom*** denir.

C(x,y) nin derecesi 3 + 5 = 8 dir.

★ The polynomials in the form of $C(x,y) = 4x^3y^5 - 3x^2y^2 + 6xy + 7$ are called **polynomials in two variables.** The degree of C(x,y) is 3 + 5 = 8

SABİT POLİNOM / CONSTANT POLYNOMIAL

▼ $a_0 \neq 0$ ve $a_1 = a_2 = ... = a_{n-1} = a_n = 0$ ise, $P(x) = a_0$ sabit polinomdur.

P(x) =3, Q(x) = –1, ... gibi.

▼ If $a_0 \neq 0$ and $a_1 = a_2 = ... = a_n = 0$, $P(x) = a_0$ is constant polynomial.

like P(x) =3, Q(x) = –1, ...

Önemli / Important

- Sabit polinomun derecesi sıfırdır.
- The degree of a constant polynomial is zero.

SIFIR POLİNOMU / ZERO POLYNOMIAL

▼ $a_n = a_{n-1} = \dots = a_0 = 0$ ise, $P(x) = 0$ sıfır polinomudur.

▼ If $a_n = a_{n-1} = \dots = a_0 = 0$; $P(x)$ is zero polynomial.

Önemli / Important

- Sıfır polinomunun derecesi belirsizdir.
- The degree of a zero polynomial is indefinite.

POLİNOMLARIN EŞİTLİĞİ / EQUALITY of POLYNOMIALS

▼ Dereceleri aynı olan polinomların eşit olması için, aynı dereceli terimlerin katsayıları eşit olmalıdır.

▼ To the polynomials with same degree be equal, the coefficients of the terms with the same degrees must be equal.

$P(x) = a_n \cdot x^n + a_{n-1} \cdot x^{n-1} + \dots + a_1 \cdot x + a_0$

$Q(x) = b_n \cdot x^n + b_{n-1} \cdot x^{n-1} + \dots + b_1 \cdot x + b_0$

$P(x) = Q(x) \Rightarrow \boldsymbol{a_n = b_n \, , a_{n-1} = b_{n-1} \, , \dots , a_1 = b_1 \, , a_0 = b_0}$

POLİNOMLARDA İŞLEMLER / OPERATIONS in POLYNOMIALS

TOPLAMA ve ÇIKARMA İŞLEMİ / ADDITION and SUBSTRACTION OPERATIONS

▼ İki polinom toplanırken (veya çıkarılırken) dereceleri aynı olan terimlerin katsayıları toplanır (veya çıkarılır) ve toplam (veya fark) polinomu elde edilir.

▼ To find the sum (or difference) of two polynomials, we add (or subtract) the coeffients of the like terms of the same degree.

Önemli / Important

- $p > q$

  $der(P(x)) = p$

  $der(Q(x)) = q$

  $\Rightarrow der(P(x) \mp Q(x)) = p$

ÇARPMA İŞLEMİ / MULTIPLICATION OPERATION

▼ İki polinomun çarpımı; polinomlardan birinin her teriminin diğer polinomun her bir terimi ile ayrı ayrı çarpımlarından elde edilen terimlerin toplamına eşittir.

▼ The product of two polynomials is obtained by collecting multiplication of each terms of one polynomial by each term of the other polynomial.

Önemli / Important

der(P(x)) = p , der(Q(x)) = q

- $der(P(x) \cdot Q(x)) = p + q$
- $der(P^m(x)) = m \cdot p$
- $der(P(Q(x))) = p \cdot q$
- $der(P(x^m)) = m \cdot p$
- $der(x^m \cdot P(x)) = m + p$

BÖLME İŞLEMİ / DIVISION OPERATION

▼ **Bölme Özdeşliği** / Identity of Division

$$\begin{array}{r|l} P(x) & \underline{Q(x)} \\ \underline{-\quad\quad} & B(x) \\ K(x) & \end{array} \Rightarrow$$

P(x) = Bölünen polinom / Dividend polynomial
Q(x) = Bölen polinomu / Divisor polynomial
B(x) = Bölüm polinomu / Quotient polynomial
K(x) = Kalan polinomu / Remainder polynomial

★ $der(P(x)) \geq der(Q(x))$
★ $der(K(x)) < der(Q(x))$
★ ***$P(x) = Q(x) \cdot B(x) + K(x)$***

Önemli / Important

der(P(x)) = p , der(Q(x)) = q

- $der\left(\frac{P(x)}{Q(x)}\right) = p - q$

BÖLME YÖNTEMLERİ / THE DIVISION METHODS

▼ **Polinom Bölmesi:** Sayılardaki bölme işlemine benzer olarak yapılır. En büyük dereceli terim yok edilerek sonuca ulaşılır.

▼ **Polynomial Division:** This method is similar to the division of the numbers. The term with the maximum degree is simplified, in order to find the result.

Önemli / Important

- Horner yöntemi; böleni yalnızca x – a şeklinde olan bir polinom için kullanılabilir. Bu yöntem, böleni $x^2 - 7$ şeklinde olan ikinci dereceden bir polinom için kullanılamaz.
- Horner's Method works only for divisors of the form x – a. You cannot use this method to divide a polynomial by a polynomial of degree two such as $x^2 - 7$.

P(x) POLİNOMUNUN (ax + b) İLE BÖLÜMÜNDEN ELDE EDİLEN KALAN THE REMAINDER of the POLYNOMIAL P(x) WHEN DIVIDED by (ax + b)

▼ Bir P(x) polinomunun (ax + b) ile bölümünden kalanı bulmak için $ax + b = 0 \Rightarrow x = -\frac{b}{a}$ belirlenir. P(x) polinomunda, x'in yerine $-\frac{b}{a}$ yazılırsa işlemin sonucu kalanı verir. ax + b birinci dereceden bir polinom olduğu için kalan sıfırıncı dereceden bir polinom, yani reel sayı olmak zorundadır.

▼ To find the remainder of P(x) divided by (ax + b) the following is determined: $ax + b = 0 \Rightarrow x = -\frac{b}{a}$. If $-\frac{b}{a}$ is substituted for of x in P(x), then the result gives the remainder. Since ax + b is a first order polynomial, the remainder must always be equal to real number, which is a zero degree polynomial.

$$\begin{array}{r|l} P(x) & \underline{ax + b} \\ \underline{-\quad\quad} & T(x) \\ K(x) & \end{array} \Rightarrow P(x) = \underbrace{(ax + b)}_{=0} \cdot T(x) + K(x) \Rightarrow \mathbf{K(x) = P\left(-\frac{b}{a}\right)}$$

Önemli / Important

- $(x - a) \cdot P(x) = Q(x) \Rightarrow Q(a) = 0$

P(x) POLİNOMUNUN $(x^n \pm a)$ İLE BÖLÜMÜNDEN ELDE EDİLEN KALAN
THE REMAINDER of P(x) WHEN DIVIDED by $(x^n \pm a)$

▼ Bir P(x) polinomunun $(x^n \pm a)$ ile bölümünden kalanını bulmak için $x^n \pm a = 0$'dan $x^n = \mp a$ bulunur. Polinomda x^n yerine $\mp a$ değeri yerine yazılarak kalan bulunur.

▼ To find the remainder of P(x) divided by $(x^n \pm a)$, insert $(x^n = \mp a)$ in the polynomial P(x), and substitute $\pm a$ for of x^n in the polynomial.

P(x) POLİNOMUNUN $(x - a) \cdot (x - b) \cdot (x - c) \cdot \ldots$ ÇARPIMI İLE BÖLÜNEBİLMESİ
DIVISIBILITY of the POLYNOMIAL P(x) by $(x - a) \cdot (x - b) \cdot (x - c) \cdot \ldots$

▼ P(x) polinomu $(x - a) \cdot (x - b) \cdot (x - c) \cdot \ldots$ çarpımı ile tam bölünebiliyorsa $(x - a), (x - b), (x - c), \ldots$ çarpanlarıyla da ayrı ayrı tam bölünür.

▼ If the polynomial P(x) is divisible by $(x - a) \cdot (x - b) \cdot (x - c) \cdot \ldots$, it is divisible by the factors $(x - a) \cdot (x - b) \cdot (x - c) \cdot \ldots$

Önemli / Important

- P(x) ve Q(x) polinomları x – a'ya bölündüğünde kalanlar sırayla A ve B olsun.
  a. $P(x) \pm Q(x)$ polinomu $(x - a)$ ile bölündüğünde kalan $A \pm B$ dir.
  b. $P(x) \cdot Q(x)$ polinomu $(x - a)$ ile bölündüğünde kalan $A \cdot B$'dir.
- If the remainder of P(x) and Q(x) divided by (x – a) are equal to A and B respectively, then.
  a. The remainder of $P(x) \pm Q(x)$ divided by $(x-a)$ is equal to $A \pm B$.
  b. The remainder of $P(x) \cdot Q(x)$ divided by $(x-a)$ equal to $A \cdot B$.
- P(x) polinomu $(x - a)$ ile bölündüğünde kalan A, $(x - b)$ ile bölündüğünde kalan B dir.
  P(x) polinomu $(x - a) \cdot (x - b)$'ye bölündüğünde kalan $K(x) = mx + n$ biçimindedir.
- If remainders of P(x) when divided by $(x - a)$ and $(x - b)$ are equal to A and B, respectively, Then the remainder of P(x) divided by $(x - a) \cdot (x - b)$ is in the form of $K(x) = mx + n$.

P(x) POLİNOMUNUN $(ax + b)^n$ ile BÖLÜNEBİLMESİ
DIVISIBILITY of POLYNOMIAL P(x) by $(ax + b)^n$

▼ P(x) polinomu $(ax + b)^n$ polinomu ile tam bölünebiliyorsa, $P'(x), P''(x), P'''(x), \ldots, P^{(n-1)}(x)$ polinomları da $(ax + b)$ ile tam bölünebilir. Burada $P'(x), P''(x), P'''(x), \ldots, P^{(n-1)}(x)$ polinomları P(x) polinomunun 1., 2., 3. ,..., (n – 1). mertebeden türevleridir.

▼ If the polynomial P(x) is divisible by $(ax + b)^n$, the polynomials $P'(x), P''(x), P'''(x) \ldots P^{(n-1)}(x)$ are divisible by $ax + b$; where $P'(x), P''(x), P'''(x) \ldots P^{(n-1)}(x)$ are 1st, 2nd, ... (n – 1)th. order derivatives of the polynomial P(x).

$$\begin{array}{r|l} P(x) & (ax+b)^n \\ \cline{2-2} - \quad & \\ \hline 0 & \end{array} \Rightarrow \begin{array}{l} P\left(-\frac{b}{a}\right) = 0 \\ P'\left(-\frac{b}{a}\right) = 0 \\ P''\left(-\frac{b}{a}\right) = 0 \\ \vdots \\ P^{(n-1)}\left(-\frac{b}{a}\right) = 0 \end{array}$$

POLİNOMLARIN BASİT KESİRLERE AYRILMASI
FACTORIZATION of POLYNOMIALS into PARTIAL FRACTIONS

▼ Payın derecesinin paydanın derecesinden küçük ve paydanın çarpanlara ayrıldığı durumlarda basit kesirlere ayırma yöntemi kullanılır.

▼ This method could be used if the degree of numerator is less than the degree of denominator and the denominator is factorizable into simple partial fractions of polynomials.

1. $P(x-1) = 2x^2 - 3x - 1 \Rightarrow \mathbf{P(1) - P(-1) = ?}$

A) – 2 B) 0 C) 1 D) 2 E) 4

2. $P(5x-8) = 4x^2 + 6x + 9 \Rightarrow \mathbf{P(-3) = ?}$

A) 13 B) 17 C) 19 D) 24 E) 28

3. $P(2-x) = 3x + 5 \Rightarrow \mathbf{P(P(0)) = ?}$

A) – 6 B) –10 C) –14 D) –18 E) –22

4. $P(x-1) = x^3 - x^2 + x + 1 \Rightarrow \mathbf{P(0) = ?}$

A) 2 B) 3 C) 4 D) 5 E) 6

5. $P(x) = 2x^3 - x^2 + 3x + a$,

$P(-1) = 0 \Rightarrow \mathbf{a = ?}$

A) 2 B) 3 C) 4 D) 5 E) 6

6. $P(x) = x^2 + 4x \Rightarrow \mathbf{P(P(-4)) = ?}$

A) 0 B) 1 C) 2 D) 3 E) 4

7. $P(x) = x^2 - 2x - 1 \Rightarrow \mathbf{P(3x+1) = ?}$

A) $9x^2 - 6x + 5$ B) $9x^2 - 8x - 1$

C) $9x^2 + 5x + 2$ D) $9x^2 + 3$

E) $9x^2 - 2$

8. $P(x) = x^2 - 1 \Rightarrow \mathbf{P(2x+1) = ?}$

A) $4x^2$ B) $4x^2 + 4x + 1$

C) $4x^2 + 4x$ D) $4x^2 + 2x + 2$

E) $4x^2 + 2x + 1$

9. $P(2x-1) = ax^2 + 3x - 4,$

$P(3) = -2 \Rightarrow$ **a = ?**

A) 1   B) 0   C) –1   D) –2   E) –3

10. $P(x) = x^2 + 3x - 7$

$\Rightarrow$ **P(2x + 1) = ?**

A) $2x^2 + 10x - 3$ B) $4x^2 - 10x + 3$

C) $4x^2 + 10x + 3$ D) $4x^2 - 10x - 3$

E) $4x^2 + 10x - 3$

11. $\left.\begin{array}{l} P(x)=ax^2+bx+c \\ P(0)=2 \\ P(1)=8 \end{array}\right\} \Rightarrow$ **a + b – c = ?**

A) 8   B) 6   C) 4   D) 2   E) 0

12. $P(2x-3) = 4x - 6 \Rightarrow$ **P(2x) = ?**

A) 2x   B) 4x   C) 6x   D) 8x   E) 9x

13. $P(1-x) = 2x^2 + 2 \Rightarrow$ **P(x) = ?**

A) $2x^2 - 4x + 4$ B) $2x^2 - x + 4$

C) $2x^2 - 2x + 4$ D) $x^2 - 3x + 8$

E) $2x^2 + 3x + 8$

14. $P(x-1) = x^2 - 4 \Rightarrow$ **P(x) = ?**

A) $x^2 + x - 2$ B) $x^2 + 2x - 3$

C) $x^2 + 3x - 4$ D) $x^2 + 5x - 8$

E) $x^2 + 5x - 4$

15. $\left.\begin{array}{l} P(x-1)\ \ ax^2+6x+2 \\ P(1)=3a+8 \end{array}\right\} \Rightarrow$ **P(–2) = ?**

A) – 10   B) – 8   C) – 6   D) – 4   E) – 2

16. $P(x) = (x-1) \cdot (x^2 + ax - 6) - 3$

$P(-3) = -3 \Rightarrow$ **P(a) = ?**

A) –3   B) 0   C) 3   D) 9   E) 75

17. $(x-2) \cdot P(x) = x^2 + ax + 4 \Rightarrow$ **P(3) = ?**

A) –13   B) –11   C) –10   D) –8   E) 1

YANITLAR / ANSWERS

1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
D	C	E	A	E	A	E	C	C
10	**11**	**12**	**13**	**14**	**15**	**16**	**17**	
E	C	B	A	B	A	A	E	

1. P(x) bir polinom / is a polynomial

$$P(x)=x^{\frac{n+8}{n}}+3x^{n}-7 \Rightarrow \sum n = ?$$

A) 6 B) 9 C) 12 D) 15 E) 18

2. $\dfrac{3x-4}{x^3+x}=\dfrac{A}{x}+\dfrac{Bx+C}{x^2+1}$

$\Rightarrow$ **A + B + C = ?**

A) –3 B) –1 C) 3 D) 5 E) 6

3. der[$P(x^3)\cdot Q^3(x)$] = 27,

der[$P^2(x^2)\cdot Q(x^3)$] = 32

$\Rightarrow$ **der[2·P(x) + 3·Q(x)] = ?**

A) 4 B) 5 C) 8 D) 9 E) 10

4. der[$P^2(x)\cdot Q(x^3)$] = 18

$$\text{der}\left[\frac{P^3(x)}{Q^2(x)}\right]=14$$

$\Rightarrow$ **der[5·P(x) + 4·Q(x)]^2 = ?**

A) 12 B) 13 C) 15 D) 16 E) 18

5. $(x-2)\cdot P(x) = ax^2 + 3x + 4$

$$\begin{array}{l|l} P(x) & \underline{x+1} \\ \underline{-} & \\ K(x) & \end{array} \Rightarrow \mathbf{K(x) = ?}$$

A) –1 B) $-\frac{1}{2}$ C) 0 D) $\frac{1}{2}$ E) 1

6.

$$\begin{array}{l|l} P(x) & \underline{x+1} \\ \underline{-} & \\ -3 & \end{array} \qquad \begin{array}{l|l} Q(x) & \underline{x+1} \\ \underline{-} & \\ -4 & \end{array} \qquad \begin{array}{l|l} T(x) & \underline{x+1} \\ \underline{-} & \\ 2 & \end{array}$$

$$\begin{array}{l|l} m\cdot P(x)\cdot Q(x) - T(x) & \underline{x+1} \\ \underline{-} & \\ 0 & \end{array} \Rightarrow \mathbf{m = ?}$$

A) $-\frac{1}{8}$ B) $-\frac{1}{6}$ C) $-\frac{1}{3}$ D) $\frac{1}{3}$ E) $\frac{1}{6}$

7.

$$\begin{array}{l|l} P(x) & \underline{x+1} \\ \underline{-} & \\ A & \end{array} \qquad \begin{array}{l|l} P(x) & \underline{x-1} \\ \underline{-} & \\ B & \end{array}$$

$$\begin{array}{l|l} P(x) & \underline{x^2-1} \\ \underline{-} & \\ 3x+7 & \end{array} \Rightarrow \mathbf{A + B = ?}$$

A) 10 B) 12 C) 14 D) 16 E) 18

8. $P(2x-1) + P(x-2) + P(3x) = x^2 + 8$

$\Rightarrow$ **P(–3) = ?**

A) –3 B) –2 C) 0 D) 2 E) 3

9. $P(x) = x^3 - 4x^2 + mx + n$

$$\begin{array}{l|l} P(x) & (x-1)^2 \\ \cline{2-2} - & \\ \cline{1-1} 0 & \end{array} \Rightarrow \mathbf{m \cdot n = ?}$$

A) –10 B) –8 C) –6 D) 6 E) 8

10.

$$\begin{array}{l|l} P(x) & x-1 \\ \cline{2-2} - & x^2-x+3 \\ \cline{1-1} k & \end{array} \qquad \begin{array}{l|l} P(x) & x+2 \\ \cline{2-2} - & \\ \cline{1-1} 5 & \end{array}$$

$\Rightarrow$ **k = ?**

A) 16 B) 32 C) 36 D) 38 E) 40

11. $P(x) = 2x^3 - mx^2 + nx - 11$

$$\begin{array}{l|l} P(x) & x^2+1 \\ \cline{2-2} - & \\ \cline{1-1} 4x-3 & \end{array} \Rightarrow \mathbf{m + n = ?}$$

A) 10 B) 11 C) 14 D) 15 E) 16

12. $P(x^3 - 2) = x^6 + 3x^3 + 4$

$$\begin{array}{l|l} P(x^2) & x-\sqrt{3} \\ \cline{2-2} - & \\ \cline{1-1} \mathbf{K(x) = ?} & \end{array}$$

A) 40 B) 44 C) 46 D) 48 E) 52

13. $P(x) = (a-2)x$

$(x-3) \cdot Q(x) = x^2 - ax + 12$

$$\begin{array}{l|l} (x+2) \cdot Q(x-1) + (x-1) \cdot P(x+2) & x-3 \\ \cline{2-2} - & \\ \cline{1-1} K(x) & \end{array} \Rightarrow \mathbf{K(x) = ?}$$

A) 24 B) 28 C) 32 D) 36 E) 40

14.

$$\begin{array}{l|l} P(x-2) & x-3 \\ \cline{2-2} - & \\ \cline{1-1} 7 & \end{array} \qquad \begin{array}{l|l} P(2x+1) & x+1 \\ \cline{2-2} - & \\ \cline{1-1} 5 & \end{array}$$

$$\begin{array}{l|l} P(x) & x^2-1 \\ \cline{2-2} - & \\ \cline{1-1} \mathbf{K(x) = ?} & \end{array}$$

A) x – 3 B) x – 4 C) x – 6

D) x + 4 E) x + 6

15. $P(x) = x^4 - mx^3 + 4x^2 - nx + 7$

$$\begin{array}{l|l} P(x) & (x-1)^2 \\ \cline{2-2} - & \\ \cline{1-1} 0 & \end{array} \Rightarrow \mathbf{m - n = ?}$$

A) –9 B) –10 C) –12 D) –15 E) –17

16.

$$\begin{array}{l|l} P(x) & x^2-x+3 \\ \cline{2-2} - & \\ \cline{1-1} x-4 & \end{array} \qquad \begin{array}{l|l} P^2(x) & x^2-x+3 \\ \cline{2-2} - & \\ \cline{1-1} \mathbf{K(x) = ?} & \end{array}$$

A) –7x + 13 B) –7x + 10 C) –7x + 7

D) x – 4 E) x – 5

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1	2	3	4	5	6	7	8
D	C	B	A	D	B	C	E
9	**10**	**11**	**12**	**13**	**14**	**15**	**16**
A	B	C	B	E	E	C	A

1. $P(-2x+5) = -x^2 + x - 6 \Rightarrow$ **$P(3) = ?$**

A) -5 B) -6 C) -7 D) -8 E) -9

2. $P(x+1) = x^2 - 4x + 7 \Rightarrow$ **$P(1) = ?$**

A) 9 B) 8 C) 7 D) 6 E) 5

3. $P(x) = x^3 - 3x^2 + 3x - 1 \Rightarrow$ **$P(x+1) = ?$**

A) x^3 B) $2x - x^3$ C) $x^3 - 1$ D) $x^3 + 2x$ E) $1 - x^3$

4. $P(x) = x^2 - tx - 2$

$\Rightarrow$ **$P(-1) + P(0) + P(1) = ?$**

A) -5 B) -4 C) -3 D) -2 E) -1

5. $P(x+2) = x^2 + 2x - 5 \Rightarrow$ **$P(x-2) = ?$**

A) $x^2 - 6x + 3$ B) $x^2 - 5x - 2$ C) $x^2 - 8x - 1$ D) $x^2 + 4x + 3$ E) $x^2 + x - 4$

6.

$$\left.\begin{array}{l} P(x) = x^2 + ax + b + 4 \\ P(1) = 2 \\ P(2) = 6 \end{array}\right\} \Rightarrow \mathbf{a = ?}$$

A) 0 B) 1 C) 2 D) 3 E) 4

7. $P(x+1) = 3x + 4 \Rightarrow$ **$P(x) = ?$**

A) $2x + 1$ B) $6x + 4$ C) $3x + 8$ D) $3x + 7$ E) $3x + 1$

8. $P(3+x) = x^2 + 3x - 2 \Rightarrow$ **$P(2x) = ?$**

A) $2x^2 - 5x - 2$ B) $4x^2 - 6x$ C) $4x^2 - 6x - 2$ D) $4x^2 - 6x + 2$ E) $4x^2 + 6x + 2$

9. $P(1-x) = 7x + 1 \Rightarrow$ **$P(2x-3) = ?$**

A) $7x - 8$ B) $7x - 7$ C) $5 - 7x$ D) $8 - 7x$ E) $29 - 14x$

10. $P(x) = (x^2 - 3x + 2)$,

$Q(x) = 4x^3 + x^2$

$\Rightarrow$ **$P(1) + Q(2) = ?$**

A) 32 B) 36 C) 47 D) 54 E) 60

11. $\left.\begin{array}{l} P(x) = 4x + 3 \\ Q(x) = 2x + 1 \end{array}\right\} \Rightarrow \mathbf{P(x) \cdot Q(x) = ?}$

A) $2x^2 + 5x + 4$

B) $8x^2 + 10x + 3$

C) $5x^2 - 2x + 3$

D) $-8x^2 - 5x - 4$

E) $2x^2 - 10x + 5$

12. $P(x) = -4x^3 + 2x^2 - x + 3$

$Q(x) = x^4 - x^2 + 2x - 10$

$\Rightarrow$ **$P(x) + Q(x) = ?$**

A) $x^4 - x^2 + 3$

B) $x^4 - 4x^3 + 2x^2 + 3$

C) $x^4 - 4x^3 + 2x^2 - x + 13$

D) $x^4 - 4x^3 + x^2 - x + 7$

E) $x^4 - 4x^3 + x^2 + x - 7$

13. $(3x + 1)^2 - (x - 2)^2 = ax^2 + bx + c \Rightarrow$ **$a + b + c = ?$**

A) 12 B) 13 C) 14 D) 15 E) 16

14. $\dfrac{-x+2}{x^2 - x - 6} = \dfrac{A}{x-3} + \dfrac{B}{x+2} \Rightarrow \mathbf{\dfrac{A}{B} = ?}$

A) $\frac{1}{10}$ B) $\frac{1}{6}$ C) $\frac{1}{5}$ D) $\frac{1}{4}$ E) $\frac{1}{2}$

15. $\left.\begin{array}{l} d[P(x)] = 3 \\ d[Q(x)] = 4 \end{array}\right\} \Rightarrow \mathbf{d[P(x^2) \cdot Q(x^2)] = ?}$

A) 14 B) 36 C) 108 D) 144 E) 586

16. $\left.\begin{array}{l} d[P(x)] = 3 \\ d[Q(x)] = 2 \end{array}\right\} \Rightarrow \mathbf{d[P(Q(x)]^4 = ?}$

A) 12 B) 16 C) 18 D) 24 E) 36

17. $\left.\begin{array}{l} d[P(x) \cdot Q(x)] = 8 \\ d\left[\dfrac{P(x^2)}{Q(x^3)}\right] = 6 \end{array}\right\} \Rightarrow \mathbf{d[P(x) + Q(x)] = ?}$

A) 4 B) 6 C) 7 D) 8 E) 12

18. $\left.\begin{array}{l} d[x^2 \cdot P(x^2) \cdot Q(x)] = 13 \\ d\left[\dfrac{P(x)}{Q(x)}\right] = 4 \end{array}\right\} \Rightarrow \mathbf{d[P(x) + Q(x)] = ?}$

A) 4 B) 5 C) 6 D) 7 E) 8

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1	2	3	4	5	6	7	8	9
B	C	A	B	A	B	E	C	E
10	11	12	13	14	15	16	17	18
B	B	E	D	D	A	D	B	B

1. $P(x+2) = ax^3 + x^2 + 4x + b,$

$$\begin{array}{r|l} P(x) & x-2 \\ \hline 4 & \end{array} \Rightarrow \mathbf{b = ?}$$

A) 2 B) 3 C) 4 D) 5 E) 6

2.

$$\begin{array}{r|l} P(x) & (x+2)^2 \cdot (x-2) \\ \hline 2x^2 - x - 5 & \end{array}$$

$$\begin{array}{r|l} P(x) & x^2 - 4 \\ \hline \mathbf{K(x) = ?} & \end{array}$$

A) 0 B)2 C) 3

D) $-x+2$ E) $-x+3$

3. $P(x) = x^4 + 2ax^3 + bx + c,$

$$\begin{array}{r|l} P(x) & (x+1)^3 \\ \hline 0 & \end{array} \Rightarrow \mathbf{a = ?}$$

A) –4 B) –2 C) –1 D) 1 E) 2

4. P(x) bir polinom / is a polynomial

$$P(x) = 5 \cdot x^{\frac{m^2-4}{m}} - 4x^m + 8 \Rightarrow \mathbf{\sum m = ?}$$

A) 2 B) 4 C) 6 D) 8 E) 10

5. $\operatorname{der}[P^3(x) \cdot Q^2(x)] = 8$

$$\operatorname{der}\left[\frac{P^4(x)}{Q(x)}\right] = 7 \Rightarrow \mathbf{der[P(x) \cdot Q(x)] = ?}$$

A) 3 B) 5 C) 6 D) 7 E) 8

6. $\dfrac{2x+3}{2x^2-x-1} = \dfrac{A}{x-1} + \dfrac{B}{2x+1}$

$\Rightarrow$ **A + B = ?**

A) $-\frac{1}{2}$ B) $-\frac{1}{3}$ C) $-\frac{1}{4}$ D) $\frac{1}{2}$ E) $\frac{1}{3}$

7. $P(x) = \dfrac{x^2-(m-1)x+1}{x-1} - (x^2-x+1)$

$$\begin{array}{r|l} P(x) & x-1 \\ \hline \mathbf{K(x) = ?} & \end{array}$$

A) –2 B) –1 C) 0 D) 1 E) 2

8. $P(x) = ax^5 - 2x^5 + 8x^n - 6x^2 + (b+1)\sqrt{x} - 5$

$\operatorname{der}[P(x)] = 4 \Rightarrow$ **a + b + n = ?**

A) 3 B) 4 C) 5 D) 6 E) 7

9. $$\begin{array}{r|l} P(2x-3) & x-1 \\ \cline{2-2} - \qquad & \\ \cline{1-1} 3 & \end{array} \qquad \begin{array}{r|l} Q(x+3) & x+1 \\ \cline{2-2} - \qquad & \\ \cline{1-1} -2 & \end{array}$$

$$\frac{k\cdot Q(x+1)+x-3}{x\cdot P(x-2)+x^2} = k^2x-4 \Rightarrow \sum k = ?$$

A) $-\frac{5}{2}$ B) $-\frac{3}{2}$ C) $-\frac{1}{2}$ D) $\frac{3}{2}$ E) $\frac{5}{2}$

10. $$\begin{array}{r|l} P(x) & x^2-2x+4 \\ \cline{2-2} - \qquad & B(x) \\ \cline{1-1} 3x+1 & \end{array} \qquad \begin{array}{r|l} B(x) & x^2-4 \\ \cline{2-2} - \qquad & \\ \cline{1-1} 2x-3 & \end{array}$$

$$\begin{array}{r|l} P(x) & x^3+8 \\ \cline{2-2} - \qquad & \\ \cline{1-1} K(x) = ? & \end{array}$$

A) $-7x^2 + 17x - 27$

B) $7x^2 - 17x$

C) $-7x^2 + 17x - 29$

D) $6x^2 - 13x + 2$

E) $6x^2 + 13x - 2$

11. $$\frac{4x-3}{x^2+x-6} = \frac{A}{x+3} + \frac{B}{x-2}$$

$\Rightarrow$ **A – B = ?**

A) –8 B) –6 C) –2 D) 2 E) 6

12. $(x + 2)\cdot P(2x - 1) = 4x^3 - 3x^2 - (m - 1)x + 4$

$$\begin{array}{r|l} P(x) & x+2 \\ \cline{2-2} - \qquad & \\ \cline{1-1} K(x) = ? & \end{array}$$

A) $\frac{11}{2}$ B) $\frac{13}{2}$ C) $\frac{15}{2}$ D) $\frac{17}{2}$ E) $\frac{19}{2}$

13. $P(x) = x^3 - 3x^2 + mx + n$

$$\begin{array}{r|l} P(x) & (x-1)^2 \\ \cline{2-2} - \qquad & \\ \cline{1-1} 0 & \end{array} \Rightarrow \mathbf{n = ?}$$

A) –3 B) –1 C) 1 D) 2 E) 3

14. $(x^2 - x + 3)\cdot P(2x + 1) = P(2x - 3) + x^4 + 3x^2 - 4x + 5$

$$\begin{array}{r|l} P(x) & x-5 \\ \cline{2-2} - \qquad & \\ \cline{1-1} 7 & \end{array} \qquad \begin{array}{r|l} P(x) & x-1 \\ \cline{2-2} - \qquad & \\ \cline{1-1} K(x) = ? & \end{array}$$

A) 8 B) 10 C) 14 D) 15 E) 16

15. $P(x) = (x + 2) \cdot Q(x) + 5$

$Q(x) = (x-2) \cdot B(x) - 3$

$$\begin{array}{r|l} P(x) & x^2-4 \\ \cline{2-2} - \qquad & \\ \cline{1-1} K(x) = ? & \end{array}$$

A) $3x + 1$ B) $3x + 2$ C) $3x + 3$ D) $-3x - 2$ E) $-3x - 1$

16. $P(x) = 2x^3 - ax^2 + bx + c$

$$\begin{array}{r|l} P(x) & (x-2)^3 \\ \cline{2-2} - \qquad & \\ \cline{1-1} 0 & \end{array} \Rightarrow \mathbf{c = ?}$$

A) –18 B) –16 C) –12 D) 12 E) 16

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1	2	3	4	5	6	7	8
C	E	D	C	A	E	B	C
9	**10**	**11**	**12**	**13**	**14**	**15**	**16**
C	A	D	D	B	B	E	B

1. $P(x, y) = x^3y - x^2y^2 + 2y^4 \Rightarrow$ **$P(1, -1) = ?$**

A) –4 B) –2 C) 0 D) 2 E) 4

2. $P(x-1) = (x - 3) \cdot Q(x) + 2x - 1 \Rightarrow$ **$P(2) = ?$**

A) 1 B) 2 C) 3 D) 4 E) 5

3. $P(x) = (2x^2 - 3x + 1) \cdot Q(x - 3)$

$Q(1) = 2 \Rightarrow$ **$P(4) = ?$**

A) 42 B) 38 C) 32 D) 21 E) 16

4. $P(x^2) = x^4 + 5x^2 + 8 \Rightarrow$ **$P(-3) = ?$**

A) 1 B) 2 C) 3 D) 4 E) 6

5. $2x^2 + x - 6 = a \cdot (x + 1)^2 + bx + c \Rightarrow$ **$a + b + c = ?$**

A) –6 B) –7 C) –8 D) –9 E) –10

6. $(2x + 1)^2 \cdot (x - 3) = ax^3 + bx^2 + cx + d \Rightarrow$ **$a + c = ?$**

A) – 8 B) – 7 C) 3 D) 4 E) 7

7. $P(x) = (3x^3 - 4x^2 + x - 5) \cdot (4x^4 + x^3 + x^2 - 1),$

$P(x) = Ax^7 + Bx^6 + Cx^5 + Dx^4 + Ex^3 + Fx^2 + Kx + L$

$\Rightarrow$ **$C = ?$**

A) 1 B) 2 C) 3 D) 4 E) 5

8. $\left.\begin{aligned} P(x+1) &= x^2 + ax + b \\ P(x-1) &= x^2 - 4x + 3 \end{aligned}\right\} \Rightarrow$ **$a + b = ?$**

A) – 1 B) – 2 C) – 3 D) – 4 E) – 5

9. $\dfrac{3x+8}{x^2+3x-10} = \dfrac{A}{x-2} + \dfrac{B}{x+5} \Rightarrow$ **$2B - A = ?$**

A) –1 B) 0 C) 1 D) 2 E) 3

10. $P(x) = (x + 1) \cdot Q(x - 1) + x^2 + 3$

$P(x) = (x - 2) \cdot (ax - b)$

$\Rightarrow$ **$Q(1) = ?$**

A) $-\frac{7}{3}$ B) $-\frac{7}{2}$ C) 0 D) $\frac{7}{2}$ E) $\frac{7}{3}$

11. $P(x) = x^3 - 6x^2 + 12x - 6$

$\Rightarrow$ **$P(\sqrt[3]{2} + 2) = ?$**

A) 2 B) 4 C) 6 D) 8 E) 10

12. $P(x) = (x - 2)^3 + ax^2$

$Q(x) = x^3 + 6x^2 + 12x + b$

$P(x) = Q(x) \Rightarrow$ **$a - b = ?$**

A) 20 B) 18 C) 16 D) 14 E) 12

13. $\left.\begin{array}{l} P(x) = x^3 - 1 \\ Q(x) = x^2 + x + 1 \end{array}\right\} \Rightarrow \mathbf{\frac{P(x)}{(x-1) \cdot Q(x)} = ?}$

A) $x + 1$ B) $x - 1$ C) 1 D) $x^2 + x$ E) x^2

14. $P(x) + P(3x) = 4x + 6 \Rightarrow$ **$P(-3) = ?$**

A) –4 B) –2 C) 0 D) 2 E) 4

15. $P(x) + P(2x) = 6x + 4 \Rightarrow$ **$P(2) = ?$**

A) 4 B) 5 C) 6 D) 8 E) 10

16. $\left.\begin{array}{l} P(x^2) = x^4 - 1 \\ Q(\sqrt{x}) = x^3 + x + 1 \end{array}\right\} \Rightarrow \mathbf{P(x) \cdot Q(x) = ?}$

A) $x^5 + x^2 - x + 1$

B) $x^8 + x^6 + x^4 - 1$

C) $x^8 - x^6 + 1$

D) $x^8 - x^6 + x^4 - 1$

E) $x^8 + x^6 + 1$

17. $P(x) + P(x - 4) = 2x^2 + 18 \Rightarrow$ **$P(2) = ?$**

A) 11 B) 13 C) 15 D) 17 E) 21

18. $P(x - 1) + P(x + 1) = 4x^2 - 2x + 10 \Rightarrow$ **$P(x) = ?$**

A) $2x^2 - x - 3$ B) $2x^2 + x - 3$

C) $2x^2 - x + 3$ D) $4x^2 + x - 1$

E) $2x^2 - 1$

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1	2	3	4	5	6	7	8	9
C	E	A	B	D	B	C	A	B
10	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**
A	B	A	C	C	C	D	E	C

1. $P(x+1)=\dfrac{x^2+2x+3}{Q(x+2)}+3x-2,\ P(4)=6$

$\Rightarrow$ **$Q(5)=?$**

A) – 18 B) – 15 C) – 9 D) – 6 E) – 3

2. $P(x-4)=(x^2+3x+2)\cdot Q(x)+x^2+5$

$\Rightarrow$ **$P(-6)=?$**

A) 10 B) 9 C) 7 D) 4 E) 1

3. $\dfrac{3\cdot P(x-2)-2x}{Q(x+3)}=x+6$

$Q(2)=4\ \Rightarrow$ **$P(-3)=?$**

A) 2 B) 3 C) 4 D) 5 E) 6

4. $\dfrac{P(x+3)+x^2}{x-2}=(x^3-1)\cdot Q(x)+4$

$P(2)=5\ \Rightarrow$ **$Q(-1)=?$**

A) 1 B) 2 C) 3 D) 4 E) 5

5. $\left.\begin{array}{l}P(x)=(x+1)\cdot Q(x)+7x\\ P(x)=x^2+5a\end{array}\right\}\Rightarrow$ **$a=?$**

A) $-\frac{8}{5}$ B) $-\frac{6}{5}$ C) $-\frac{4}{5}$ D) $-\frac{2}{5}$ E) 0

6. $P(\sqrt[3]{x^2})=x^4-x^2+3\ \Rightarrow$ **$P(x)=?$**

A) x^6-x^3+3

B) x^6+x^3+3

C) x^6-x^3+1

D) x^3+2x^3+1

E) x^3-x+3

7. $P(x)=(x^2-5x-6)\cdot Q(x)+3x+m$

$P(-1)=-2\ \Rightarrow$ **$P(6)=?$**

A) 15 B) 16 C) 18 D) 19 E) 21

8. $P(4-x)=(x^2-8)\cdot Q(x+10)-2mx-12,$

$P(7)=Q(7)\Rightarrow$ **$m=?$**

A) 6 B) 5 C) 4 D) 3 E) 2

9. $P(x+1) - 4 = Q(2x-1) \cdot (x^2 - 4)$

$Q(-3) = 2 \Rightarrow \mathbf{P(0) = ?}$

A) -6 B) -4 C) -3 D) -2 E) 1

10. $P(x+1) = Q(x) \cdot (x^3 - 2x + k) + 3$

$Q(2) = 3,\ P(3) = -1 \Rightarrow \mathbf{k = ?}$

A) -6 B) $-\frac{16}{3}$ C) -5 D) $-\frac{14}{3}$ E) $-\frac{10}{3}$

11.

$$\begin{array}{r|l} x^3+x^2-x-1 & x-1 \\ \cline{2-2} - & Q(x) \\ \hline 0 & \end{array} \Rightarrow \mathbf{Q(x) = ?}$$

A) $x^2 + 4x + 1$ B) $x^2 + 2x + 1$

C) $3x^2 - 4x + 3$ D) $x^2 + 4x + 3$

E) $3x^2 + 3$

12.

$$\begin{array}{r|l} 4x^4+x^3-1 & x^2+1 \\ \cline{2-2} - & Q(x) \\ \hline -x+3 & \end{array} \Rightarrow \mathbf{Q(x) = ?}$$

A) $4x^2 + x - 4$ B) $2x^2 + x - 1$

C) $4x^2 + 2x - 1$ D) $4x^2 + x + 4$

E) $4x^2 - x + 4$

13.

$$\begin{array}{r|l} P(x) & x+2 \\ \cline{2-2} - & x^2-3 \\ \hline 13 & \end{array} \Rightarrow \mathbf{\frac{P(0)}{P(-2)} = ?}$$

A) $\frac{7}{13}$ B) 3 C) $\frac{10}{3}$ D) $\frac{13}{2}$ E) 10

14. $P(x) = x^4 + x^3 + ax^2 + x + 6$

$$\begin{array}{r|l} P(x) & (x-2) \\ \cline{2-2} - & \\ \hline 0 & \end{array} \Rightarrow \mathbf{a = ?}$$

A) -2 B) -4 C) -6 D) -8 E) -10

15. $P(x+1) = x^2 + 2x + a$

$$\begin{array}{r|l} P(2x-3) & x-4 \\ \cline{2-2} - & \\ \hline 11 & \end{array} \Rightarrow \mathbf{a = ?}$$

A) -6 B) -10 C) -13 D) -17 E) -20

16.

$$\begin{array}{r|l} P(x) & x-2 \\ \cline{2-2} - & \\ \hline 2 & \end{array} , \quad \begin{array}{r|l} Q(x) & x-2 \\ \cline{2-2} - & \\ \hline 4 & \end{array}$$

$$\begin{array}{r|l} 2xP(x)+Q(x) & x-2 \\ \cline{2-2} - & \\ \hline K(x) & \end{array} \Rightarrow \mathbf{K(x) = ?}$$

A) 12 B) 15 C) 17 D) 18 E) 22

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1	2	3	4	5	6	7	8
A	B	E	C	A	A	D	E
9	**10**	**11**	**12**	**13**	**14**	**15**	**16**
D	B	B	A	A	D	C	A

1. $$\begin{array}{r|l} 3x^3-4x^2+2x+k & x-2 \\ \cline{2-2} & Q(x) \\ - & \\ \hline 0 & \end{array} \Rightarrow \mathbf{k=?}$$

A) −12 B) −10 C) − 9 D) − 8 E) − 6

2. $P(-x+1)=2x^2-5x-1$

$$\begin{array}{r|l} P(x) & 3x-6 \\ \cline{2-2} - & \\ \hline K(x) & \end{array} \Rightarrow \mathbf{K(x)=?}$$

A) 6 B) 8 C) 12 D) 15 E) 17

3. $P(x)=x^3+1$

$$\begin{array}{r|l} P(x) & x+1 \\ \cline{2-2} & Q(x) \\ - & \\ \hline 0 & \end{array} \Rightarrow \mathbf{Q\,(0)=?}$$

A) $\frac{1}{2}$ B) 1 C) $-\frac{1}{2}$ D) − 1 E) − 2

4. $P(1-x)=\dfrac{x}{Q(x-2)}+2x-1$

$$\begin{array}{r|l} P(x-4) & x-2 \\ \cline{2-2} - & \\ \hline -1 & \end{array}, \quad \begin{array}{r|l} Q(x) & x-1 \\ \cline{2-2} - & \\ \hline K(x) & \end{array} \Rightarrow \mathbf{K(x)=?}$$

A) −1 B) $-\frac{1}{2}$ C) 0 D) $\frac{1}{2}$ E) 1

5. $P(x)=x^3+ax^2+bx+1$

$$\begin{array}{r|l} P(x) & x^2-1 \\ \cline{2-2} - & \\ \hline 0 & \end{array} \Rightarrow \mathbf{a+b=?}$$

A) −1 B) −2 C) −3 D) −4 E) −5

6. $P(x+2)=x+a$

$$\begin{array}{r|l} P(x) & x-b \\ \cline{2-2} - & \\ \hline 1 & \end{array} \Rightarrow \mathbf{a=?}$$

A) $\frac{4+b}{2}$ B) $\frac{1+3b}{2}$ C) $b+1$ D) $2-2b$ E) $3-b$

7. $P\left(\frac{x-1}{4}\right)=x^2\cdot Q(x-6)+x$

$P(1)=30,$

$$\begin{array}{r|l} Q(x) & x+1 \\ \cline{2-2} - & \\ \hline K(x) & \end{array} \Rightarrow \mathbf{K(x)=?}$$

A) −2 B) −1 C) 0 D) 1 E) 2

8. $P(x)=x^7-3x^6+2ax+b-4$

$$\begin{array}{r|l} P(x) & x-1 \\ \cline{2-2} - & \\ \hline 0 & \end{array} \Rightarrow \mathbf{2a+b=?}$$

A) 4 B) 5 C) 6 D) 7 E) 8

9.

$$\begin{array}{l|l} 2x^3-mx^2+nx+1 & x^2+2x-3 \\ \cline{2-2} & P(x) \\ - & \\ \hline 0 & \end{array} \Rightarrow \mathbf{3m+n=?}$$

A) $-\frac{53}{3}$ B) $-\frac{58}{3}$ C) $-\frac{61}{3}$

D) $-\frac{64}{3}$ E) $-\frac{68}{3}$

10.

$$\begin{array}{l|l} 2x^2-mx+n & (x-1)\cdot(x+1) \\ \cline{2-2} & Q(x) \\ - & \\ \hline 0 & \end{array} \Rightarrow \mathbf{n=?}$$

A) –2 B) –1 C) 0 D) 1 E) 2

11. $P(x) = -x^{32} + x^{24} - 2$

$$\begin{array}{l|l} P(x) & x^8-a^2 \\ \cline{2-2} - & \\ \hline K(x) & \end{array} \Rightarrow \mathbf{K(x)=?}$$

A) $-a^8 + a^6 - 2$ B) $-a^8 + a^4 - 2$

C) $-a^8 + 2a^6 + 1$ D) $-a^8 - a^5 + 2$

E) $-a^8 + a^7 + 2$

12. $P(x) = -x^4 + x^3 + ax^2 + bx + 2,$

$$\begin{array}{l|l} P(x) & x^2-1 \\ \cline{2-2} - & \\ \hline 12x-9 & \end{array} \Rightarrow \mathbf{a+b=?}$$

A) 0 B) 1 C) 2 D) 3 E) 4

13.

$$\begin{array}{l|l} P(x) & x-\frac{1}{2} \\ \cline{2-2} - & \\ \hline 1 & \end{array} \qquad \begin{array}{l|l} P(x) & x+\frac{1}{2} \\ \cline{2-2} - & \\ \hline -1 & \end{array}$$

$$\begin{array}{l|l} P(x) & x^2-\frac{1}{4} \\ \cline{2-2} - & \\ \hline K(x) & \end{array} \Rightarrow \mathbf{K(x)=?}$$

A) $\frac{x}{2}$ B) x C) $\frac{3x}{2}$ D) 2x E) $\frac{5x}{2}$

14.

$$\begin{array}{l|l} 3x^3-4x^2+ax-b & x^2-x+3 \\ \cline{2-2} & Q(x) \\ - & \\ \hline 0 & \end{array} \Rightarrow \mathbf{a\cdot b=?}$$

A) –30 B) –20 C) 10 D) 20 E) 30

15. $P(0) = 3,$

$$\begin{array}{l|l} P(x) & 2x-1 \\ \cline{2-2} - & \\ \hline 4 & \end{array} \qquad \begin{array}{l|l} P(x) & 2x^2-x \\ \cline{2-2} - & \\ \hline K(x) & \end{array} \Rightarrow \mathbf{K(x)=?}$$

A) 2x + 3 B) 2x + 2 C) 2x – 1

D) 2x – 4 E) 2x – 5

16. $P(x-2) = x^3 - ax^2 + 2x + 2$

$$\begin{array}{l|l} P(x-1) & x^3-1 \\ \cline{2-2} - & \\ \hline x^2-3 & \end{array} \Rightarrow \mathbf{a=?}$$

A) 1 B) 3 C) 4 D) 6 E) 7

YANITLAR / ANSWERS

1	2	3	4	5	6	7	8
A	A	B	B	B	E	D	C
9	**10**	**11**	**12**	**13**	**14**	**15**	**16**
A	A	A	B	D	E	A	C

BÖLÜM
CHAPTER
17

TARİHSEL NOT / HISTORICAL NOTE

MERYEM MİRZAKHANİ [1977 – 2017]

2008'den bu yana Stanford Üniversitesi'nde matematik profesörü olarak çalışan MİRZAKHANİ, 2014 yılında "Matematiğin Nobeli " sayılan Fields madalyasını alarak bu ödülü kazanan ilk kadın olmuştur. Çalışmalarında özellikle hiperbolik geometri, ergodik teori, simplektik geometri ve Teichmüller teorisine odaklanmaktadır.

MIRZAKHANI, who had been working as a professor of mathematics at Stanford University since 2008, has awarded the Fields Medal as the Nobel Prize in Mathematics in 2014 and became the first woman to win this award. She focused on hyperbolic geometry, ergodic theory, symplectic geometry and Teichmüller theory in her works.

İKİNCİ DERECEDEN DENKLEMLER
QUADRATIC EQUATIONS

Bu bölüm, 87 test sorusu içermektedir.
This chapter includes 87 test questions.

BÖLÜM / CHAPTER 17

İKİNCİ DERECEDEN DENKLEMLER / QUADRATIC EQUATIONS

- İkinci Dereceden Denklemler / Quadratic Equations .... 307 – 322

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BÖLÜM
17
CHAPTER

İKİNCİ DERECEDEN DENKLEMLER
QUADRATIC EQUATIONS

İKİNCİ DERECEDEN DENKLEMLER / QUADRATIC EQUATIONS

Tanım: a, b, $c \in R$, $a \neq 0$ olmak üzere, $ax^2 + bx + c = 0$ biçimindeki eşitliklere ***ikinci dereceden denklem*** denir.

▼ a, b, c sayılarına ***denklemin katsayıları*** denir. Denklemi sağlayan sayılara ***denklemin kökleri,*** köklerin kümesine de ***çözüm kümesi*** denir.

Definition: An equation in the form of $ax^2 + bx + c = 0$ is called a ***quadratic equation*** where a, b, $c \in R$, $a \neq 0$.

▼ The numbers a,b and c are ***coefficients*** *of the equation.* Numbers satisfying the equation are called ***roots*** *of the equation,* the set containing the roots is called *the* ***solution set.***

İKİNCİ DERECEDEN DENKLEMLERİN ÇÖZÜM KÜMESİNİN BULUNMASI
FINDING the SOLUTION SET of the QUADRATIC EQUATIONS

ÇARPANLARA AYIRMA YÖNTEMİ / FACTORIZATION METHOD

▼ a, b, $c \in R$, $a \neq 0$ olmak üzere, $ax^2 + bx + c = 0$ denklemi çarpanlarına ayrılıyorsa denklemin kökleri çarpanlara ayrılarak bulunur.

▼ If the equation $ax^2 + bx + c = 0$ is factorizable, the roots of equation are found by the factorization method.

DİSKRİMİNANT YÖNTEMİ / THE DISCRIMINANT METHOD

▼ Çarpanlara ayırma yöntemi, çarpanlara ayrılabilen ikinci dereceden denklemlerde kullanılır. Fakat birçok ikinci dereceden denklem, çarpanlara ayrılmayabilir. Bu tür denklemleri çözmek için, genel bir formül ***diskriminant yöntemi***dir.

▼ The factorization method is used for the factorisable quadratic equations. But a lot of quadratic equations are not factorisable. For these types of quadratic equations, there is a general formula called **the discriminant method**.

▼ $a \neq 0$, $ax^2 + bx + c = 0$ denklemi için;

$\Delta = b^2 - 4 \cdot a \cdot c$ reel sayısına denklemin ***diskriminantı*** denir.

▼ For the equation $ax^2 + bx + c = 0$,

$\Delta = b^2 - 4 \cdot a \cdot c$ is called the ***discriminant*** of the equation.

Özellik / Property

$a \neq 0$, $ax^2 + bx + c = 0$ denkleminde,

In equation $ax^2 + bx + c = 0$,

★ **$\Delta > 0$ ise, denklemin birbirinden farklı iki reel kökü vardır.**

Bu kökler aşağıdaki gibidir:

★ If $\Delta > 0$, the equation has two distinct real solutions.

These roots are as follows:

$$x_1 = \frac{-b-\sqrt{\Delta}}{2a}$$

$$x_2 = \frac{-b+\sqrt{\Delta}}{2a}$$

★ **$\Delta = 0$ ise, denklemin birbirine eşit iki kökü vardır.**

Bu kökler aşağıdaki gibidir.

★ If $\Delta = 0$, there are two equal solutions.

These roots are as follows:

$$x_1 = x_2 = -\frac{b}{2a}$$

★ **$\Delta < 0$ ise, denklemin reel kökü yoktur. Çözüm kümesi boş kümedir.**

★ If $\Delta < 0$, the equation has no real solutions. The solution set is an empty set.

Önemli / Important

- $x_1 = x_2 \Rightarrow$ çift katlı kök (çakışık kök)
- $x_1 = -x_2 \Rightarrow$ simetrik kök $(b = 0)$
- $x_1 = x_2 \Rightarrow$ double root (eclipsed root)
- $x_1 = -x_2 \Rightarrow$ symmetric root $(b = 0)$

Önemli / Important

- İkinci dereceden bir denklemin kökleri rasyonel değil ise, kökler **eşlenik ifade**dir.
- If the roots of a second order equation are not rational, then they are conjugate.

$a \neq 0$, $ax^2 + bx + c = 0$, ÇK(SS) = $\{x_1 , x_2\}$

$x_1 = \sqrt{a} - \sqrt{b} \Rightarrow x_2 = \sqrt{a} + \sqrt{b}$

İKİNCİ DERECEDEN DENKLEME DÖNÜŞTÜRÜLEBİLEN DENKLEMLER
THE EQUATIONS that CAN be CONVERTED into a QUADRATIC EQUATION

$f(x) \cdot g(x) = 0 \Rightarrow f(x) = 0$ veya $g(x) = 0$

DEĞİŞKEN DEĞİŞTİREREK ÇÖZÜLEBİLEN DENKLEMLER
THE EQUATIONS that CAN be SOLVED by CHANGING the VARIABLE

KÖKLÜ DENKLEMLER / THE EQUATIONS INCOLVING RADICALS

▼ Bu tür denklemler çözülürken, verilen denklem $\sqrt[n]{f(x)} = g(x)$ biçiminde yazılır ve her iki tarafın n. kuvveti alınarak kökten kurtarılır. Eğer kök kuvveti çift ise bulunan x'ler denklemde yerine yazılarak denklemi sağlayıp sağlamadığına bakılır. Denklemi sağlayan x'ler çözüm kümesini oluşturur.

▼ In order to solve these type of equations, the given equation is written in the form of $\sqrt[n]{f(x)} = g(x)$, and then the radical is reduced by taking both sides to the same power. If the index is odd, the founded values are the roots of the equation. But if the index is even, we need to substitute the founded values in the original equation. The values which are satisfying the equation build up the solution set.

KÖKLER ile KATSAYILAR ARASINDAKİ BAĞINTILAR
RELATIONS BETWEEN ROOTS and COEFFICIENTS

▼ $a \neq 0$, $ax^2 + bx + c = 0$ denkleminde $\triangle \geq 0$ ise,

▼ If $\triangle > 0$ in the equation, $ax^2 + bx + c = 0$, $a \neq 0$

$$x_1 = \frac{-b+\sqrt{\triangle}}{2a}, \quad x_2 = \frac{-b-\sqrt{\triangle}}{2a}$$

★ $x_1 + x_2 = -\frac{b}{a}$

★ $x_1 \cdot x_2 = \frac{c}{a}$

★ $|x_1 - x_2| = \frac{\sqrt{\triangle}}{|a|}$

Önemli / Important

- $\frac{1}{x_1} + \frac{1}{x_2} = -\frac{b}{c}$

İspat / Proof:

$$\frac{1}{\underset{(x_2)}{x_1}} + \frac{1}{\underset{(x_1)}{x_2}} = \frac{x_2 + x_1}{x_1 \cdot x_2}$$

$$= \frac{-\frac{b}{a}}{\frac{c}{a}} = -\frac{b}{c}$$

KÖKLERİ VERİLEN İKİNCİ DERECEDEN DENKLEMİN YAZILMASI
WRITING QUADRATIC EQUATION with the GIVEN ROOTS

▼ $a \neq 0$, $ax^2 + bx + c = 0$

$$a\left(x^2 + \frac{b}{a}x + \frac{c}{a}\right) = 0 \Rightarrow x^2 + \frac{b}{a}x + \frac{c}{a} = 0$$

$$x_1 + x_2 = -\frac{b}{a}, \quad x_1 \cdot x_2 = \frac{c}{a}$$

Önemli / Important

- Kökleri x_1 ve x_2 olan ikinci dereceden bir denklem genel olarak aşağıdaki gibi yazılır:
- In general, we can write a quadratic equation with the roots x_1 and x_2 as follows:

$$a \cdot (x - x_1) \cdot (x - x_2) = 0 \quad (a \neq 0,\ a \in R)$$

ÜÇÜNCÜ DERECEDEN DENKLEMLER / CUBIC EQUATIONS

Tanım: a, b, c, d $\in$ R, $a \neq 0$ olmak üzere $ax^3 + bx^2 + cx + d = 0$ biçimindeki eşitliklere ***üçüncü dereceden denklem*** denir.

Definition: a, b, c, d $\in$ R, $a \neq 0$ the equations in the form of $ax^3 + bx^2 + cx + d = 0$ are called cubic ,equations.

▼ a, b, c, d sayılarına **denklemin katsayıları** denir. Denklemi sağlayan sayılara **denklemin kökleri**, köklerin kümesine de **çözüm kümesi** denir.

▼ the numbers a, b, c, d are called the ***coefficients*** of the equation. The values satisfying the equation are called the ***roots of that equation***, the set of these roots is called the ***solution set***.

Özellik / Property

$a \neq 0,\ ax^3 + bx^2 + cx + d = 0,\ ÇK(SS) = \{x_1, x_2, x_3\}$

★ $x_1 + x_2 + x_3 = -\frac{b}{a}$

★ $x_1 \cdot x_2 + x_1 \cdot x_3 + x_2 \cdot x_3 = \frac{c}{a}$

★ $x_1 \cdot x_2 \cdot x_3 = -\frac{d}{a}$

1. $2x^2 - 3x - 20 = 0 \Rightarrow$ **ÇK(SS) = ?**

A) $\left\{4, -\frac{5}{2}\right\}$ B) $\left\{\frac{5}{2}, 4\right\}$ C) $\left\{-\frac{5}{2}, -4\right\}$

D) $\{-2, 5\}$ E) $\{-5, 2\}$

2. $3x^2 - 26x + 40 = 0 \Rightarrow$ **ÇK(SS) = ?**

A) $\{2, 3\}$ B) $\{-3, 3\}$ C) $\left\{\frac{20}{3}, 2\right\}$

D) $\left\{\frac{10}{3}, 1\right\}$ E) $\left\{\frac{40}{3}, 4\right\}$

3. $x^2 - 3x + 5 = 0 \Rightarrow$ **ÇK(SS) = ?**

A) $\{-1, 2\}$ B) $\{0\}$ C) $\{2, 3\}$

D) R E) $\varnothing$

4. $2x^2 + ax - 3 = 0$

$x_1 = -2 \Rightarrow$ **a = ?**

A) $\frac{1}{2}$ B) 1 C) $\frac{3}{2}$ D) 2 E) $\frac{5}{2}$

5. $x^3 - x^2 - x + 1 = 0 \Rightarrow$ **ÇK(SS) = ?**

A) $\{1, -1\}$ B) $\{-1, 0, 1\}$ C) $\{-2, -1\}$

D) $\{-2, -1, 0\}$ E) $\{1, 3\}$

6. $(x^2 + 3x)\cdot(x^2 - 2x + 5) = 0 \Rightarrow$ **ÇK(SS) = ?**

A) $\varnothing$ B) $\{-3, 3\}$ C) $\{0, 3\}$

D) $\{-3, 0\}$ E) $\{1, 3\}$

7. $\frac{3x^2 - 8x + 4}{x - 2} = 0 \Rightarrow$ **ÇK(SS) = ?**

A) $\left\{2, \frac{2}{3}\right\}$ B) $\left\{-\frac{2}{3}, \frac{2}{3}\right\}$ C) $\{-2, 2\}$

D) $\left\{-2, \frac{2}{3}\right\}$ E) $\left\{\frac{2}{3}\right\}$

8. $\frac{1}{x - 1} + \frac{2}{x + 3} = \frac{7}{5} \Rightarrow$ **ÇK(SS) = ?**

A) $\left\{-\frac{13}{7}, 1\right\}$ B) $\left\{\frac{13}{7}\right\}$ C) $\left\{-1, \frac{13}{7}\right\}$

D) $\left\{-\frac{13}{7}, 2\right\}$ E) $\left\{-2, \frac{13}{7}\right\}$

9. $2x^3 + 3x^2 - 18x - 27 = 0 \Rightarrow$ **ÇK(SS) = ?**

A) $\{-3, 3\}$ B) $\left\{-3, -\frac{3}{2}, 3\right\}$ C) $\{-3, 0, 3\}$

D) $R - \{-3\}$ E) $\varnothing$

10. $\frac{x-1}{x-2}-\frac{8}{3}=\frac{x-2}{x-1} \Rightarrow$ **ÇK(SS) = ?**

A) $\left\{-2,\frac{5}{4},\frac{5}{3}\right\}$
B) $\left\{-1,\frac{5}{4},\frac{5}{2},2\right\}$
C) {2, 1}
D) $\left\{\frac{5}{4},\frac{5}{3}\right\}$
E) $\left\{\frac{5}{4},\frac{5}{2}\right\}$

11. $x^6+26x^3-27=0 \Rightarrow$ **ÇK(SS) = ?**

A) {–3, 3}
B) {1, –1}
C) {–3, –1, 1, 3}
D) {–1, 3}
E) {–3, 1}

12. $(x^2-3x)^2-22(x^2-3x)+72=0 \Rightarrow$ **ÇK(SS) = ?**

A) {–3, –1, 4, 6}
B) {4, 18}
C) {6, –3}
D) {1, 4}
E) {4, 6}

13. $9x^3-4x=0$

$x_1 < x_2 < x_3 \Rightarrow$ **x_1 = ?**

A) 0
B) $-\frac{2}{3}$
C) $-\frac{4}{9}$
D) $-\frac{4}{9}$
E) $-\frac{5}{9}$

14. $x^4-5x^2-36=0 \Rightarrow$ **ÇK(SS) = ?**

A) {–3, 3}
B) {–3, –2, 2, 3}
C) {–2, 2}
D) {–1, 1}
E) {2, 3}

15. $\frac{x^2}{2x+3}+2-3\cdot\left(\frac{2x+3}{x^2}\right)=0 \Rightarrow$ **ÇK(SS) = ?**

A) {–3, 1}
B) {3, –1}
C) {–3, 3}
D) {–3, 1, 3}
E) {–3, –1, 3}

16. $\sqrt{1-2x}-7=x \Rightarrow$ **ÇK(SS) = ?**

A) {–1, 4}
B) {–4}
C) {–4, 4}
D) {–4, –12}
E) {–12}

17. $\sqrt{x}+\sqrt{x+2}+2=0 \Rightarrow$ **ÇK(SS) = ?**

A) {–2}
B) {0}
C) ∅
D) {1}
E) {2}

18. $\sqrt[3]{3x-\sqrt{x-2}}=2 \Rightarrow$ **ÇK(SS) = ?**

A) {3}
B) $\left\{3,\frac{22}{9}\right\}$
C) $\left\{\frac{22}{9}\right\}$
D) $\left\{\frac{22}{9},4\right\}$
E) ∅

YANITLAR / ANSWERS

1	2	3	4	5	6	7	8	9
A	C	E	E	A	D	E	D	B
10	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**
E	E	A	B	A	E	B	C	B

1. $3x^2 - 8x + 2 = 0 \Rightarrow \mathbf{x_1 + x_2 = ?}$

A) $-\frac{8}{3}$ B) $-\frac{2}{3}$ C) $\frac{8}{3}$ D) $\frac{2}{3}$ E) 8

2. $2x^2 - 4(m-1)x + 3 = 0$

$x_1 + x_2 = 2 \Rightarrow \mathbf{m = ?}$

A) – 1 B) 0 C) 1 D) 2 E) 3

3. $x^2 - 4x + m + 1 = 0$

$x_1 = 3x_2 \Rightarrow \mathbf{m = ?}$

A) –1 B) 0 C) 2 D) 4 E) 8

4. $mx^2 + (m-1)x - 4m = 0$

$x_1 = -x_2 \Rightarrow \mathbf{m = ?}$

A) –3 B) –1 C) 1 D) 2 E) 4

5. $3x^2 + 4x + 6 \Rightarrow \mathbf{x_1 \cdot x_2 = ?}$

A) $-\frac{4}{3}$ B) $\frac{4}{3}$ C) –2 D) 2 E) $\frac{3}{2}$

6. $3x^2 - 3x + 4 = 0 \Rightarrow \mathbf{(x_1 - 3)\cdot(x_2 - 3) = ?}$

A) $\frac{11}{3}$ B) 4 C) $\frac{13}{3}$ D) 5 E) $\frac{22}{3}$

7. $2x^2 - mx + 3m - 2 = 0$

$(x_1 + x_2) = 4(x_1 \cdot x_2) \Rightarrow \mathbf{m = ?}$

A) $\frac{8}{11}$ B) $\frac{7}{11}$ C) $\frac{6}{11}$ D) $\frac{5}{11}$ E) $\frac{3}{11}$

8. $2x^2 - 6x + 4 = 0$

$\Rightarrow \mathbf{\frac{1}{x_1 + 3} + \frac{1}{x_2 + 3} = ?}$

A) $-\frac{3}{20}$ B) $\frac{1}{20}$ C) $\frac{3}{20}$ D) $\frac{5}{20}$ E) $\frac{9}{20}$

9. $x^2 + (a-5)x - 4 = 0$

$2x_1 = x_2^2 \Rightarrow \mathbf{a = ?}$

A) 8 B) 5 C) 4 D) 3 E) 2

10. $x^2 - 12x + a + 17 = 0$

$x_1 = 3x_2 \Rightarrow \mathbf{a = ?}$

A) 20 B) 18 C) 16 D) 12 E) 10

11. $x^2 - 12x + 144 = 0$

$\Rightarrow \sqrt{x_1} + \sqrt{x_2} = ?$

A) –6 B) –3 C) 0 D) 3 E) 6

12. $x^2 - (m-2)x + m - 8 = 0$

$\frac{1}{x_1+2} + \frac{1}{x_2+2} = \frac{4}{5} \Rightarrow \mathbf{m = ?}$

A) 10 B) 8 C) 7 D) 6 E) 5

13. $2x^2 + (3m-1)x + m - 1 = 0$

$x_1 = \frac{3}{x_2} \Rightarrow \mathbf{x_1 + x_2 = ?}$

A) 10 B) 8 C) 5 D) –10 E) –15

14. $(a-2)x^2 - (2a+3)x + a + 1 = 0$

$x_1 = x_2 \Rightarrow \mathbf{a = ?}$

A) 3 B) 2 C) $\frac{7}{15}$ D) $-\frac{4}{5}$ E) $-\frac{17}{16}$

15. $x^2 + (a-3)x + b - 2 = 0$

$x_1 - x_2 = 1,$

$x_1^2 - x_2^2 = 5$

$\Rightarrow \mathbf{a - b = ?}$

A) –6 B) –10 C) 6 D) 4 E) 2

16. $3x^2 - 7x - 9 = 0 \Rightarrow \mathbf{6x_1^2x_2 + 6x_1x_2^2 + 9x_1 + 9x_2 = ?}$

A) –21 B) –12 C) 7 D) 18 E) 21

17. $(m-1)x^2 - 3mx - 4 = 0$

$x_1 + x_2 = 2 \Rightarrow \mathbf{m = ?}$

A) 2 B) 1 C) 0 D) –1 E) –2

18. $p \in R^+$

$x^2 + px + 12 = 0$

$x_1 - x_2 = 1 \Rightarrow \mathbf{p = ?}$

A) 7 B) 6 C) $3\sqrt{5}$ D) $\frac{3}{2}$ E) $\sqrt{6}$

YANITLAR / ANSWERS

1	2	3	4	5	6	7	8	9
C	D	C	C	D	E	A	E	B
10	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**
E	E	D	D	E	B	A	E	A

1. $\frac{1}{x+1}-\frac{3x}{1-x}=5 \Rightarrow \mathbf{x_1+x_2=?}$

A) –2 B) –1 C) 1 D) 2 E) 3

2. $\frac{1-\frac{1}{x}}{\frac{1-x}{x}}=x^2-5 \Rightarrow \mathbf{x_1 \cdot x_2=?}$

A) –5 B) – 4 C) – 2 D) 2 E) 4

3. $m<0$,

$x^2+2mx-3=0$,

$x_1=-3x_2 \Rightarrow \mathbf{m-3=?}$

A) –4 B) –3 C) –2 D) 3 E) 4

4. $3x^2-ax+b=0$,

$x_1+x_2=2x_1\cdot x_2 \Rightarrow \mathbf{\frac{a-b}{a+b}=?}$

A) $\frac{1}{2}$ B) $\frac{1}{3}$ C) $-\frac{1}{2}$ D) $-\frac{1}{3}$ E) 2

5. $a<14$,

$4x^2-(a-2)x+a-5=0$,

$\Delta=0 \Rightarrow \mathbf{a=?}$

A) –4 B) –2 C) 0 D) 2 E) 6

6. $x^2+(2m-1)x+m-1=0$,

$\frac{x_1+x_2}{2}=-\frac{5}{2} \Rightarrow \mathbf{x_1^2\cdot x_2+x_1\cdot x_2^2=?}$

A) –6 B) –10 C) –4 D) 3 E) 6

7. $2x^2-3x(a-x)+2a+1=0$,

$x_1\cdot x_2=-1 \Rightarrow \mathbf{x_1+x_2=?}$

A) $\frac{5}{6}$ B) $-\frac{5}{6}$ C) $\frac{6}{5}$ D) $-\frac{6}{5}$ E) $-\frac{9}{5}$

8. $(m-1)x^2-(2m+1)x+m+1=0$,

ÇK(SS) = ∅ ⇒ **m < ?**

A) –5 B) –4 C) $-\frac{5}{4}$ D) $-\frac{6}{7}$ E) 4

9. $2x^2 - 6x + m = 0,\quad x_1 - 3x_2 + 1 = 0 \Rightarrow \mathbf{m = ?}$

A) –2 B) –1 C) 1 D) 2 E) 4

10. $x^2 - x - 4 = 0 \Rightarrow \dfrac{\mathbf{2}}{\mathbf{3x_1}} + \dfrac{\mathbf{2}}{\mathbf{3x_2}} = \mathbf{?}$

A) $\frac{2}{3}$ B) $-\frac{1}{6}$ C) $-\frac{2}{5}$ D) $-\frac{3}{7}$ E) $-\frac{1}{4}$

11. $f(x) = x^2 - 6x + 2,$

$f(x_1) = f(x_2) = 0$

$\Rightarrow \mathbf{(x_1)^{-1} + (x_2)^{-1} = ?}$

A) –1 B) 1 C) 2 D) 3 E) 4

12. $f(x) = x^2 + 3x - 4$

$f(x_1) = f(x_2) = 0$

$\Rightarrow \dfrac{\mathbf{x_1}}{\mathbf{x_2}} + \dfrac{\mathbf{x_2}}{\mathbf{x_1}} = \mathbf{?}$

A) $-\frac{21}{4}$ B) $-\frac{17}{4}$ C) 1 D) $\frac{17}{4}$ E) $\frac{21}{4}$

13. $f(x) = x^2 - 4x + 2$

$f(x_1) = f(x_2) = 0$

$\Rightarrow \mathbf{x_1^3 + x_2^3 = ?}$

A) 36 B) 38 C) 40 D) 42 E) 46

14. $(x + a)^2 = 1 - 2a,\quad x_1 = x_2 \Rightarrow \mathbf{a = ?}$

A) $-\frac{1}{2}$ B) 2 C) 3 D) $\frac{7}{2}$ E) 4

15. $\sqrt{2x+13} = x + 5 \Rightarrow \mathbf{x_1 + x_2 = ?}$

A) –2 B) –8 C) –10 D) 10 E) 12

16. $f(x) = x^2 - (m - 3)x + n + 15,$

$f(m) = f(n) = 0$

$\Rightarrow \mathbf{m - n} = ?$

A) –7 B) –3 C) –1 D) 1 E) 7

17. $x^2 = |x| + 2 \Rightarrow \mathbf{x_1 \cdot x_2 = ?}$

A) –9 B) –4 C) –1 D) 1 E) 9

18. $f(x) = x^2 + bx + c,$

$f(2-\sqrt{3}) = f(2+\sqrt{3}) = 0$

$\Rightarrow \mathbf{c - b = ?}$

A) –3 B) –2 C) –1 D) 1 E) 5

YANITLAR / ANSWERS

1	2	3	4	5	6	7	8	9
D	B	A	B	E	B	E	C	E
10	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**
B	D	B	C	A	B	C	B	E

1. $m > 0$

$x^2 - (m + 1)x + 9 = 0,$

$ÇK(SS) = \{x_1 , x_2\},$

$\frac{x_1}{x_2} + \frac{x_2}{x_1} = 2 \Rightarrow \mathbf{m = ?}$

A) 3 B) 4 C) 5 D) 6 E) 7

2. $x^2 - (m - 3)x - 27 = 0$

$ÇK(SS) = \{x_1, x_1^2\} \Rightarrow \mathbf{m = ?}$

A) –9 B) –8 C) –1 D) 8 E) 9

3. $m > 0,$

$(m - 3)x^2 + 5x + 2 = 0,\ ÇK(SS) = \{x_1, x_2\},$

$6x^2 + nx + 3m = 0,\ ÇK(SS) = \{x_1, x_2\}$

$\Rightarrow \mathbf{\frac{n-m}{2} = ?}$

A) 12 B) 13 C) 15 D) 16 E) 18

4. $9^x - 4 \cdot 3^{x+1} + 27 = 0$

$ÇK(SS) = \{x_1, x_2\} \Rightarrow \mathbf{x_1^2 + x_2^2 = ?}$

A) 5 B) 6 C) 8 D) 9 E) 12

5. $x^3 + \frac{1}{x-2} = 4x - \frac{1}{2-x} \Rightarrow$ **ÇK (SS) = ?**

A) {–2, 0, 2} B) {–2, 0, 1} C) {–4, 0, 2)

D) {–4, 0} E) {–2, 0)

6. $\left.\begin{matrix} y + 2x = x^2 \\ x + y = 2 \end{matrix}\right\} \Rightarrow$ **ÇK(SS) = ?**

A) {(0, –1), (2, 0)} B) {(–2, –1), (–1,–3)}

C) {(–1, 3), (2, 0)} D) {(–1, 3), (–2, 0)}

E) {(–1, –3), (2, 0)}

7. $x^2 + 2x + k = 0,\ ÇK(SS) = \{x_1, 1\}$

$x^2 - x + m = 0,\ ÇK(SS) = \{x_1, 2\}$

$\Rightarrow \mathbf{\frac{k}{m} = ?}$

A) $-\frac{3}{2}$ B) – 1 C) $-\frac{1}{2}$ D) $\frac{1}{2}$ E) $\frac{3}{2}$

8. $\left(1-\frac{3}{x}\right)^2 - 2\left(\frac{3}{x}-1\right) - 8 = 0$

$ÇK(SS) = \{x_1, x_2\} \Rightarrow \mathbf{x_1 + x_2 = ?}$

A) $-\frac{18}{5}$ B) $-\frac{12}{5}$ C) –2 D) $\frac{12}{5}$ E) $\frac{18}{5}$

9. $(3m-1)x^2 - mx + 2m - 1 = 0$

$\text{ÇK(SS)} = \{x_1, x_2\}, \; x_1 \cdot x_2 = x_1 + x_2$

$\Rightarrow$ **$m = ?$**

A) -2 B) -1 C) 0 D) 1 E) 2

10. $x^2 + mx + n = 0, \; \text{ÇK(SS)} = \{x_1, -6\}$

$x^2 - ax - b = 0, \; \text{ÇK(SS)} = \{x_1, 2\}$

$\Rightarrow$ **$m + a + \frac{n}{b} = ?$**

A) 15 B) 11 C) 9 D) 7 E) 5

11. $\sqrt[3]{x} + \sqrt[6]{x} = 6 \Rightarrow$ **ÇK(SS) = ?**

A) $\{1, 8\}$ B) $\{64\}$ C) $\{729\}$
D) $\{(-2, 64)\}$ E) $\{(-16, 64)\}$

12. $ax^2 + bx + c = 0, \; \text{ÇK(SS)} = \{x_1, x_2\}$

$2x_1 + 2x_2 - 3x_1 \cdot x_2 = 13$

$3x_1 + 3x_2 + x_1 \cdot x_2 = 3$

$\Rightarrow$ **$\min(a + b - c) = ?$**

A) -4 B) -3 C) -2 D) 2 E) 4

13. $x^2 + 2mx + m - 5 = 0, \; \text{ÇK(SS)} = \{x_1, x_2\}$

$x_2x_1 + 2x_2 + x_1 \cdot x_2 + 2x_1 = 0$

$\Rightarrow$ **$m = ?$**

A) -6 B) -5 C) -3 D) 3 E) 5

14. $x^2 + mx + 3 = 0, \; \text{ÇK(SS)} = \{x_1, x_2\}$

$x^2 + (m + 2n)x + n = 0, \; \text{ÇK(SS)} = \{x_1 + 1, x_2 + 1\}$

$\Rightarrow$ **$n = ?$**

A) -2 B) -1 C) 1 D) 2 E) 3

15. $x^2 - 3x - 1 = 0, \; \text{ÇK(SS)} = \{x_1, x_2\}$

$$\Rightarrow \left(x_1 - \frac{1}{x_1}\right) \cdot \left(x_2 - \frac{1}{x_2}\right) = ?$$

A) 3 B) 4 C) 5 D) 8 E) 9

16. $\dfrac{\sqrt{x+5} - 1}{\sqrt{2x+3}} = 1 \Rightarrow$ **ÇK(SS) = ?**

A) $\{-1, 1\}$ B) $\{-1, 11\}$ C) $\{-1\}$
D) $\{11\}$ E) $\varnothing$

YANITLAR / ANSWERS

1	2	3	4	5	6	7	8	9
C	E	B	A	E	C	E	B	D
10	11	12	13	14	15	16		
B	B	D	B	B	E	C		

1. $x^2 = 8(x + 1) \Rightarrow$ **ÇK (SS) = ?**

A) $\{2 - \sqrt{2}, 2 + \sqrt{2}\}$

B) $\{2 - 2\sqrt{6}, 2 + 2\sqrt{6}\}$

C) $\{2 - \sqrt{6}, 2 + \sqrt{6}\}$

D) $\{4 - \sqrt{2}, 4 + \sqrt{2}\}$

E) $\{4 - 2\sqrt{6}, 4 + 2\sqrt{6}\}$

2. $(m + 1)x^2 - 6mx + m = 0$

$x_1 = x_2 \Rightarrow \sum$ **m = ?**

A) –4 B) $-\frac{1}{8}$ C) $\frac{1}{8}$ D) 4 E) 8

3. $2x^2 - (2m - 1)x + 2 = 0$

$x_1 = 2 \Rightarrow$ **x_2 = ?**

A) $\frac{1}{4}$ B) $\frac{1}{2}$ C) 1 D) 4 E) 8

4. $f(x) = x^2 + bx + c$

$f\left(\frac{1}{2-\sqrt{3}}\right) = f\left(\frac{1}{2+\sqrt{3}}\right) = 0 \Rightarrow$ **f(x) = ?**

A) $x^2 - 4x + 1$ B) $x^2 - 4x + 2$

C) $x^2 - 3x + 2$ D) $x^2 - 6x - 8$

E) $x^2 - 2x - 1$

5. $x + \frac{m+2}{x-2} = -1$

$x_1 \cdot x_2 = 4 \Rightarrow$ **m = ?**

A) 1 B) 2 C) 3 D) 4 E) 5

6. $x^2 - ax - 3x - 1 - a^2 = 0$

$x_1 + x_2 = 0 \Rightarrow$ **$x_1 \cdot x_2$ = ?**

A) –16 B) –10 C) 8 D) 10 E) 16

7. $x^2 + x + m = 0$

$(1 - x_1)^2 \cdot (1 - x_2)^2 = (2x_1 \cdot x_2)^2$

$m > 0 \Rightarrow$ **m = ?**

A) 2 B) 3 C) 4 D) 5 E) 6

8. $f(x) = x^2 + bx + c$,

$f(x_1) = f(x_2) = 0$,

$2 \cdot x_1 + 2 \cdot x_2 = 8 + 4 \cdot x_1 \cdot x_2$

$x_1 + x_2 + x_1 \cdot x_2 = 1 \Rightarrow$ **f(x) = ?**

A) $x^2 - 2x + 2$ B) $x^2 - 2x + 1$

C) $x^2 - 2x - 1$ D) $x^2 - 3x - 2$

E) $x^2 - 4x + 1$

9. $x > y$

$x \cdot y = 2$

$x + y = -4 \Rightarrow$ **$x = ?$**

A) $-2-\sqrt{3}$ B) $-2-\sqrt{2}$ C) $-2+\sqrt{2}$ D) $-3+\sqrt{2}$ E) $-4+\sqrt{2}$

10. $\frac{2}{x^2-4}=\frac{1}{x^2-2x}-\frac{x-4}{x^2+2x} \Rightarrow$ **$x_1 + x_2 = ?$**

A) -5 B) -3 C) -2 D) 3 E) 5

11. $\left.\begin{matrix} 2x+y=-1 \\ x^2+y^2=13 \end{matrix}\right\} \Rightarrow$ **ÇK(SS) = ?**

A) $\{(1,-3),(-1,1)\}$ B) $\left\{(-1,1),\left(\frac{2}{5},\frac{-7}{5}\right)\right\}$

C) $\left\{(-2,3),\left(\frac{6}{5},\frac{-17}{5}\right)\right\}$ D) $\left\{(-1,1),\left(\frac{3}{2},-4\right)\right\}$

E $\{(2,-5),(3,-7)\}$

12. $\left.\begin{matrix} x^3-y^3=35 \\ x-y=5 \end{matrix}\right\} \Rightarrow$ **ÇK(SS) = ?**

A) $\{(-3,2),(2,-3)\}$ B) $\{(1,-4),(4,-1)\}$

C) $\{(2,3),(3,2)\}$ D) $\{(4,-1),(1,-4)\}$

E) $\{(3,-2),(2,-3)\}$

13. $f(x) = ax^2 + bx + c$,

$f(x_1) = f(x_2) = 0$,

$x_1 = 3-\sqrt{5},\ x_1 + x_2 = A \cdot (x_1 \cdot x_2) \Rightarrow$ **$A = ?$**

A) $\frac{1}{2}$ D) 1 C) $\frac{4}{3}$ D) $\frac{3}{2}$ E) 2

14. $3x^2 - 5x + 2m - 8 = 0$

$2x_1 + x_2 = -\frac{1}{3} \Rightarrow$ **$m = ?$**

A) -11 B) -9 C) -7 D 1 E) 3

15. $2x^2 - x - 1 = 0 \Rightarrow$ **$x_1^3 + x_2^3 = ?$**

A) $\frac{3}{8}$ B) $\frac{1}{2}$ C) $\frac{5}{8}$ D) $\frac{3}{4}$ E) $\frac{7}{8}$

16. $b < 0$

$f(x) = x^2 + bx + c$,

$f(x_1) = f(x_2) = 0$,

$x_1 \cdot x_2 = 4,\ x_1^2+x_2^2=8 \Rightarrow$ **$f(x) = ?$**

A) $x^2 + 4x + 8$ B) $x^2 + 4x + 2$

C) $x^2 - 4x + 8$ D) $x^2 - 4x + 2$

E) $x^2 - 4x + 4$

17. $\frac{x-2}{x-3}+\frac{x-2}{x-4}=0$

$\Rightarrow$ **$x_1 + x_2 = ?$**

A) 4 B) $\frac{9}{2}$ C) 5 D) $\frac{11}{2}$ E) 6

YANITLAR / ANSWERS

1	2	3	4	5	6	7	8	9
E	C	B	A	D	B	A	C	C
10	**11**	**12**	**13**	**14**	**15**	**16**	**17**	
D	C	E	D	C	E	E	D	

BÖLÜM
CHAPTER 18

TARİHSEL NOT / HISTORICAL NOTE

GABRIEL CRAMER [1704 - 1752]

İsviçreli matematikçi. 18 yaşında doktorasını aldı ve 20 yaşında Cenevre Üniversitesi'nde matematik başkanlığı yaptı. 1750'de, Cramer'in kuralını yayınladı ve sistem tarafından ima edilen belirleyiciler açısından, benzersiz bir çözüme sahip doğrusal denklem sisteminde herhangi bir bilinmeyenin çözümü için genel bir formül verdi. Bu kural hala standarttır.

Swiss mathematician. He received his doctorate's degree at the age of 18, and at the age of 20, he became the head of mathematics at the University of Geneva. In 1750, he published Cramer's rule and gave a general formula for the solution of any unknown in the system of linear equations with a unique solution, for the determinants implied by the system. This rule is still standard.

İKİNCİ DERECEDEN EŞİTSİZLİKLER
QUADRATIC INEQUALITIES

Bu bölüm, 50 test sorusu içermektedir.
This chapter includes 50 test questions.

BÖLÜM / CHAPTER 18

İKİNCİ DERECEDEN EŞİTSİZLİKLER / QUADRATIC INEQUALITIES

- İkinci Dereceden Eşitsizlikler / Quadratic Inequalities .... 323 – 332

BÖLÜM

18

CHAPTER

İKİNCİ DERECEDEN EŞİTSİZLİKLER
QUADRATIC INEQUALITIES

İKİNCİ DERECEDEN EŞİTSİZLİKLER / QUADRATIC INEQUALITIES

Tanım: $f(x) \geq 0$, $f(x) \leq 0$, $f(x) > 0$, $f(x) < 0$ şeklindeki ifadelere ***eşitsizlik*** denir.

Definition: The expressions in the form of $f(x) \geq 0$, $f(x) \leq 0$, $f(x) > 0$, $f(x) < 0$ are called *inequalities*.

▼ $f(x) = ax + b = 0 \Rightarrow x = -\frac{b}{a}$

x	$-\infty$	$-\frac{b}{a}$	$+\infty$
ax +b	a'nın işaretinin tersi opposite of a's sign	0	a'nın işareti ile aynı same as a's sign

Önemli / Important

- $\{x \mid m \leq x \leq n;\ m, n \in R\} = [m, n]$
- $\{x \mid m \leq x < n;\ m, n \in R\} = [m, n)$
- $\{x \mid m < x \leq n;\ m, n \in R\} = (m, n]$
- $\{x \mid m < x < n;\ m, n \in R\} = (m, n)$

Tablo oluşturma:

▼ f(x)'in pay ve paydasını sıfır yapan değerler sayı doğrusuna yerleştiriliyormuş gibi küçükten büyüğe doğru sıralanır.

▼ Pay ve paydayı sıfır yapan değerlerden tek sayıda olan kökler ***tek katlı kök***, çift sayıda olan kökler ***çift katlı kök*** olarak adlandırılır.

▼ Pay ve paydanın en büyük dereceli terimlerinin işareti bulunarak f(x)'in işareti belirlenir.

▼ En sağ kısma f(x)'in işareti yazılır.

▼ Tek katlı köklerde sağdaki işaretle zıt; çift katlı köklerde ise sağdaki işaretle aynı işaret yazılarak tablo sağdan sola doğru oluşturulur.

Making Table:

▼ The values that make numerator and denominator of f(x) equal to zero should be ordered from smallest to largest.

▼ If the roots are single roots, they are named as odd root, and if they are double roots, they are named as even roots.

▼ The sign of f(x) is determined by the sign of the expression with maximum degree.

▼ The sign of f(x) is written to most right side.

▼ moving from right to left, signs of intervals are written by opposing the sign in odd root and maintaining it unchanged in double roots.

▼ $f(x) = ax^2 + bx + c$

i. $\Delta > 0 \Rightarrow x_1 = \dfrac{-b+\sqrt{\Delta}}{2a}$ ve/and $x_2 = \dfrac{-b-\sqrt{\Delta}}{2a}$

x	$-\infty$		x_1		x_2		$+\infty$
ax^2+bx+c		a'nın işareti ile aynı same as a's sign	○	a'nın işaretinin tersi opposite of a's sign	○	a'nın işareti ile aynı same as a's sign	

ii. $\Delta = 0 \Rightarrow x_1 = x_2 = -\dfrac{b}{2a}$

x	$-\infty$		$x_1 = x_2 = -\dfrac{b}{2a}$		$+\infty$
ax^2+bx+c		a'nın işareti ile aynı Same sign with a	○	a'nın işareti ile aynı Same sign with a	

Önemli / Important

- $f(x) \geq g(x)$, $f(x) > g(x)$, $f(x) < g(x)$, $f(x) \leq g(x)$ biçimindeki eşitsizlikler, $f(x) - g(x) \geq 0$, $f(x) - g(x) > 0$, $f(x) - g(x) < 0$, $f(x) - g(x) \leq 0$ biçimine dönüştürüldükten sonra bilinen yöntem uygulanır.
- Inequalities in the form of $f(x) \geq g(x)$, $f(x) > g(x)$, $f(x) < g(x)$, $f(x) \leq g(x)$ will be solved using known methods after transformation to the form of $f(x) - g(x) \geq 0$, $f(x) - g(x) > 0$, $f(x) - g(x) < 0$, $f(x) - g(x) \leq 0$.

Önemli / Important

$\forall x \in R$ için/for

- $ax^2 + bx + c > 0 \Rightarrow a > 0$ ve/and $\Delta < 0$
- $ax^2 + bx + c < 0 \Rightarrow a < 0$ ve/and $\Delta < 0$

Önemli / Important

$a \neq 0,\ ax^2 + bx + c = 0$

- $\Delta = 0 \Rightarrow x_1 \cdot x_2 = \dfrac{c}{a} > 0$

  $x_1 + x_2 = -\dfrac{b}{a} > 0\ (0 < x_1 = x_2)$

  $x_1 + x_2 = -\dfrac{b}{a} < 0\ (x_1 = x_2 < 0)$
- $\Delta > 0 \Rightarrow$

  i. $x_1 \cdot x_2 < 0 \Rightarrow x_1 < 0 < x_2$

  $|x_2| > |x_1| \Rightarrow x_1 + x_2 = -\dfrac{b}{a} > 0$

  $|x_1| > |x_2| \Rightarrow x_1 + x_2 = -\dfrac{b}{a} < 0$

  ii. $x_1 \cdot x_2 > 0$

  $\Rightarrow x_1 + x_2 > 0 \Rightarrow 0 < x_1 < x_2$

  $\Rightarrow x_1 + x_2 < 0 \Rightarrow x_1 < x_2 < 0$

EŞİTSİZLİK SİSTEMİ / SYSTEM of INEQUALITY

▼ Birden fazla eşitsizliğin oluşturduğu sisteme ***eşitsizlik sistemi*** denir. Sistemi oluşturan eşitsizliklerin çözüm kümelerinin kesişimine ise, ***eşitsizlik sisteminin çözüm kümesi*** denir.

▼ The system of more than one inequalities is called a ***system of inequality***. The intersection of the solution sets of the inequalities is called the ***solution set of inequality system***.

1. $x^2 < 2x + 3 \Rightarrow \mathbf{? < x < ?}$

A) $3 < x < 4$
B) $-1 < x < 3$
C) $-3 < x < -2$
D) $-2 < x < -1$
E) $4 < x < 5$

2. $(x - 3)\cdot(5 - 2x)\cdot(x^2 - 1) \geq 0 \Rightarrow$ **ÇK(SS) = ?**

A) $[-6, 0]$
B) $[-1, 0]$
C) $(-\infty, -1]$
D) $[-1,1]\cup[\frac{5}{2},3]$
E) $(-\infty, 0]$

3. $x(x^2 - 3x - 4) \geq 0 \Rightarrow$ **ÇK(SS) = ?**

A) $(-\infty, -1) \cup [0, \infty)$
B) $(-1, 4]$
C) $[-1, 0] \cup [4, \infty)$
D) $(-\infty, -1) \cup (0, 4]$
E) $(-\infty, -1) \cup (4, \infty)$

4. $x^2 - 7x \leq -10 \Rightarrow$ **ÇK(SS) = ?**

A) $(-\infty, 2)$
B) $(5, \infty)$
C) $(-\infty, 2] \cup [5, \infty)$
D) $(2, 5)$
E) $[2, 5]$

5. $(x + 6)(x - 2)^2 \geq 0 \Rightarrow$ **ÇK(SS) = ?**

A) $[-6, \infty)$
B) $(-5, \infty)$
C) $(-1, 2)$
D) $(2, \infty)$
E) $(-6, 2)$

6. $\dfrac{(1-x)\cdot(x+2)}{(x-2)\cdot(3-x)} < 0 \Rightarrow$ **ÇK(SS) = ?**

A) $[-2, 1] \cup (2, 3)$
B) $(-2, 1) \cup (2, 3)$
C) $(-\infty, 2) \cup (3, \infty)$
D) $(-\infty, -2) \cup (1, 2)$
E) $(-\infty, 1) \cup (2, 3)$

7. $\dfrac{x^2 - 4x}{(1+x)\cdot(4-x^2)} \geq 0 \Rightarrow$ **ÇK(SS) = ?**

A) $[0, 1]$
B) $[1, 2]$
C) $[1, 4]$
D) $[2, 4]$
E) $(-\infty, -2) \cup (-1, 0] \cup (2, 4]$

8. $\left.\begin{matrix} x^2 \leq x+2 \\ 3x \geq x^2+2 \end{matrix}\right\} \Rightarrow$ **ÇK(SS) = ?**

A) $[-1, 1]$
B) $[-2, 0]$
C) $[1, 2] \cup \{-1\}$
D) $[-1, 2]$
E) $[-1, 1] \cup \{2\}$

9. $\frac{x+1}{x+4} \le \frac{1}{x} \Rightarrow$ **ÇK(SS) = ?**

A) [0, 4]

B) [2, 4]

C) [0, 2]

D) (0, 4]

E) (–4, –2] ∪ (0, 2]

10. $\frac{x^2-2x-3}{(x-2)^2} < 0 \Rightarrow$ **ÇK(SS) = ?**

A) (–∞, –1)

B) (3, ∞)

C) (– 1, 3) – {2}

D) (–1, 3)

E) (2, ∞)

11. $x \in Z$

$\frac{(x^2-2x-8)(x^2+2)}{1-x^2} \ge 0 \Rightarrow \sum x = ?$

A) 9

B) 7

C) 6

D) 5

E) 4

12. $\left.\begin{matrix} x^2-3x-28 \le 0 \\ x^2+2x-15 \ge 0 \end{matrix}\right\} \Rightarrow$ **ÇK(SS) = ?**

A) [–5, –4]

B) [–4, 3]

C) (5, 7)

D) [–5, 3]

E) [3, 7]

13. $x \in Z$

$\frac{(2-x)(x+1)(x^2+4)}{(x^2-3x)(5-x)} \ge 0 \Rightarrow \min(x) = ?$

A) 0

B) 1

C) –1

D) 2

E) 3

14. $\frac{5}{x^2+6x} < -1 \Rightarrow$ **ÇK(SS) = ?**

A) (–∞, –6) ∪ (–5, –1) ∪ (0, +∞)

B) (–5, –1)

C) (–6, –5) ∪ (–1, 0)

D) (–1, 0)

E) (–6, –5) ∪ (0, +∞)

15. $x \in Z$

$\frac{(4x-x^2)(x+2)^2}{x^2+1} \ge 0 \Rightarrow \sum x = ?$

A) 10

B) 9

C) 8

D) 7

E) 6

16. $x \in Z^-$

$\frac{-(x+6)(x+7)^2}{x} > 0 \Rightarrow$ **ÇK(SS) = ?**

A) [–6, 0)

B) (–6, 0]

C) (–6, 0)

D) (–∞, –6) ∪ (0, ∞)

E) (–∞, –6] ∪ (0, ∞)

17. $\frac{(x-2)^2(x^2+x-6)}{x^2-9} \le 0 \Rightarrow$ **ÇK(SS) = ?**

A) (–∞, –3)

B) (–3, 2)

C) [2, 3)

D) (2, 3]

E) [3, ∞)

YANITLAR / ANSWERS

1	2	3	4	5	6	7	8
B	D	C	E	A	B	E	C
9	**10**	**11**	**12**	**13**	**14**	**15**	**16**
E	B	E	C	C	C	C	C

1. $\left(\dfrac{x^3+8}{x^3-1}\right)^3 \ge 0 \Rightarrow$ **ÇK(SS) = ?**

A) $(-\infty, -2) \cup (1, \infty)$
B) $(-\infty, -2] \cup (1, \infty)$
C) $(-\infty, -2] \cup [1, \infty)$
D) $(-\infty, -1] \cup [2, \infty)$
E) $\varnothing$

2. $x \in Z$,

$\dfrac{x^2-8x+7}{(x+2)^2} < 0 \Rightarrow \sum x = ?$

A) 10
B) 15
C) 20
D) 25
E) 30

3. $\dfrac{x}{x-3} \le \dfrac{1}{x} \Rightarrow$ **ÇK(SS) = ?**

A) $[0, 3]$
B) $(-\infty, 0) \cup (3, \infty)$
C) $[3, \infty)$
D) $[0, 3)$
E) $(0, 3)$

4. $\dfrac{x\cdot(7-x)^2}{(4-x)\cdot(9+x^2)} \ge 0 \Rightarrow$ **ÇK(SS) = ?**

A) $[0, 4) \cup \{7\}$
B) $[0, 7)$
C) $(-\infty, 0] \cup [4, 7)$
D) $[0, 4] \cup \{7\}$
E) $\varnothing$

5. $\dfrac{x^2(x^2+2x+1)}{2-x} < 0 \Rightarrow$ **ÇK(SS) = ?**

A) $[0, 2)$
B) $(-\infty, 2)$
C) $(2, \infty)$
D) $[-1, 2]$
E) $[-1, 0]$

6. $x \in N$,

$\dfrac{(2-x)^2(x^2+x)}{(x^2+4)(1-x)} > 0 \Rightarrow$ **ÇK(SS) = ?**

A) $(0, 1)$
B) $[0, 1]$
C) $(-\infty, -1) \cup [0, 1)$
D) $[0, 1)$
E) $\varnothing$

7. $\left.\begin{array}{l} x^2-3x-4<0 \\ x^2-8x+15<0 \end{array}\right\} \Rightarrow$ **ÇK(SS) = ?**

A) $(-1, 3) \cup (4, 5)$
B) $(3, 4)$
C) $(-1, 5)$
D) $(3, 5)$
E) $(-1, 4)$

8. $\dfrac{(x-2)^{2009}\cdot(-x+4)^{200}}{(x-7)^{2001}} \ge 0 \Rightarrow$ **ÇK(SS) = ?**

A) $(2, 7)$
B) $[2, 7)$
C) $[2, 7] - \{4\}$
D) $(-\infty, 2] \cup (7, \infty) \cup \{4\}$
E) $(-\infty, 2] \cup [4, 7)$

9. $x \in Z$,

$$\frac{4^x \cdot (x-5) \cdot (x^2+x+1)}{x-2} \geq 0 \Rightarrow \sum x = ?$$

A) –9 B) –10 C) –12 D) –14 E) –15

10. $x \in Z$,

$$\frac{-x^2+4x+5}{|x-2|} > 0 \Rightarrow \sum x = ?$$

A) 7 B) 8 C) 9 D) 12 E) 15

11. $(x-3)\cdot(x+1)\cdot(-x-4) > 0 \Rightarrow$ **ÇK(SS) = ?**

A) $(-4, -1) \cup (3, \infty)$

B) $(-4, -1)$

C) $(-4, 3) - \{-1\}$

D) $(-\infty, -4) \cup (-1, 3)$

E) $\varnothing$

12. $\left.\begin{matrix} x-3<0 \\ x-5 \geq 0 \end{matrix}\right\} \Rightarrow$ **ÇK(SS) = ?**

A) $(3, \infty)$

B) $(3, 5]$

C) $(-\infty, 5]$

D) $[3, \infty)$

E) $\varnothing$

13. $x \leq \frac{1}{x} \Rightarrow$ **ÇK(SS) = ?**

A) $(-\infty, -1] \cup (0, 1]$

B) $(-\infty, 1] \cup (0, 1]$

C) $[-1, 0]$

D) $[-1, 1)$

E) $[-\infty, 1]$

14. $x \in Z$,

$$\frac{x^2-11x+24}{(x-5)^2} < 0 \Rightarrow \sum x = ?$$

A) 9 B) 10 C) 12 D) 15 E) 17

15. $\sqrt{x^2-5x} < 6 \Rightarrow$ **ÇK(SS) = ?**

A) $(-2, 7)$

B) $(-\infty, -4) \cup (5, \infty)$

C) $(-4, 0] \cup [5, 9)$

D) $(-4, -1) \cup (2, 6)$

E) $(-6, 0) \cup (2, 9)$

16. $a > 0$,

$$\left.\begin{matrix} x^2-a^2<0 \\ x^2-ax>0 \end{matrix}\right\} \Rightarrow \textbf{ÇK(SS) = ?}$$

A) $(-a, 0)$

B) $(0, a)$

C) (a, ∞)

D) $(1, a)$

E) $(-a, 1)$

17. $\frac{3^x \cdot |x-2|}{\sqrt{x-3}} < 0 \Rightarrow$ **ÇK(SS) = ?**

A) $(-\infty, 2)$

B) $(-\infty, 3)$

C) $(3, \infty)$

D) $[3, \infty)$

E) $\varnothing$

YANITLAR / ANSWERS

1	2	3	4	5	6	7	8	9
B	C	E	A	C	A	B	D	A
10	11	12	13	14	15	16	17	
B	D	E	A	E	C	A	E	

1. $\dfrac{x^2-3x-10}{(4-x^2)\cdot(x-5)} \geq 0 \Rightarrow$ **ÇK(SS) = ?**

A) $(-\infty, 2)$ B) $(-\infty, -2)$ C) $(-\infty, 0]$
D) $(-\infty, 5)$ E) $(-\infty, 2) - \{-2\}$

2. $\dfrac{x^2(x^2+2x-3)}{x^2+4x+3} \geq 0,$

$x \in Z \Rightarrow \sum$ **x = ?**

A) –4 B) –3 C) 3 D) 4 E) 6

3. $\sqrt{x^2-x-6} \leq x+2$

$\Rightarrow$ **ÇK(SS) = ?**

A) $(-\infty, -2)$ B) $[3, \infty)$ C) $[-2, 3)$
D) $(-\infty, -3]$ E) $[3, \infty) \cup \{-2\}$

4. $\dfrac{|x+2|\cdot(x^2+3x-10)}{(x^2+4)\cdot(|x+7|+1)} < 0$

$x \in Z \Rightarrow \sum$ **x = ?**

A) –7 B) –5 C) –3 D) 3 E) 5

5. $(m-2)x^2 - (2m+6)x - m + 1 = 0$

$x_1 < 0 < x_2,\ m \in Z \Rightarrow \sum$ **m = ?**

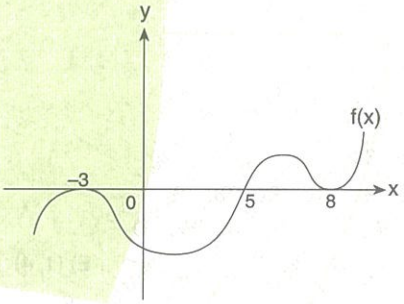
A) –5 B) –3 C) –1 D) 3 E) 5

6. $\dfrac{x^5\cdot(x-2)^2\cdot(3-x)}{|2-x|} \leq 0$

$x \in Z \Rightarrow \sum$ **x = ?**

A) –7 B) –5 C) –3 D) –1 E) 0

7.



$(x^2+1)\cdot f(x) \leq 0,\ x \in N^+ \Rightarrow \sum$ **x = ?**

A) 15 B) 17 C) 19 D) 21 E) 23

8. $x \in Z^-,$

$\dfrac{(1-x^2)^4\cdot(x+2)}{x^2-x-2} \geq 0 \Rightarrow$ **max(x) = ?**

A) –8 B) –6 C) –4 D) –2 E) –1

9. $\dfrac{(5^{x-2}-1)\cdot|x-1|}{-x^2+4x-4}<0 \Rightarrow$ **ÇK(SS) = ?**

A) $(2, \infty)$
B) $(-1, 0)$
C) $(-2, 1)$
D) $(-\infty, 2)$
E) $(2, 3)$

10. $x^2-(m+1)x-2m=0$

$x_1\cdot x_2^2+x_1^2\cdot x_2\geq 0 \Rightarrow$ **? < m < ?**

A) $-2\leq m<-1$
B) $-1\leq m\leq 0$
C) $-1\leq m\leq 1$
D) $1\leq m\leq 2$
E) $-1\leq m\leq 2$

11. $x^2-2x-8<0$

$x^2+2x-3>0 \Rightarrow$ **ÇK(SS) = ?**

A) $(-\infty, 1)$
B) $(-3, -2)$
C) $(-\infty, 4)$
D) $(-2, 4)$
E) $(1, 4)$

12. $\dfrac{x^2-4x-12}{|x-3|}\leq 0,$

$x\in Z \Rightarrow \sum x = ?$

A) 9
B) 12
C) 13
D) 15
E) 16

13. $x\in R^{+},$

$\dfrac{x^2-3x-4}{x+2}\geq 4-x \Rightarrow$ **ÇK(SS) = ?**

A) $(4, \infty)$
B) $[4, \infty)$
C) $\left[-\dfrac{3}{2}, 4\right]$
D) $[-2, 4]$
E) $(-\infty, -2]$

14. $2-\dfrac{1}{x+1}<\dfrac{x-1}{x}$

$x\in Z \Rightarrow \sum x = ?$

A) 0
B) 1
C) 2
D) 3
E) 4

15. $\dfrac{1}{x}-\dfrac{2}{x-2}>\dfrac{2}{x+2} \Rightarrow$ **ÇK(SS) = ?**

A) $(2, \infty)$
B) $(-\infty, -2)$
C) $(-2, 0)\cup(2, \infty)$
D) $(-\infty, -2)\cup(0, 2)$
E) $(-2, 2)$

16. $\dfrac{(x^2-1)\cdot(x^2+3x+2)}{(-x^2+3x+4)}\leq 0$

$x\in Z \Rightarrow \sum x = ?$

A) -10
B) -9
C) -8
D) -6
E) -4

YANITLAR / ANSWERS

1	2	3	4	5	6	7	8
E	D	E	A	B	C	E	D
9	10	11	12	13	14	15	16
A	B	E	D	B	A	D	C

BÖLÜM
CHAPTER
19

TARİHSEL NOT / HISTORICAL NOTE

HÂRİZMÎ [850-780 BC]

El-Harezmi'nin hayatına dair kesin olarak bilinen ayrıntı az sayıdadır. İranlı bir ailede Büyük Horasan'ın Harezm şehrinde (modern Hive,Harezm bölgesi,Özbekistan) doğmuştur. Matematik, coğrafya, astronomi ve haritacılığa katkısı, cebir ve trigonometride yeniliğin temelini oluşturdu. Doğrusal ve ikinci dereceden denklemleri çözmeye yönelik sistematik yaklaşımıyla cebrin ortaya çıkmasına neden olan kitabının başlığı şöyledir, "Tamamlama ve Dengeleme ile Hesaplama Üzerine Özlü Kitap".

There is only a small number of known facts about Al-Khwarezmi's life. He was born in an Iranian family in Harezm city of Horasan (modern Hive, Harezm region, Uzbekistan). He has contribution to mathematics, geography, astronomy and cartography and constituted the basis of innovation in algebra and trigonometry. The title of his book, which leads to the emergence of algebra with the systematic approach to solve linear and quadratic equations, is as follows: "The Compendious Book on Calculation by Completion and Balancing".

İKİNCİ DERECEDEN FONKSİYONLAR
QUADRATIC FUNCTIONS

Bu bölüm, 47 test sorusu içermektedir.

This chapter includes 47 test questions.

BÖLÜM / CHAPTER 19

İKİNCİ DERECEDEN FONKSİYONLAR / QUADRATIC FUNCTIONS

- İkinci Dereceden Fonksiyonlar / Quadratic Functions . . . . 333 – 348

BÖLÜM 19 CHAPTER

İKİNCİ DERECEDEN FONKSİYONLAR
QUADRATIC FUNCTIONS

İKİNCİ DERECEDEN FONKSİYONLAR / QUADRATIC FUNCTIONS

Tanım: a, b, c $\in$ R ve a $\neq$ 0 olmak üzere, $y = f(x) = ax^2 + bx + c$ şeklindeki ikinci dereceden fonksiyonların grafiğine ***parabol*** denir.

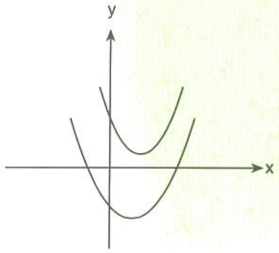
Definition: Provided that a, b, c $\in$ R and a $\neq$ 0, the graph of a quadratic function $y = f(x) = ax^2 + bx + c$ is called a ***parabola***.

PARABOL ÇİZİMİ / DRAWING a PARABOLA

$y = f(x) = ax^2 + bx + c$ fonksiyonunun grafiği çizilirken aşağıdaki sıra izlenir:

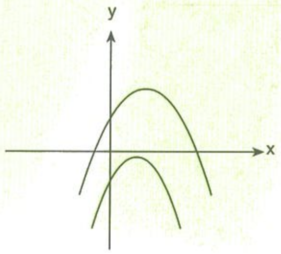
To draw the graph of a quadratic function $y = f(x) = ax^2 + bx + c$, follow the steps below:

▼



a > 0 ise grafiğin kolları yukarı doğrudur.

If a > 0, then the graph opens upward.



a < 0 ise grafiğin kolları aşağı doğrudur.

If a < 0, then the graph opens downward.

▼ **Tepe Noktası / The Vertex:**

$$T(r,k) = \left(-\frac{b}{2a}, \frac{4ac-b^2}{4a}\right)$$

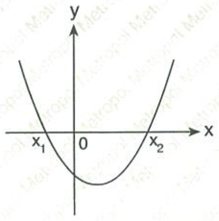
veya/or

$$T(r,k) = \left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right)$$

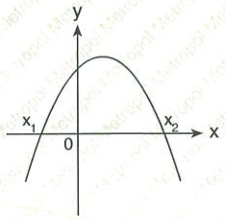
▼ Parabolün x Eksenini Kestiği Noktalar / x – Intercepts of a Parabola :

$y = f(x) = ax^2 + bx + c = 0$

i. $\Delta > 0 \Rightarrow$

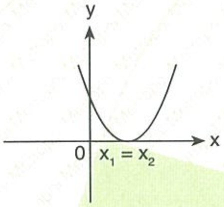


a > 0

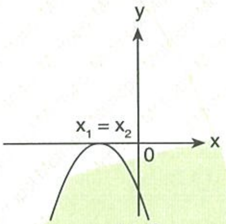


a < 0

ii. $\Delta = 0 \Rightarrow$

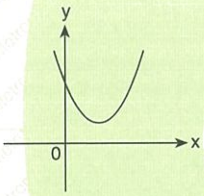


a > 0

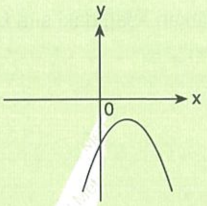


a < 0

iii. $\Delta < 0 \Rightarrow$



a > 0

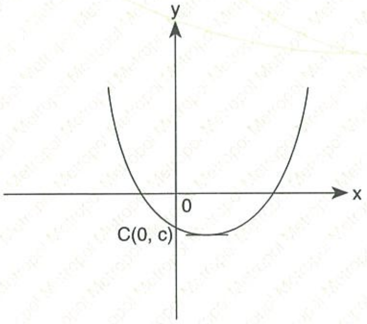


a < 0

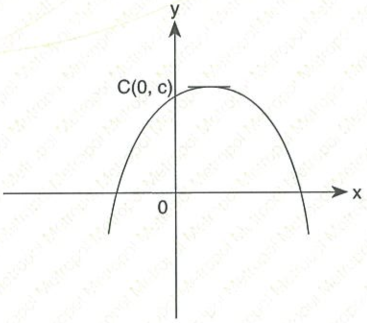
▼ Parabolün y Eksenini Kestiği Noktalar / y – Intercept of a Parabola :

$f(x) = ax^2 + bx + c$ parabolünün y eksenini kestiği nokta $x = 0$ için $f(0) = c$ dir.

y-intercept of the parabola $f(x) = ax^2 + bx + c$, for $x = 0$, $f(0) = c$.



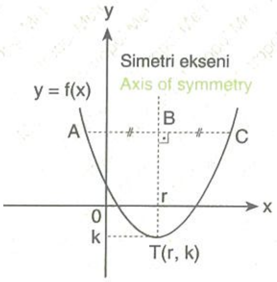
a > 0



a > 0

Özellik / Property

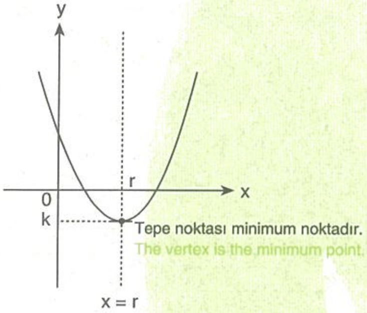
★ $a \neq 0, \quad y = f(x) = ax^2 + bx + c$


y
Simetri ekseni
Axis of symmetry
y = f(x)
A
B
C
r
x
0
k
T(r, k)

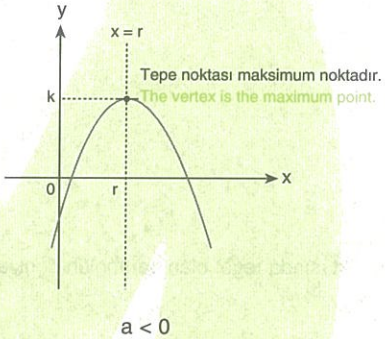
$|AB| = |BC|$

★ Grafik, $x = r$ doğrusuna göre simetrik olduğundan $x = r$ doğrusuna parabolün ***simetri ekseni*** denir. k değerine; $a > 0$ ise parabolün **minimum değeri**, $a < 0$ ise parabolün **maksimum değeri** denir.

★ Since the graph is symmetrical with respect to the line $x = -\frac{b}{2a}$, then line $x = -\frac{b}{2a}$ is called ***axis of symmetry***. If $a > 0$, then k is called **the minimum value** of the parabola, and if $a < 0$, then **k is called the maximum value** of the parabola.

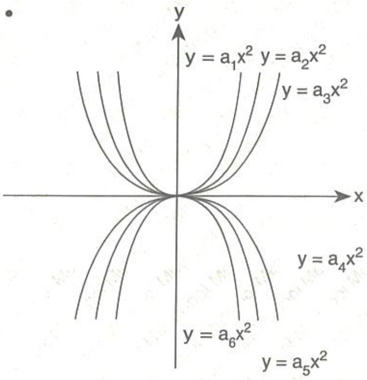

y
r
x
0
k
Tepe noktası minimum noktadır.
The vertex is the minimum point.
x = r

$a > 0$


y
x = r
Tepe noktası maksimum noktadır.
The vertex is the maximum point.
k
x
0
r

$a < 0$

Önemli / Important

- 
y
y = a₁x²
y = a₂x²
y = a₃x²
x
y = a₄x²
y = a₆x²
y = a₅x²

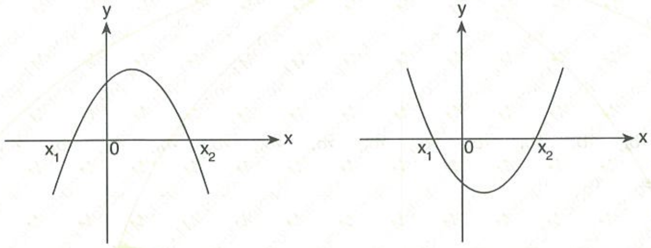
$\Rightarrow a_1 > a_2 > a_3 > a_4 > a_5 > a_6$

PARABOL DENKLEMİNİN YAZILMASI / WRITING the EQUATION of a PARABOLA

EKSENLERİ KESTİĞİ NOKTALAR ile BİR NOKTASI BİLİNEN PARABOLÜN DENKLEMİNİN BULUNMASI
FINDING EQUATION of the PARABOLA with one POINT and the X and Y INTERCEPTS KNOWN

▼ x eksenini $(x_1,0)$, $(x_2,0)$ noktalarında kesen parabolün denklemi:

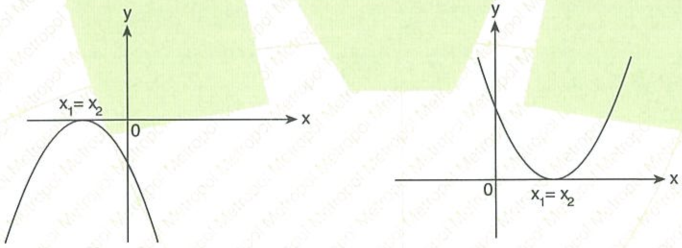
The equation of the parabola with x-intercepts known: $(x_1, 0)$ and $(x_2, 0)$:



$$y = a\cdot(x - x_1)\cdot(x - x_2), \quad [a \neq 0, a \in R]$$

▼ x eksenine $(x_1, 0) = (x_2, 0)$ noktasında teğet olan parabolün denklemi:

The equation of the parabola that is tangent to the x-axis at the point $(x_1, 0) = (x_2, 0)$:



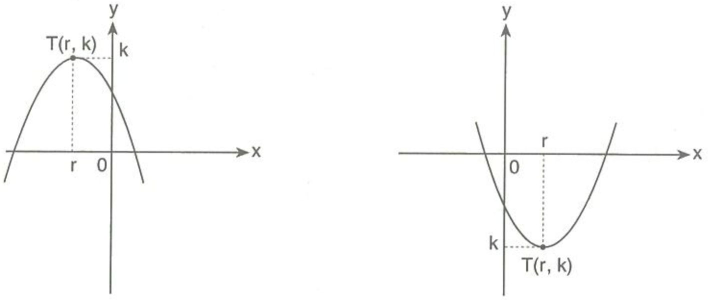
$$y = a\cdot(x - x_1)\cdot(x - x_1) = a\cdot(x - x_1)^2, \quad [a \neq 0, a \in R]$$

Önemli / Important

- Parabol denklemindeki a (başkatsayı) gerçel sayısının bulunması için, fonksiyonun x eksenini kestiği noktalar dışında geçtiği bir nokta gereklidir.
- In order to find the leading coefficient (a) of the parabola's equation, one point of its graph other than its x-intercepts is needed.

TEPE NOKTASI T(r, k) OLAN PARABOLÜN DENKLEMİ
THE EQUATION of the PARABOLA with the VERTEX T(r, k)

▼



$$y = a\cdot(x - r)^2 + k, \quad [a \neq 0,\ a \in R]$$

Önemli / Important

- Parabol denklemindeki a (başkatsayı) gerçel sayısının bulunması için, fonksiyonun geçtiği herhangi bir nokta gereklidir.
- In order to find the leading coefficient (a) of the parabola's equation, one point of its graph is needed.

BİR PARABOL ve BİR DOĞRUNUN BİRBİRİNE GÖRE DURUMLARI / THE RELATED POSITION of a LINE and a PARABOLA

▼ $y = ax^2 + bx + c$ parabolü ile $y = mx + n$ doğrusunun durumunu belirtmek için ortak çözümden yararlanılır.

▼ We use common solution to the relationship between the parabola $y = ax^2 + bx + c$ and the line $y = mx + n$.

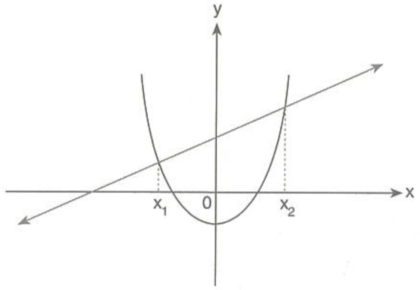
▼ **$ax^2 + bx + c = mx + n$ denkleminde,**

▼ **In equation $ax^2 + bx + c = mx + n$,**

i. $\Delta > 0$ ise, doğru parabolü iki noktada keser.

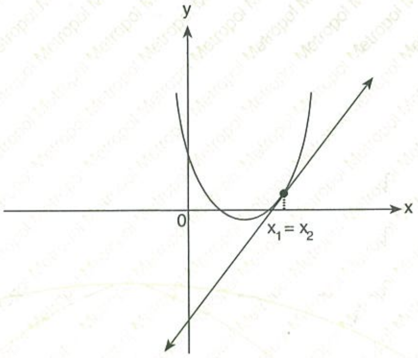
If $\Delta > 0$, then line cuts the parabola at two distinct point.

⇒



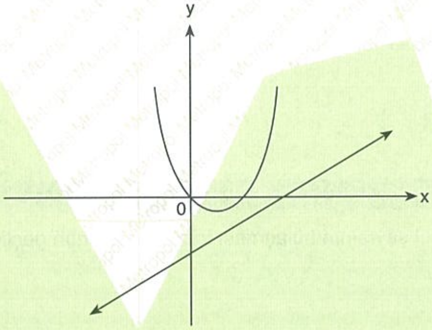
ii. **$\Delta = 0$** ise, doğru parabole bir noktada teğettir. ⇒

If $\Delta = 0$, the line is tangent to the parabola at one point.



iii. **$\Delta < 0$** ise, parabol ile doğru kesişmez. ⇒

If $\Delta < 0$, the line do not intersect with the parabola.

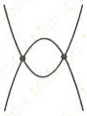


Önemli / Important

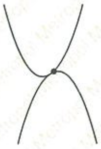
- İki parabolün birbirine göre durumları:
- The related position of two parabolas:

$a_1 \neq 0,\ f(x) = a_1x^2 + b_1x + c_1$
$a_2 \neq 0,\ g(x) = a_2x^2 + b_2x + c_2$
$f(x) = g(x) \Rightarrow$
$(a_1 - a_2)x^2 + (b_1 - b_2)x + c_1 - c_2 = 0$

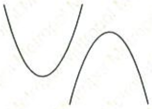
- $\Delta > 0$ ⇒



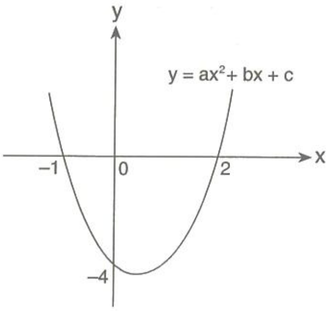
- $\Delta = 0$ ⇒



- $\Delta < 0$ ⇒

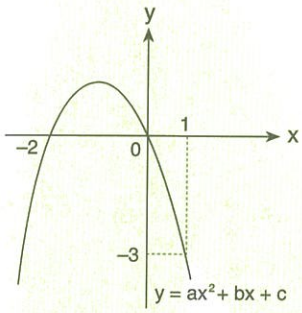


1. $a + b + c = ?$



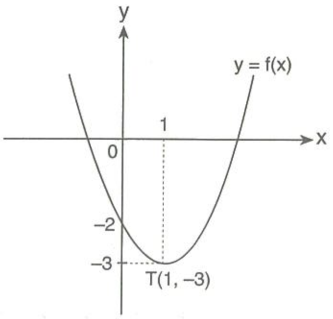
A) –4 B) –2 C) 0 D) 2 E) 4

2. $\dfrac{a+c}{b} = ?$



A) – 1 B) $-\frac{1}{2}$ C) 0 D) $\frac{1}{2}$ E) 2

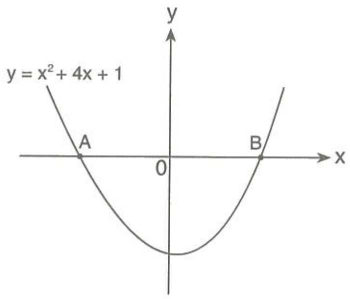
3. $y = f(x) = ?$



A) $x^2 - 2x - 1$ B) $x^2 - 2x - 2$ C) $\frac{3}{4}x^2 - 3x + 3$

D) $x^2 - 4x + 3$ E) $\frac{3}{4}x^2 + 3x + 3$

4.

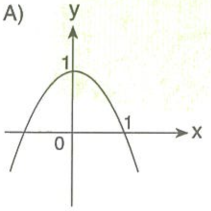


$\Rightarrow |AB| = ?$ br(u)

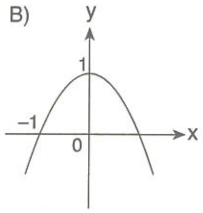
A) $3\sqrt{2}$ B) $2\sqrt{3}$ C) 4 D) $4\sqrt{2}$ E) $4\sqrt{3}$

5. $\left.\begin{array}{l} f(x) = x - 1 \\ g(x) = x^2 + 1 \end{array}\right\} \Rightarrow (gof)(x) = ?$

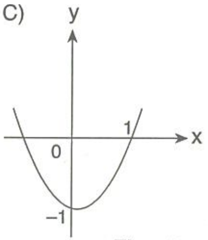
A)



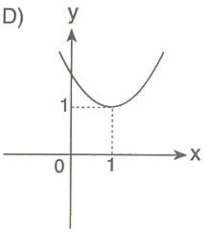
B)



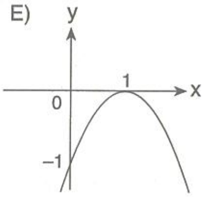
C)



D)

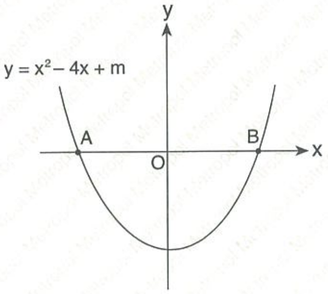


E)



6. $|OB| = 5|OA|$

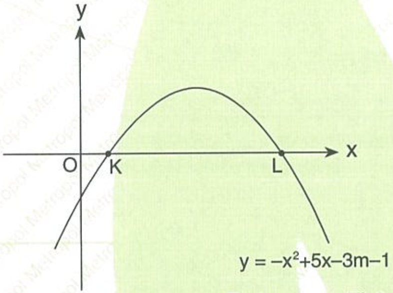
$\Rightarrow$ **m = ?**


y
y = x² – 4x + m
A
B
O
x

A) –5 B) –3 C) –1 D) 1 E) 5

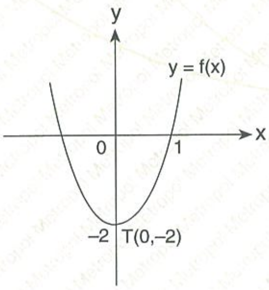
7. $|OL| = 4|OK|$

$\Rightarrow$ **m = ?**


y
O
K
L
x
y = –x²+5x–3m–1

A) –3 B) –2 C) 1 D) 2 E) 3

8. **f(3) = ?**

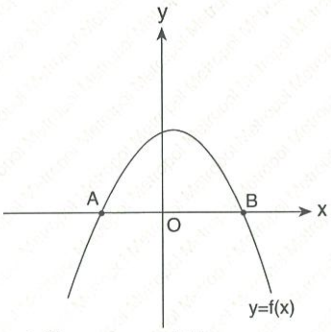

y
y = f(x)
0
1
x
–2
T(0,–2)

A) 12 B) 13 C) 14 D) 15 E) 16

9. $f(x) = -x^2 + 9x + 2m + 8$,

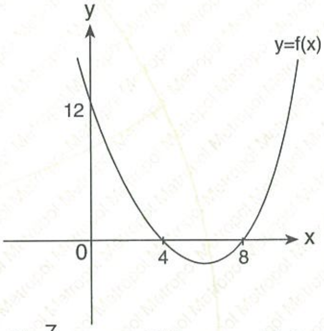
$|OB| = 4|OA|$

$\Rightarrow$ **max(f(x)) = ?**



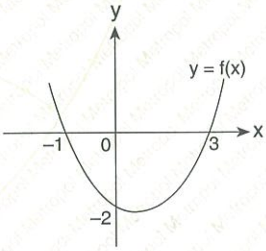
A) 56 B) $\frac{225}{4}$ C) $\frac{245}{4}$ D) $\frac{405}{8}$ E) 66

10. **f(2) = ?**


y
y=f(x)
12
0
4
8
x

A) $\frac{9}{2}$ B) 4 C) $\frac{7}{2}$ D) 3 E) 2

11. **min(y) = ?**

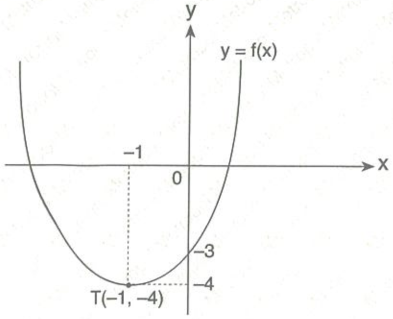


A) $-\frac{10}{3}$ B) – 3 C) $-\frac{8}{3}$ D) $-\frac{7}{3}$ E) – 2

YANITLAR / ANSWERS

1	2	3	4	5	6	7	8	9	10	11
A	D	B	B	D	A	C	E	B	A	C

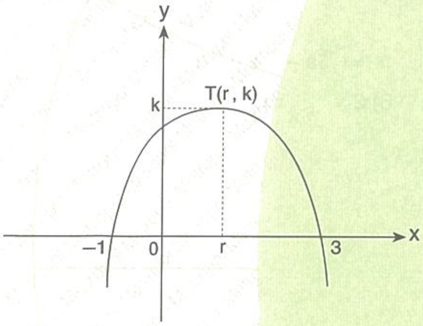
1.


y
y = f(x)
–1
0
x
–3
–4
T(–1, –4)

$f(x) = ax^2 + bx + c \Rightarrow$ **f(1) = ?**

A) –1 B) 0 C) 1 D) 2 E) 3

2.

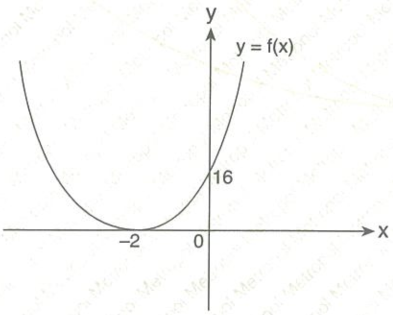

y
k
T(r , k)
–1
0
r
3
x

$\Rightarrow$ **r = ?**

A) $\frac{1}{3}$ B) $\frac{1}{2}$ C) 1 D) $\frac{3}{2}$ E) 2

3. $f(x) = (x + 2)\cdot(ax + b)$

$\Rightarrow$ **a·b = ?**

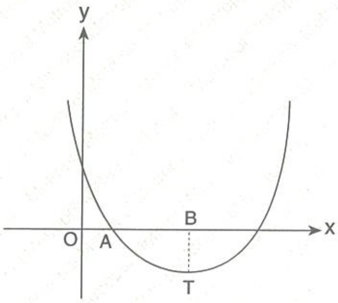


A) –32 B) –16 C) 0 D) 16 E) 32

4. $y = x^2 - 12x + m + 8$

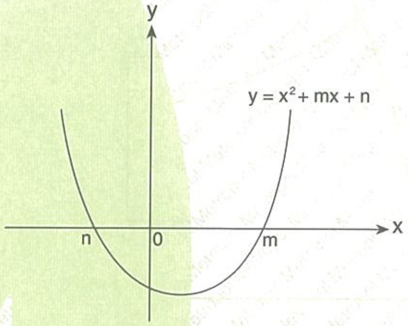
$|AB| = 2|OA|$

$\Rightarrow$ **m = ?**



A) –8 B) –6 C) 18 D) 12 E) 19

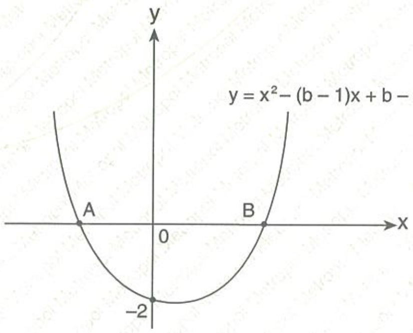
5.


y
y = x² + mx + n
n
0
m
x

$\Rightarrow$ **m + n = ?**

A) $-\frac{1}{2}$ B) $-\frac{3}{4}$ C) –1 D) –2 E) –3

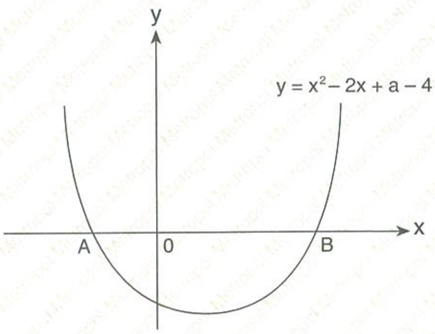
6.


y
y = x² – (b – 1)x + b – 4
A
B
0
x
–2

$\Rightarrow$ **|AB| = ? br (u)**

A) $\sqrt{13}$ B) $\sqrt{17}$ C) 2 D) 3 E) 5

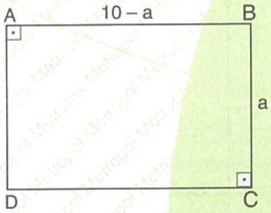
7.


y
$y = x^2 - 2x + a - 4$
x
A
0
B

$|AB| = 6$ br (u) $\Rightarrow$ **a = ?**

A) 3 B) 2 C) 0 D) –3 E) –4

8.



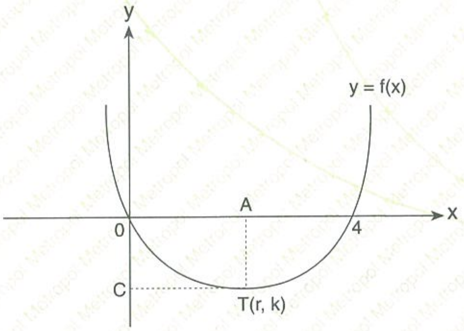
ABCD bir dikdörtgendir. / ABCD is a rectangle.

$|AB| = (10 - a)$ cm, $|BC| = (a)$ cm

$\Rightarrow$ **max(A(ABCD)) = ? cm^2**

A) 16 B) 25 C) 36 D) 49 E) 64

9.


y
y = f(x)
A
x
0
4
C
T(r, k)

$2|CT| = 3|AT|$ $\Rightarrow$ **f(x) = ?**

A) $y = x^2 - 4x$

B) $y = \frac{1}{2}x^2 - \frac{3}{2}x$

C) $y = \frac{1}{3}x^2 - \frac{4}{3}x$

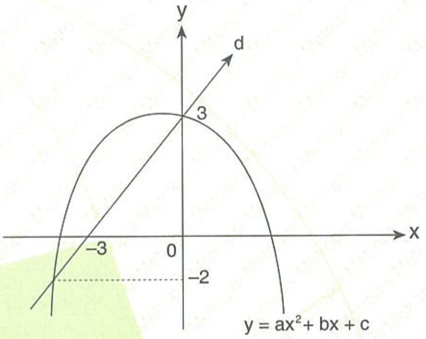
D) $y = 2x^2 - 8x$

E) $y = \frac{1}{4}x^2 - \frac{5}{4}x$

10. $y = 3x^2 - 2x + 1 \Rightarrow$ **min(y) = ?**

A) $-\frac{3}{2}$ B) $-\frac{1}{2}$ C) 0 D) $\frac{2}{3}$ E) $\frac{1}{2}$

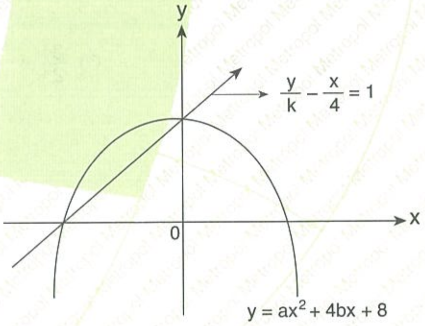
11.


y
d
3
x
–3
0
–2
$y = ax^2 + bx + c$

$\Rightarrow$ **5a – b + c = ?**

A) 1 B) 2 C) 3 D) 4 E) 5

12.


y
$\frac{y}{k} - \frac{x}{4} = 1$
x
0
$y = ax^2 + 4bx + 8$

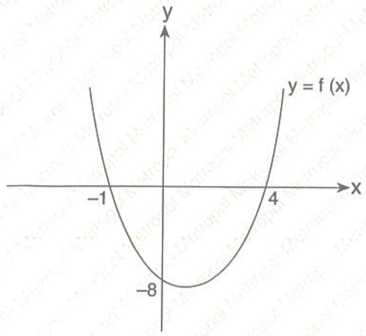
$\Rightarrow$ **a – b + k = ?**

A) $\frac{3}{2}$ B) $\frac{8}{3}$ C) $\frac{13}{2}$ D) $\frac{15}{2}$ E) $\frac{17}{2}$

YANITLAR / ANSWERS

1	2	3	4	5	6	7	8	9	10	11	12
B	C	E	D	C	D	E	B	C	D	B	D

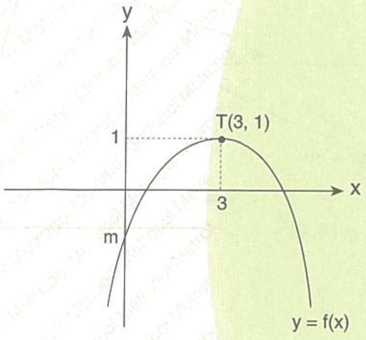
1.



$\Rightarrow$ **f(2) + |f(3)| = ?**

A) –20 B) –12 C) –4 D) 12 E) 20

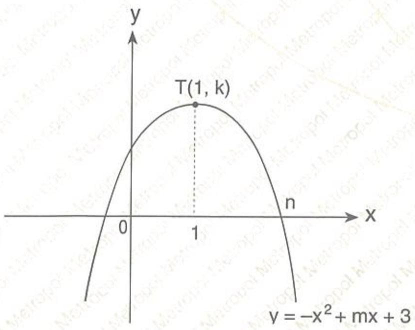
2.


y
T(3, 1)
1
3
x
m
y = f(x)

f(5) = –3 $\Rightarrow$ **m = ?**

A) –10 B) –9 C) –8 D) –7 E) –6

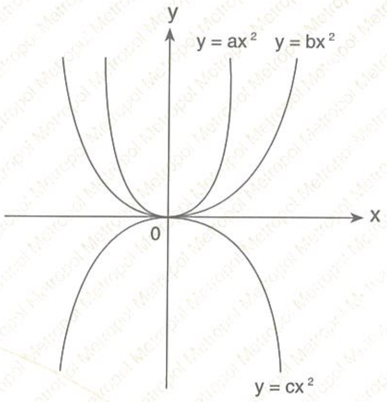
3.


y
T(1, k)
n
x
0
1
y = –x² + mx + 3

$\Rightarrow$ **n = ?**

A) –3 B) –1 C) 2 D) 3 E) 4

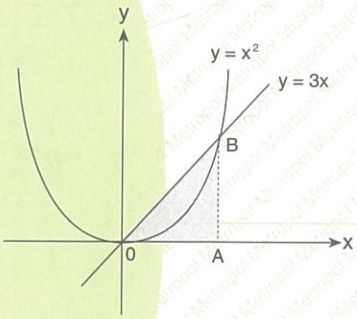
4.


y
y = ax²
y = bx²
x
0
y = cx²

$\Rightarrow$ **? < ? < ?**

A) c < a < b B) a < c < b C) c < b < a D) a < b < c E) b < a < c

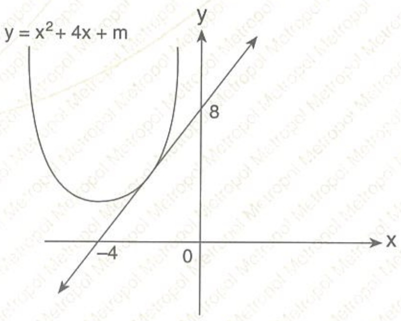
5.


y
y = x²
y = 3x
B
0
A
x

$\Rightarrow$ **$A(\widehat{AOB})$ = ? br²(u²)**

A) 27 B) 18 C) $\frac{25}{2}$ D) $\frac{27}{2}$ E) $\frac{9}{2}$

6.


y = x² + 4x + m
y
8
–4
0
x

$\Rightarrow$ **m = ?**

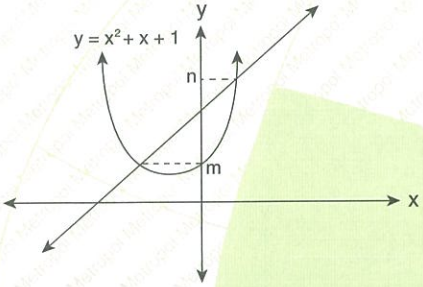
A) 7 B) 8 C) 9 D) 10 E) 12

7. $f(x) = 2x^2 - 6x + m$

$\min(f(x)) = -4 \Rightarrow$ **$m = ?$**

A) $-\frac{1}{2}$ B) 1 C) $\frac{3}{2}$ D) 2 E) $\frac{5}{2}$

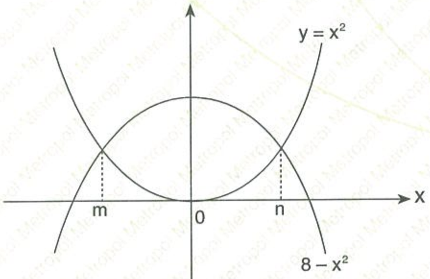
8.



$\Rightarrow$ **$m + n = ?$**

A) 19 B) 20 C) 21 D) 22 E) 23

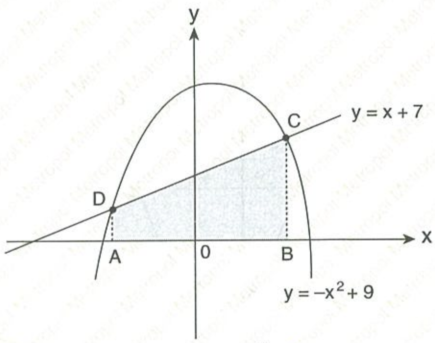
9.


y
y = x²
m
0
n
x
8 – x²

$\Rightarrow$ **$m \cdot n = ?$**

A) –8 B) –6 C) –4 D) –2 E) –1

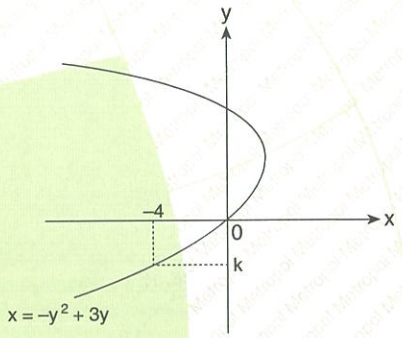
10.


y
C
y = x + 7
D
A
0
B
x
y = –x² + 9

$\Rightarrow$ **A(ABCD) = ? br² (u²)**

A) 20 B) $\frac{39}{2}$ C) 19 D) $\frac{37}{2}$ E) 18

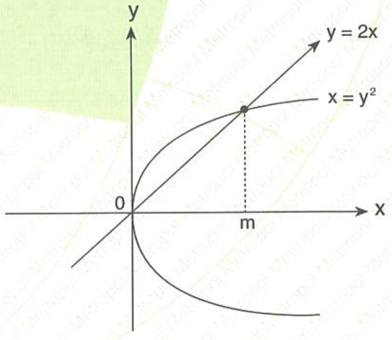
11.


y
–4
x
0
k
x = –y² + 3y

$\Rightarrow$ **$k = ?$**

A) –5 B) –4 C) –3 D) –2 E) –1

12.


y
y = 2x
x = y²
0
m
x

$\Rightarrow$ **$m = ?$**

A) $\frac{1}{4}$ B) $\frac{1}{2}$ C) $\frac{3}{2}$ D) 2 E) $\frac{5}{2}$

YANITLAR / ANSWERS

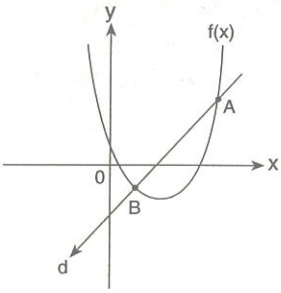
1	2	3	4	5	6	7	8	9	10	11	12
C	C	D	C	D	C	A	D	C	B	E	A

1. $d : y = 2x + n$,

$f(x) = x^2 - 3x + 2$,

$n \in Z$

$\Rightarrow$ **min(n) = ?**

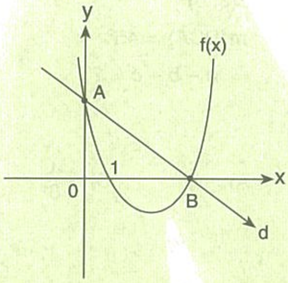


A) –6 B) –4 C) –2 D) 2 E) 3

2. $d : 2x + 3y - 6 = 0$,

$f(x) = ax^2 + bx + c$

$\Rightarrow \dfrac{\mathbf{b}}{\mathbf{c}} = \mathbf{?}$



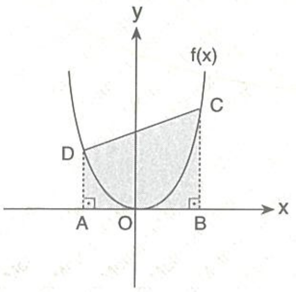
A) $-\frac{4}{3}$ B) $-\frac{10}{3}$ C) $\frac{7}{6}$ D) $\frac{3}{2}$ E) $\frac{5}{2}$

3. $f(x) = y = ax^2$,

$A(-1, 0)$, $B(5, 0)$,

$A(ABCD) = 104\ br^2\ (u^2)$

$\Rightarrow$ **a = ?**

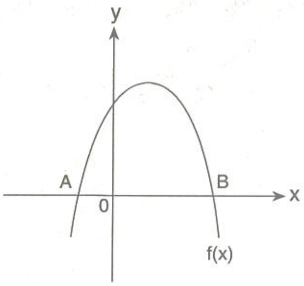


A) $\frac{2}{3}$ B) 1 C) $\frac{4}{3}$ D) $\frac{5}{3}$ E) 2

4. $|AB| = 10$ br (u),

$f(x) = y = -x^2 + 6x + c$

$\Rightarrow$ **c = ?**



A) 6 B) 9 C) 12 D) 14 E) 16

5. OABC bir dikdörtgen.

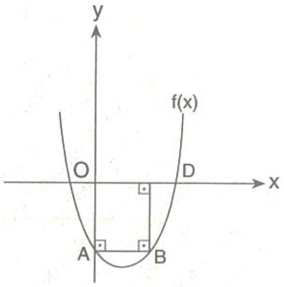
OABC is a rectangle.

$f(x) = x^2 + bx + c$,

$D(2, 0)$,

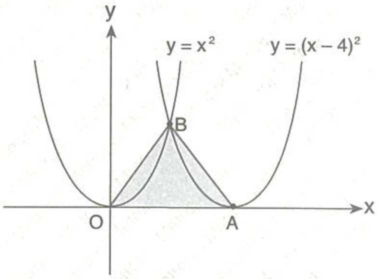
Ç(OABC) = 14 br (u)

$\Rightarrow$ **b + c = ?**



A) –7 B) –5 C) –3 D) 5 E) 7

6. $\mathbf{A(\widehat{OAB}) = ?\,br^2\,(u^2)}$

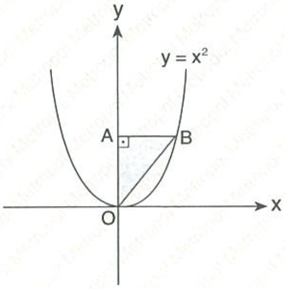

y
y = x²
y = (x – 4)²
B
x
O
A

A) 4 B) 6 C) 8 D) 10 E) 16

7. $|AB| = |AO|$

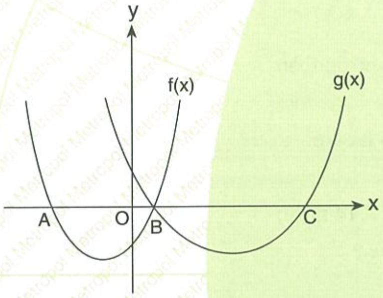
$[AB] \perp Oy$

$\Rightarrow$ **$A(\widehat{OAB})$ = ? br^2 (u^2)**



A) $\frac{\sqrt{2}}{2}$ B) $\frac{\sqrt{3}}{2}$ C) 1 D) $\sqrt{2}$ E) $\frac{1}{2}$

8.


y
f(x)
g(x)
A
O
B
C
x

$f(x) = x^2 + ax - 2$, $g(x) = x^2 + mx + n$

$A(-2, 0)$, $B(x, 0)$, $C(5, 0) \Rightarrow$ **$m \cdot n \cdot a$ = ?**

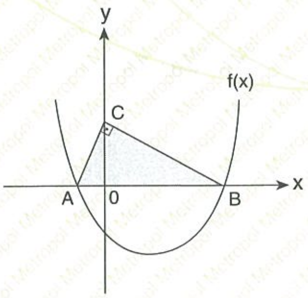
A) –48 B) –45 C) –36 D) –30 E) –20

9. $f(x) = ax^2 + bx + c$,

$A(-2, 0)$,

$B(8, 0)$

$\Rightarrow$ **$A(\widehat{ABC})$ = ? br^2(u^2)**

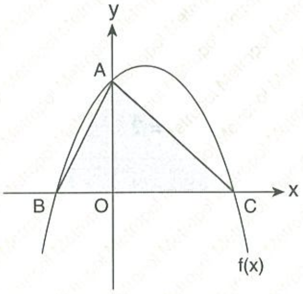


A) $4\sqrt{2}$ B) $6\sqrt{2}$ C) $8\sqrt{2}$ D) 20 E) 24

10. $f(x) = -x^2 + 3x + m$,

$4 \cdot |OB| = |OC|$

$\Rightarrow$ **$A(\widehat{ABC})$ = ? br^2 (u^2)**



A) $5\sqrt{2}$ B) 8 C) 10 D) $6\sqrt{2}$ E) $8\sqrt{2}$

11. $f(x) = ax^2 + bx + c$,

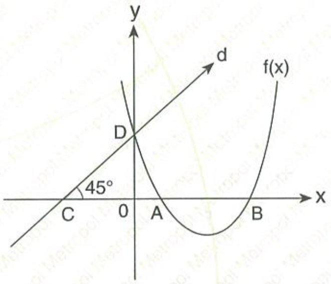
$C(-2, 0)$

$A(1, 0)$

$B(3, 0)$

$m(\widehat{DCA}) = 45°$

$\Rightarrow$ **$a - b - c$ = ?**

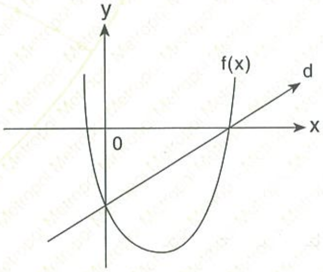

y
d
f(x)
D
45°
C
0
A
B
x

A) $-\frac{5}{3}$ B) $-\frac{4}{3}$ C) $\frac{4}{3}$ D) $\frac{5}{3}$ E) $\frac{7}{3}$

12. $f(x) = ax^2 - 3bx - 9$,

$d: mx + 3y - 3m = 0$

$\Rightarrow$ **$a - b + m$ = ?**



A) –8 B) –6 C) 6 D) 8 E) 9

YANITLAR / ANSWERS

1	2	3	4	5	6	7	8	9	10	11	12
B	A	C	E	A	C	E	D	D	C	C	A

BÖLÜM
CHAPTER
20

TARİHSEL NOT / HISTORICAL NOTE

EBU'L VEFA EL-BUZCANİ [998 – 940 BC]

İranlı matematikçi ve astronom. Bilim sahasında, matematik bilimini tahsil etti ve özellikle trigonometri üzerinde çalışmalar yaptı. Trigonometrinin altı esas oranı arasındaki trigonometrik ilişkileri ilk defa ortaya koymuştur. Bu oranlar günümüzde aynen kullanılmaktadır.

Iranian mathematician and astronomer. In the field of science, he studied mathematics and made contributions to trigonometry in particular. he revealed the trigonometric relations under the ratios of trigonometry for the first time. These rates are used as is today.

TRİGONOMETRİ I / TRIGONOMETRY I

Bu bölüm, 109 test sorusu içermektedir.
This chapter includes 109 test questions.

BÖLÜM / CHAPTER 20

TRİGONOMETRİ I / TRIGONOMETRY I

- Trigonometri I / Trigonometry I . . . . 349 – 372

BÖLÜM

20

CHAPTER

TRİGONOMETRİ I
TRIGONOMETRY I

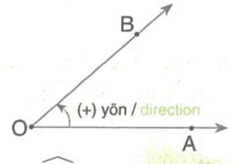
TRİGONOMETRİ / TRIGONOMETRY

YÖNLÜ AÇILAR / DIRECTED ANGLES

Tanım: Bir açının kenarlarından birini başlangıç kenarı; diğerini bitim kenarı olarak alırsak elde edilen açıya ***yönlü açı*** denir.

Definition: If we take one of the sides of an angle as initial side and other side as terminal side, we obtain a ***directed angle***.

▼



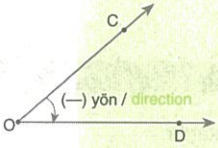
$\widehat{AOB}$ pozitif yönlü açı

$\widehat{AOB}$ is positive directional angle

[OA, başlangıç kenarı; [OB, bitim kenarı olmak üzere saatin dönme yönünün tersi yönde alınan AOB açısına ***pozitif yönlü açı*** denir.

If the angle AOB rotates in a counter-clockwise direction, the rotation is called *positive* where [OA is the initial side, [OB is the terminal side.

▼



$\widehat{COD}$ negatif yönlü açı

$\widehat{COD}$ is negative directional angle

[OC, başlangıç kenarı; [OD, bitim kenarı olmak üzere saatin dönme yönünde alınan COD açısına ***negatif yönlü açı*** denir.

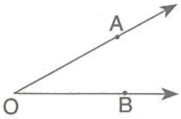
If the angle COD rotates in a clockwise direction, the rotation is called *negative* where [OC is the initial side, [OD is the terminal side.

Önemli / Important

Tanım: Başlangıç noktaları ortak olan iki ışının birleşim kümesine ***açı*** denir.

Definition: The union set of two half-lines with congruent initial points is congruent is called *angle*.

$[OA \cup [OB = \widehat{AOB} = \widehat{BOA} = \widehat{O}$



AÇI ÖLÇÜ BİRİMLERİ / ANGLE MEASUREMENT UNITS

Derece: Tüm çember yayının 360 da birine 1 derecelik yay, bu yayı gören merkez açısının ölçüsüne de ***1 derece (1°)*** denir. Tüm çember yayının ölçüsü 360° dir.

Degree: Arc with one degree is $\frac{1}{360}$ of the round arc and the measure of the corresponding central angle is called *one degree* (1°). One revolution is called 360°.

Radyan: Yarıçap uzunluğundaki çember yayına bir radyanlık yay, bu yayı gören merkez açısının ölçüsüne de ***1 radyan*** denir.

Radian: If the length of the arc is equal to radius, this arc is one radian and the measure of the corresponding central angle is called a *radian*.

$$\frac{D}{180^\circ} = \frac{R}{\pi} \text{ veya/or } \frac{D}{360^\circ} = \frac{R}{2\pi}$$

ESAS ÖLÇÜ / REFERENCE MEASURE

Tanım: Verilen ölçülerin aşağıdaki denkliklerle hesaplanan pozitif yöndeki değerlerine ***esas ölçü*** denir.

Definition: Values of the positive direction calculated in the following equation is called *reference measure* of the given measure.

I. $k \cdot 2\pi + \theta \equiv \theta \pmod{2\pi}$ $(k \in Z)$

II. $k \cdot 360^\circ + \theta \equiv \theta \pmod{360^\circ}$ $(k \in Z)$

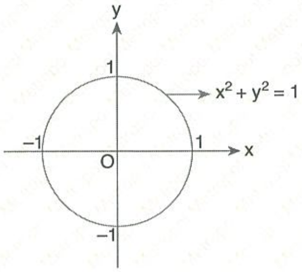
Önemli / Important

- Radyan cinsinden verilen kesirli ifadede esas ölçü aranırken paydanın çift katları paydan çıkarılır.
- In expression involving fraction in radii, reference measure is found by subtracting the numerator by the even multiples of denominator.

TRİGONOMETRİK FONKSİYONLAR / TRIGONOMETRIC FUNCTIONS

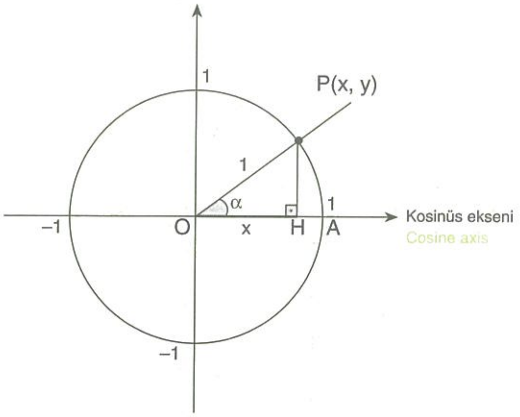
Önemli / Important

- Analitik düzlemde merkezi (0, 0) ve yarıçapı 1 birim olan çembere, birim (trigonometrik) çember denir.
- In analytic plane, the cercle with the center (0, 0) and radius of 1 is called unit (trigonometric) circle.



KOSİNÜS FONKSİYONU / COSINE FUNCTION

▼


1
P(x, y)
1
α
1
–1
O
x
H
A
Kosinüs ekseni
Cosine axis
–1

Bir α açısına birim çember üzerinde karşılık gelen P noktasının apsisine α ***açısının kosinüsü*** denir. (x = cosα)

Abscissa of point P on the unit circle of the angle α is called ***cosine of angle*** α.

$$\cos(\widehat{AOP}) = |OH| = x$$

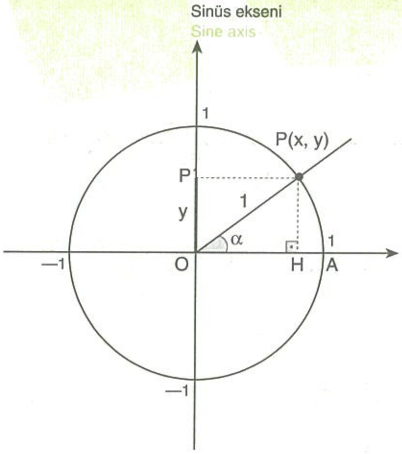
$$\cos: R \rightarrow [-1, 1]$$

$$\alpha \rightarrow \cos\alpha,\ \cos\alpha \in [-1, 1]$$

$$\Rightarrow \mathbf{-1 \le \cos\alpha \le 1}$$

SİNÜS FONKSİYONU / SINE FUNCTION

▼


Sinüs ekseni
Sine axis
1
P(x, y)
P′
1
y
α
1
–1
O
H
A
–1

Bir α açısına birim çember üzerinde karşılık gelen P noktasının ordinatına α ***açısının sinüsü*** denir.

The ordinate of point P on the unit circle of the angle α is called ***sine of angle*** α.

$$\sin(\widehat{AOP}) = |PH| = |P'O| = y$$

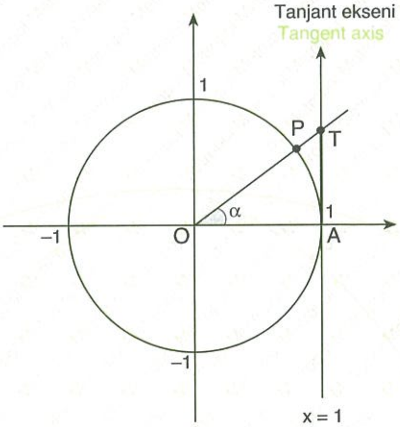
$$\sin: R \rightarrow [-1, 1]$$

$$\alpha \rightarrow \sin\alpha,\ \sin\alpha \in [-1, 1]$$

$$\Rightarrow \mathbf{-1 \le \sin\alpha \le 1}$$

TANJANT FONKSİYONU / TANGENT FUNCTION

▼



Bir α açısına birim çember üzerinde karşılık gelen P noktasından geçen [OT ışınının x = 1 doğrusunu kestiği T noktasının ordinatına α ***açısının tanjantı*** denir.

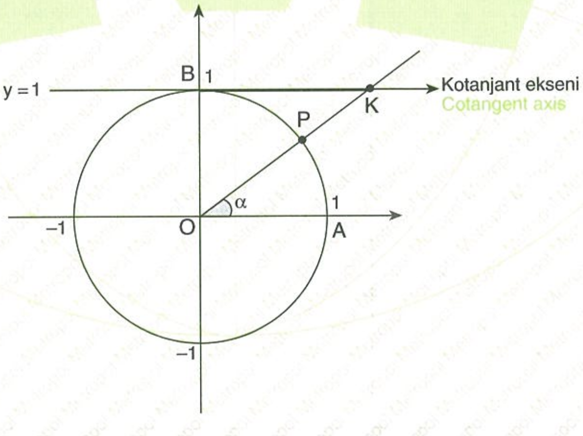
Ordinate of point T is ***tangent of the angle*** α provided that tangent axis of ray [OT passes through P on line x = 1.

$$\tan(\widehat{AOP}) = |TA|$$

$$\tan : \mathbb{R} - \left\{\frac{\pi}{2} + k\cdot 2\pi, \frac{3\pi}{2} + k\cdot 2\pi\right\} \rightarrow \mathbb{R}$$

KOTANJANT FONKSİYONU / COTANGENT FUNCTION

▼



Bir α açısına birim çember üzerinde karşılık gelen P noktasından geçen [OK ışınının y = 1 doğrusunu kestiği K noktasının apsisine α ***açısının kotanjantı*** denir.

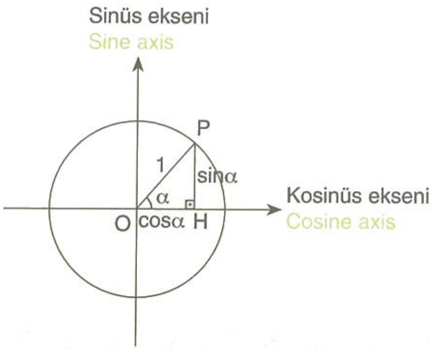
Abscissa of point K is ***cotangent of the angle*** α provided that tangent axis of ray [OK passes throung P on line y = 1.

$$\cot(\widehat{AOP}) = |BK|$$

$$\cot: \mathbb{R} - \{\pi + k\cdot 2\pi , 2\pi + k\cdot 2\pi\} \rightarrow \mathbb{R}$$

TRİGONOMETRİK ÖZDEŞLİKLER / TRIGONOMETRIC IDENTITIES

▼


Sinüs ekseni
Sine axis
P
1
sinα
α
O
cosα
H
Kosinüs ekseni
Cosine axis

$\widehat{OHP}$: $m(\widehat{OHP}) = 90°$, $\boldsymbol{\sin^2\alpha + \cos^2\alpha = 1}$
[Pisagor Teoremi / Pythagorean Theorem]

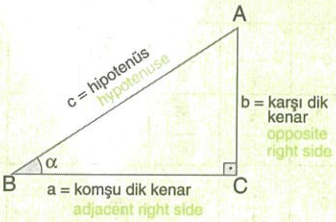
DİK ÜÇGENDE TRİGONOMETRİK ORANLAR / TRIGONOMETRIC RATIOS IN A RIGHT TRIANGLE

▼ $\sin\alpha = \dfrac{\text{karşı dik kenar opposite right side}}{\text{hipotenüs hypotenuse}} = \dfrac{b}{c}$

▼ $\cos\alpha = \dfrac{\text{komşu dik kenar adjacent right side}}{\text{hipotenüs hypotenuse}} = \dfrac{a}{c}$

▼ $\tan\alpha = \dfrac{\text{karşı dik kenar opposite right side}}{\text{komşu dik kenar adjacent right side}} = \dfrac{b}{a}$

▼ $\cot\alpha = \dfrac{\text{komşu dik kenar adjacent right side}}{\text{karşı dik kenar opposite right side}} = \dfrac{a}{b}$


A
c = hipotenüs
hypotenuse
b = karşı dik kenar
opposite right side
α
B
a = komşu dik kenar
adjacent right side
C

▼ **Trigonometrik Oranlardan Biri Verildiğinde Diğerini Bulma:** Verilen trigonometrik oran için dik üçgen çizilir. Bu üçgen yardımıyla istenen oranın değeri mutlak değerce bulunur. İstenen açının bulunduğu bölgeye göre işaret belirlenir.

▼ **Finding Other trigonometric Ratios from one of the Trigonometric Ratios:** We draw the right triangle to the given trigonometric ratio. Value of wanted ratio is found from this triangle with absolute value. And then according to quadrant, the sign is written.

Önemli / Important

- $\sec\alpha = \dfrac{1}{\cos\alpha} = \dfrac{1}{\frac{a}{c}} = \dfrac{c}{a}$
- $\operatorname{cosec}\alpha = \dfrac{1}{\sin\alpha} = \dfrac{1}{\frac{b}{c}} = \dfrac{c}{b}$

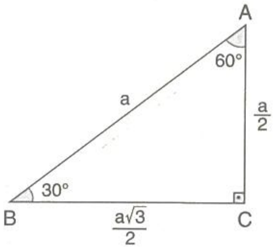
30° ve 60° nin TRİGONOMETRİK DEĞERLERİ / TRIGONOMETRIC VALUES of 30° and 60°

▼ $\sin 30° = \dfrac{1}{2}$, $\sin 60° = \dfrac{\sqrt{3}}{2}$

▼ $\cos 30° = \dfrac{\sqrt{3}}{2}$, $\cos 60° = \dfrac{1}{2}$

▼ $\tan 30° = \dfrac{\sqrt{3}}{3}$, $\tan 60° = \sqrt{3}$

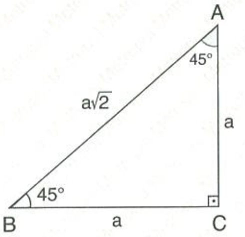
▼ $\cot 30° = \sqrt{3}$, $\cot 60° = \dfrac{\sqrt{3}}{3}$


A
60°
a
a/2
30°
B
a√3/2
C

45° nin TRİGONOMETRİK DEĞERLERİ / TRIGONOMETRIC VALUES of 45°

▼ $\sin 45° = \frac{\sqrt{2}}{2} = \cos 45°$

▼ $\tan 45° = 1 = \cot 45°$


A
45°
$a\sqrt{2}$
a
45°
B
a
C

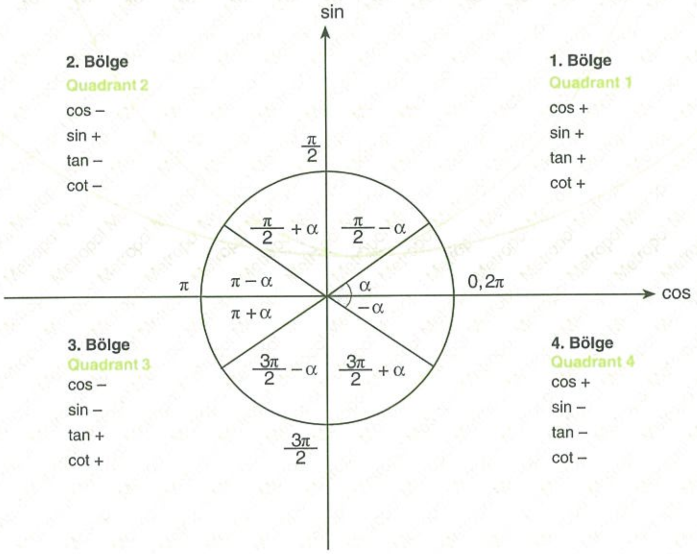
TEMEL AÇILARIN TRİGONOMETRİK DEĞERLERİ / TRIGONOMETRIC VALUES of the BASIC ANGLES

▼

x (radyan) (radian)	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
x (derece) (degree)	0°	30°	45°	60°	90°	180°	270°	360°
sinx	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	0	−1	0
cosx	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	−1	0	1
tanx	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	Belirsiz Indefinite	0	Belirsiz Indefinite	0
cotx	Belirsiz Indefinite	$\sqrt{3}$	1	$\frac{\sqrt{3}}{3}$	0	Belirsiz Indefinite	0	Belirsiz Indefinite

ÖLÇÜSÜ 90°DEN BÜYÜK OLAN AÇILARIN TRİGONOMETRİK DEĞERLERİ TRİGONOMETRİC VALUES of ANGLES with MEASURE of GREATER THAN 90°

▼ $0 < \alpha < \frac{\pi}{2}$


sin
2. Bölge
Quadrant 2
cos −
sin +
tan −
cot −
1. Bölge
Quadrant 1
cos +
sin +
tan +
cot +
$\frac{\pi}{2}$
$\frac{\pi}{2}+\alpha$
$\frac{\pi}{2}-\alpha$
π
$\pi-\alpha$
α
$0, 2\pi$
cos
$\pi+\alpha$
$-\alpha$
3. Bölge
Quadrant 3
cos −
sin −
tan +
cot +
$\frac{3\pi}{2}-\alpha$
$\frac{3\pi}{2}+\alpha$
4. Bölge
Quadrant 4
cos +
sin −
tan −
cot −
$\frac{3\pi}{2}$

▼ 1. Bölge / Quadrant 1:

$$0<\alpha<\frac{\pi}{2} \Rightarrow \sin\left(\frac{\pi}{2}-\alpha\right)=\cos\alpha$$

$$\cos\left(\frac{\pi}{2}-\alpha\right)=\sin\alpha$$

$$\tan\left(\frac{\pi}{2}-\alpha\right)=\cot\alpha$$

$$\cot\left(\frac{\pi}{2}-\alpha\right)=\tan\alpha$$

▼ 2. Bölge / Quadrant 2:

$$0<\alpha<\frac{\pi}{2} \Rightarrow \sin\left(\frac{\pi}{2}+\alpha\right)=\cos\alpha \qquad \sin(\pi-\alpha)=\sin\alpha$$

$$\cos\left(\frac{\pi}{2}+\alpha\right)=-\sin\alpha \qquad \cos(\pi-\alpha)=-\cos\alpha$$

$$\tan\left(\frac{\pi}{2}+\alpha\right)=-\cot\alpha \qquad \tan(\pi-\alpha)=-\tan\alpha$$

$$\cot\left(\frac{\pi}{2}+\alpha\right)=-\tan\alpha \qquad \cot(\pi-\alpha)=-\cot\alpha$$

▼ 3. Bölge / Quadrant 3:

$$0<\alpha<\frac{\pi}{2} \Rightarrow \sin\left(\frac{3\pi}{2}-\alpha\right)=-\cos\alpha \qquad \sin(\pi+\alpha)=-\sin\alpha$$

$$\cos\left(\frac{3\pi}{2}-\alpha\right)=-\sin\alpha \qquad \cos(\pi+\alpha)=-\cos\alpha$$

$$\tan\left(\frac{3\pi}{2}-\alpha\right)=\cot\alpha \qquad \tan(\pi+\alpha)=\tan\alpha$$

$$\cot\left(\frac{3\pi}{2}-\alpha\right)=\tan\alpha \qquad \cot(\pi+\alpha)=\cot\alpha$$

▼ 4. Bölge / Quadrant 4:

$$0<a<\frac{\pi}{2} \Rightarrow \sin\left(\frac{3\pi}{2}+\alpha\right)=-\cos\alpha \qquad \sin(2\pi-\alpha)=-\sin\alpha$$

$$\cos\left(\frac{3\pi}{2}+\alpha\right)=\sin\alpha \qquad \cos(2\pi-\alpha)=\cos\alpha$$

$$\tan\left(\frac{3\pi}{2}+\alpha\right)=-\cot\alpha \qquad \tan(2\pi-\alpha)=-\tan\alpha$$

$$\cot\left(\frac{3\pi}{2}+\alpha\right)=-\tan\alpha \qquad \cot(2\pi-\alpha)=-\cot\alpha$$

NEGATİF AÇILARIN TRİGONOMETRİK DEĞERLERİ / TRIGONOMETRIC VALUES of NEGATIVE ANGLES

▼ $\sin(-\alpha)=-\sin\alpha$

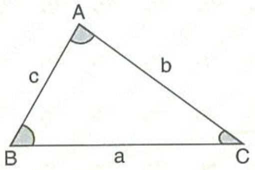
$\cos(-\alpha)=\cos\alpha$

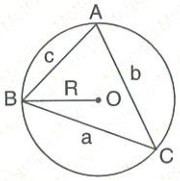
$\tan(-\alpha)=-\tan\alpha$

$\cot(-\alpha)=-\cot\alpha$

Özellik / Property

★ **Sinüs Teoremi / The Sine Theorem:**





$A(\widehat{ABC}) = \frac{1}{2}\cdot a\cdot b\cdot\sin\widehat{C} = \frac{1}{2}\cdot a\cdot c\cdot\sin\widehat{B} = \frac{1}{2}\cdot b\cdot c\cdot\sin\widehat{A}$ veya/or $A(\widehat{ABC}) = \frac{a\cdot b\cdot c}{4R}$

$$\Rightarrow \frac{a}{\sin\widehat{A}} = \frac{b}{\sin\widehat{B}} = \frac{c}{\sin\widehat{C}} = 2R$$

[R: $\widehat{ABC}$ nin çevrel çemberinin yarıçapı / radius of circumcircle of triangle ABC]

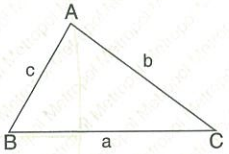
Özellik / Property

★ **Kosinüs Teoremi / The Cosine Theorem:**

$a^2 = b^2 + c^2 - 2\cdot b\cdot c\cdot\cos\widehat{A}$

$b^2 = a^2 + c^2 - 2\cdot a\cdot c\cdot\cos\widehat{B}$

$c^2 = a^2 + b^2 - 2\cdot a\cdot b\cdot\cos\widehat{C}$



Önemli / Important

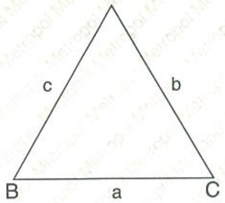
- Üçgenin kenar uzunlukları biliniyorken herhangi bir açının ölçüsünü bulmak ya da iki kenar uzunluğu ve bu iki kenar arasındaki açısı verilen üçgenin diğer kenarının uzunluğunu bulmak için kullanılır.
- Given that we know the measurement of the length of the sides, the cosine theorem is used to find the measure of any angle or the length of the other sides of triangle given by the angle between two sides.

Özellik / Property

★ **Tanjant Teoremi / The Tangent Theorem:**

$|BC| = a,\ |AC| = b,\ |AB| = c$

$$\Rightarrow \frac{b+c}{b-c} = \frac{\tan\left(\frac{m(\widehat{B})+m(\widehat{C})}{2}\right)}{\tan\left(\frac{m(\widehat{B})-m(\widehat{C})}{2}\right)}$$



1. $120° = x^R \Rightarrow x = ?$

A) $\frac{\pi}{3}$ B) $\frac{2\pi}{3}$ C) $\frac{\pi}{4}$ D) $\frac{\pi}{2}$ E) π

2. $\frac{3\pi}{4} = x° \Rightarrow D = ?$

A) 45° B) 90° C) 110° D) 120° E) 135°

3. $1390° = x + 360°k \Rightarrow x = ?$

A) 190° B) 220° C) 240° D) 290° E) 310°

4. $\frac{15\pi}{2} = x + 2k\pi \Rightarrow x = ?$

A) $\frac{\pi}{2}$ B) π C) $\frac{3\pi}{2}$ D) $\frac{4\pi}{3}$ E) $\frac{3\pi}{4}$

5. $\left(\frac{1}{\cos^2 x} - 1\right) \cdot (\cot^2 x) = ?$

A) $\cot^2 x$ B) $\tan x$ C) $\cos^2 x$ D) $\sin x$ E) 1

6. $\frac{\sin x}{\cos^3 x + \cos x \cdot \sin^2 x} = ?$

A) $\cos x$ B) $\cot x$ C) $\cos^2 x$ D) $\tan x$ E) 1

7. $\cos x - \frac{\sin^2 x}{1 - \cos x} = ?$

A) 1 B) $-\sin x$ C) $-\tan x$ D) $\tan x$ E) -1

8. $\left(\frac{\sin x}{1 - \cos^2 x}\right) \cdot \left(\frac{1 - \sin^2 x}{\cos x}\right) \cdot (\cot x) = ?$

A) $\sin x$ B) $\tan^3 x$ C) $\cot^2 x$ D) $\cos x$ E) 1

9. $\frac{1}{\sin^2 x} - \frac{1}{\tan^2 x} = ?$

A) $\sin^2 x$ B) $\tan^2 x$ C) $\cos^2 x$ D) $\cot^2 x$ E) 1

10. $\frac{\tan\theta - 1}{1 - \cot\theta} = ?$

A) $\tan\theta$ B) $\cos\theta$ C) $\cot\theta$ D) $-\tan\theta$ E) $\sin\theta$

11. $\frac{\sec x - \cos x}{\operatorname{cosec} x - \sin x} = ?$

A) $1 - \tan^2 x$ B) $1 + \tan^2 x$ C) $1 - \tan^3 x$ D) $\tan^3 x$ E) $\cot^3 x$

12. $\frac{1}{1\tan^2 x} + \frac{1}{\cot^2 x + 1} = ?$

A) $\cos^2 x - \sin^2 x$ B) $\sin^2 x - \cos^2 x$ C) $\cot x$ D) 1 E) $-\sin^2 x$

13. $(1 - \sec^2\theta)(1 - \operatorname{cosec}^2\theta) = ?$

A) $\sin\theta$ B) $\sin^2\theta$ C) $\cos\theta$ D) $\tan\theta$ E) 1

14. $\frac{\operatorname{cosec} x(\cos x + \cot x)}{\sec x + \tan x} = ?$

A) $\tan^2 x$ B) $\sin x$ C) $\cos x$ D) $\cot^2 x$ E) 1

15. $0 < x < \frac{\pi}{2}$

$\Rightarrow \sqrt{\sec^2 x - \frac{1}{\cos x}} \cdot \sqrt{\frac{1 + \sec x}{\sec x}} \cdot \cot x = ?$

A) 1 B) $\sin x$ C) $\cos x$ D) $\tan x$ E) $\cot x$

16. $\sec t - \frac{\cos x}{1 + \sin x} = a \Rightarrow 1 + \cot x = ?$

A) $a + 1$ B) $\frac{1}{a+1}$ C) $\frac{a}{a+1}$ D) $\frac{a-1}{a}$ E) $\frac{a+1}{a}$

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1	2	3	4	5	6	7	8
B	E	E	C	E	D	E	C
9	10	11	12	13	14	15	16
E	A	D	D	E	D	A	E

1. $\left(\frac{2\sin 70^\circ}{\sin 20^\circ} - \cot 20^\circ\right) \cdot \tan 20^\circ = ?$

A) sin20° B) tan20° C) tan70° D) 0 E) 1

2. $2 + \frac{\tan 40^\circ}{\cot 50^\circ} = ?$

A) 1 B) 2 C) 3 D) 4 E) 5

3. $3\tan 34^\circ - 3\cos 48^\circ + \frac{1}{\operatorname{cosec} 42^\circ} - \frac{3}{\tan 56^\circ} + 2\sin 42^\circ = ?$

A) –1 B) 0 C) 1 D) $\cos^2 24$ E) tan48

4. $\tan x + \cot x = 5 \Rightarrow \tan^2 x + \cot^2 x = ?$

A) 21 B) 23 C) 25 D) 27 E) 29

5. $\sin x + \cos x = \frac{2}{3} \Rightarrow \sin x \cdot \cos x = ?$

A) $-\frac{5}{9}$ B) $-\frac{1}{6}$ C) $-\frac{2}{9}$ D) $-\frac{5}{18}$ E) $-\frac{2}{3}$

6. $\sin x - \operatorname{cosec} x = \frac{1}{3} \Rightarrow \sin^2 x + \operatorname{cosec}^2 x = ?$

A) $\frac{7}{3}$ B) $\frac{11}{9}$ C) $\frac{19}{9}$ D) $\frac{22}{9}$ E) $\frac{11}{3}$

7. $\sin x + \operatorname{cosec} x = \frac{5}{2} \Rightarrow \sin^2 x + \operatorname{cosec}^2 x = ?$

A) $\frac{17}{4}$ B) $\frac{17}{16}$ C) $\frac{15}{4}$ D) $\frac{15}{8}$ E) $\frac{9}{4}$

8. $A = 4\sin x - 5\cos y \Rightarrow \frac{\max(A)}{\min(A)} = ?$

A) –1 B) 0 C) 1 D) 4 E) 9

9. $\frac{4\cos x - 2\sin x}{6\sin x + 2\cos x} = \frac{5}{6} \Rightarrow \cot x = ?$

A) $\frac{2}{7}$ B) $\frac{1}{6}$ C) $\frac{1}{2}$ D) $\frac{1}{3}$ E) 3

10. $0 < x < \frac{\pi}{2}$,

$\sin(2x + 10°) = \cos(3x - 20°) \Rightarrow$ **x = ?**

A) 10° B) 20° C) 30° D) 40° E) 60°

11. $\sin 2\alpha = \cos 35° \Rightarrow$ **α = ?**

A) 25,5° B) 27,5° C) 30° D) 55° E) 90°

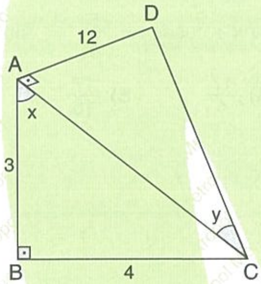
12. $m(\widehat{DAC}) = 90°$

$m(\widehat{ABC}) = 90°$

$m(\widehat{BAC}) = x$

$m(\widehat{ACD}) = y$

$\Rightarrow$ **sinx + tany = ?**



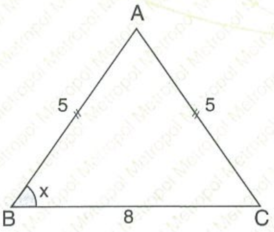
A) 3 B) $\frac{16}{5}$ C) $\frac{17}{5}$ D) $\frac{18}{5}$ E) $\frac{19}{5}$

13. $m(\widehat{ABC}) = x$,

$|AB| = |AC| = 5$ cm,

$|BC| = 8$ cm

$\Rightarrow$ **sinx = ?**



A) $\frac{1}{5}$ B) $\frac{2}{5}$ C) $\frac{3}{5}$ D) $\frac{4}{5}$ E) $\frac{3}{4}$

14. $[AB] \parallel [CD]$,

$[BD] \perp [CD]$,

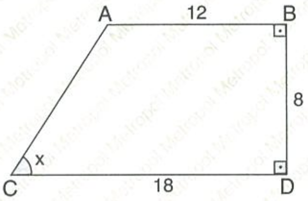
$|AB| = 12$ cm,

$|BD| = 8$ cm,

$|CD| = 18$ cm,

$m(\widehat{ACD}) = x$

$\Rightarrow$ **sinx + cotx = ?**



A) $\frac{3}{2}$ B) $\frac{5}{4}$ C) $\frac{17}{10}$ D) $\frac{21}{10}$ E) $\frac{31}{20}$

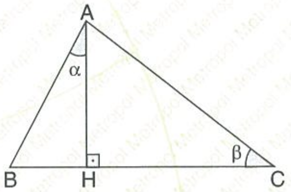
15. $m(\widehat{AHC}) = 90°$,

$|BH| = 2$ cm,

$|AH| = 3$ cm,

$|HC| = 4$ cm

$\Rightarrow$ **tanα·sinβ = ?**



A) $\frac{9}{10}$ B) $\frac{4}{5}$ C) $\frac{3}{4}$ D) $\frac{3}{5}$ E) $\frac{2}{5}$

16. $|BD| = 2$ cm,

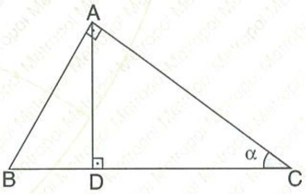
$|AD| = 4$ cm,

$m(\widehat{BAC}) = 90°$

$m(\widehat{ADC}) = 90°$

$m(\widehat{ACB}) = \alpha$

$\Rightarrow$ **tan α = ?**



A) $\frac{1}{\sqrt{3}}$ B) $\frac{1}{2}$ C) $\sqrt{3}$ D) 2 E) 3

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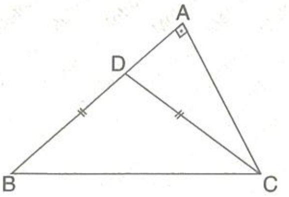
1	2	3	4	5	6	7	8
E	C	B	B	D	C	A	A
9	**10**	**11**	**12**	**13**	**14**	**15**	**16**
E	B	B	B	C	E	E	B

1. |DB| = |DC|,

$\cos(\widehat{ADC}) = \frac{5}{13}$

$\Rightarrow$ **$\sin(\widehat{DBC}) = ?$**



A) $\frac{\sqrt{3}}{13}$ B) $\frac{2\sqrt{13}}{13}$ C) $\frac{3\sqrt{13}}{13}$

D) $\frac{5\sqrt{13}}{13}$ E) $\frac{6\sqrt{13}}{13}$

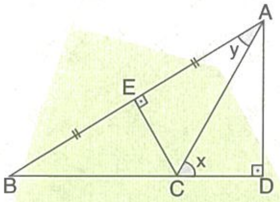
2. EC ⊥ AB,

AD ⊥ BD,

|BE| = |EA|,

$\cos x = \frac{3}{5}$

$\Rightarrow$ **$\cot y = ?$**



A) 2 B) $\frac{3}{7}$ C) $\frac{1}{2}$ D) 3 E) 1

3. $0 < x < \frac{\pi}{2}$,

$\sin x = \frac{3}{5} \Rightarrow$ **$\cot x - \tan x = ?$**

A) $-\frac{7}{12}$ B) $\frac{7}{12}$ C) $-\frac{25}{12}$ D) $\frac{25}{12}$ E) 0

4. $0 < x < \frac{\pi}{2}$,

$\frac{4}{\cos x} = \frac{5}{\sin x} \Rightarrow$ **$\cos x = ?$**

A) $\frac{\sqrt{5}}{3}$ B) $\frac{5}{3}$ C) $\frac{3}{5}$ D) $\frac{5}{\sqrt{41}}$ E) $\frac{4}{\sqrt{41}}$

5. $x \in \left(0, \frac{\pi}{2}\right)$,

$\frac{\sin x + \cos x}{7\sin x - 3\cos x} = 1 \Rightarrow$ **$\sin x = ?$**

A) $\frac{3\sqrt{13}}{13}$ B) $\frac{2\sqrt{13}}{13}$ C) $\frac{\sqrt{11}}{22}$

D) $\frac{\sqrt{15}}{6}$ E) $\frac{\sqrt{15}}{3}$

6. **$\frac{\cos 60° + \sin 60°}{\tan 45° + \cot 30°} = ?$**

A) $\frac{1}{2}$ B) $\frac{\sqrt{3}}{3}$ C) 1 D) $\sqrt{2}$ E) 2

7. $x = \frac{\pi}{3} \Rightarrow$ **$\frac{1}{\sin^4 x} - \frac{1}{\cos^4 x} = ?$**

A) $-\frac{128}{9}$ B) $-\frac{64}{9}$ C) $-\frac{32}{9}$

D) $\frac{32}{3}$ E) $\frac{64}{3}$

8. $f(x) = 1 - 2\sin^2 2x + \sin^2 x \cdot \cos^2 x \Rightarrow$ **$f\left(\frac{\pi}{6}\right) = ?$**

A) $-\frac{5}{16}$ B) $\frac{5}{6}$ C) $\frac{11}{6}$

D) $\frac{5}{8}$ E) $\frac{3}{4}$

9. sin150° , cos560° , tan342° $\Rightarrow$ **? , ? , ?**

A) – , – , +
B) – , + , +
C) + , – , +
D) + , – , –
E) – , + , –

10. $\sin\left(\frac{\pi}{2}+\alpha\right)+\cos(\pi-\alpha)=?$

A) 0
B) $\frac{1}{2}$
C) 2·sinα
D) cosα
E) 2·cosα

11. **cos(180° – x) = ?**

A) cosx
B) –cosx
C) cotx
D) –cotx
E) tanx

12. $\tan\left(\frac{22\pi}{3}\right)=?$

A) $\sqrt{3}$
B) 1
C) $\frac{\sqrt{3}}{3}$
D) – 1
E) $-\sqrt{3}$

13. $\cos(-\pi-\alpha)+\sin\left(\alpha-\frac{\pi}{2}\right)=?$

A) 0
B) –2cosα
C) cosα
D) 2cosα
E) sinα

14. $\sin\frac{7\pi}{3}\cdot\cos\frac{23\pi}{3}\cdot\tan\frac{15\pi}{4}=?$

A) $\frac{\sqrt{3}}{2}$
B) $\frac{\sqrt{3}}{4}$
C) $-\frac{\sqrt{3}}{4}$
D) $-\frac{3}{4}$
E) $-\frac{1}{2}$

15. $\dfrac{\sin\frac{35\pi}{6}+\cos\left(-\frac{20\pi}{3}\right)}{\tan\left(-\frac{67\pi}{4}\right)}=?$

A) –1
B) $-\frac{1}{2}$
C) 0
D) $\frac{1}{2}$
E) 1

16. $\dfrac{\sin\left(\frac{3\pi}{2}-x\right)+\cos\left(\frac{\pi}{2}-x\right)+\sin(\pi+x)}{\tan\left(\frac{3\pi}{2}+x\right)}=?$

A) sinx
B) –cosx
C) –sinx
D) cosx
E) 2cosx

17. $0 < x < \frac{\pi}{2}$

$$\Rightarrow \frac{\sin\left(\frac{3\pi}{2}+x\right)}{\cos\left(\frac{5\pi}{2}-x\right)}+\frac{\cos(\pi-x)}{\sin(3\pi+x)}=?$$

A) –2
B) –1
C) 0
D) 1
E) 2

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1	2	3	4	5	6	7	8	9
B	A	B	E	B	A	A	A	D
10	11	12	13	14	15	16	17	
A	B	A	B	C	A	A	C	

1. $\dfrac{\sin 300° \cdot \cos 120°}{\tan 225° \cdot \cos 240°} = ?$

A) $-\frac{\sqrt{3}}{4}$ B) $-\frac{\sqrt{3}}{2}$ C) $-\frac{3}{2}$ D) 1 E) $\frac{3}{2}$

2. **sin 330° = ?**

A) $\frac{\sqrt{3}}{2}$ B) $\frac{1}{2}$ C) $-\frac{\sqrt{3}}{2}$ D) $-\frac{1}{2}$ E) $-\frac{\sqrt{2}}{2}$

3. **cos (–120°)·sin (–135°) = ?**

A) $-\frac{\sqrt{2}}{4}$ B) $-\frac{\sqrt{3}}{4}$ C) $-\frac{1}{4}$ D) $-\frac{\sqrt{2}}{2}$ E) $\frac{\sqrt{2}}{4}$

4. $f(x) = \sin 2x + 1 \Rightarrow f\left(\frac{5\pi}{12}\right) = ?$

A) –1 B) 0 C) $\frac{1}{2}$ D) 1 E) $\frac{3}{2}$

5. $a = \sin 310°$,
$b = \cos 320°$,
$c = \sin 290°$
$d = \sin 70°$

$\Rightarrow ? < ? < ? < ?$

A) a < b < c < d B) c < a < b < d
C) b < c < a < d D) c < b < d < a
E) a < c < b < d

6. **sin1650° = ?**

A) $\frac{\sqrt{3}}{2}$ B) $\frac{1}{2}$ C) $-\frac{\sqrt{3}}{2}$ D) $-\frac{1}{2}$ E) $-\frac{\sqrt{2}}{2}$

7. $x \in \left(\frac{3\pi}{2}, 2\pi\right)$

$\sin x + \sqrt{3}\cos x = 0 \Rightarrow$ **x = ?**

A) 285° B) 300° C) 315° D) 330° E) 345°

8. $\sin(\alpha - x) = \cos(\alpha + x) \Rightarrow$ **tanα = ?**

A) 1 B) 2 C) 3 D) 4 E) 5

9. $\sin\left(x-\frac{\pi}{2}\right)=\cos x-1 \Rightarrow \mathbf{\sin x\cdot\tan x=?}$

A) $\frac{\sqrt{3}}{2}$ B) $\frac{3}{2}$ C) $\sqrt{3}$ D) 2 E) $2\sqrt{3}$

10. $\frac{\pi}{2}<\alpha<\pi$,

$\tan\alpha=-\frac{2}{3} \Rightarrow \mathbf{\cos\left(\frac{\pi}{2}+\alpha\right)=?}$

A) $-\frac{3}{\sqrt{13}}$ B) $-\frac{\sqrt{3}}{2}$ C) $-\frac{2}{\sqrt{13}}$ D) $\frac{\sqrt{13}}{2}$ E) $\frac{\sqrt{3}}{2}$

11. $\frac{\pi}{2}<x<\pi$,

$\sec x=-\frac{5}{3} \Rightarrow \mathbf{\tan\left(\frac{3\pi}{2}+x\right)=?}$

A) $-\frac{5}{3}$ B) $-\frac{4}{3}$ C) $-\frac{3}{4}$ D) $\frac{1}{5}$ E) $\frac{3}{4}$

12. $\mathbf{\sin^2\frac{7\pi}{18}-\cos^2\frac{\pi}{9}=?}$

A) 1 B) $\frac{\sqrt{2}}{2}$ C) $\frac{\sqrt{3}}{2}$ D) $\frac{1}{2}$ E) 0

13. $\frac{3\pi}{2}<x<2\pi$,

$\tan^2x+\cot^2x=2 \Rightarrow \mathbf{\tan^3x+\cot^3x=?}$

A) 4 B) 2 C) 0 D) – 2 E) – 4

14. $0<x<90°$,

$\sqrt{\frac{1+\cos x}{1-\cos x}}-\sqrt{\frac{1-\cos x}{1+\cos x}}=2\sqrt{3}$

$\Rightarrow \mathbf{x=?}$

A) 15° B) 22,5° C) 30° D) 45° E) 60°

15. $0<x<\frac{\pi}{2}$

$\tan x+\cot x=2\cdot\sec x \Rightarrow \mathbf{x=?}$

A) $\frac{\pi}{3}$ B) $\frac{\pi}{4}$ C) $\frac{\pi}{5}$ D) $\frac{\pi}{6}$ E) $\frac{\pi}{8}$

16. $\pi<x<\frac{3\pi}{2}$

$\Rightarrow \sqrt{\frac{\mathbf{5\cos^2x+\sin^2x-1}}{\mathbf{cosec^2x-1}}}\mathbf{=?}$

A) –2sinx B) –2cosx C) –2 D) 2sinx E) 2cosx

YANITLAR / ANSWERS

1	2	3	4	5	6	7	8
B	D	E	E	B	D	B	A
9	**10**	**11**	**12**	**13**	**14**	**15**	**16**
B	C	E	E	D	C	D	A

1. $0° < x < 90°$

$\tan x = \dfrac{\sqrt{5}}{2} \Rightarrow$ **$\sec x = ?$**

A) $\frac{1}{2}$ B) $\frac{2}{3}$ C) $\frac{3}{2}$ D) 2 E) 3

2. $x \in \left(\pi, \frac{3\pi}{2}\right)$

$\cot x = \frac{1}{2} \Rightarrow$ **$\sin^2 x \cdot \cos x = ?$**

A) $\frac{4}{5}$ B) $\frac{4}{5\sqrt{5}}$ C) $-\frac{4}{5}$ D) $-\frac{4\sqrt{5}}{25}$ E) $-\frac{\sqrt{5}}{5}$

3. $x \in \left(\frac{3\pi}{2}, 2\pi\right)$

$\sec x = \frac{13}{5} \Rightarrow$ **$\sin x + \cos x = ?$**

A) $-\frac{7}{13}$ B) $-\frac{3}{4}$ C) $\frac{7}{13}$ D) $\frac{5}{12}$ E) $\frac{17}{12}$

4. $\frac{\pi}{2} < x < \pi,$

$8\cos x + 15\sin x = 0 \Rightarrow$ **$\sin x + \cos x = ?$**

A) $\frac{8}{15}$ B) $-\frac{5}{13}$ C) $\frac{3}{5}$ D) $-\frac{7}{17}$ E) $-\frac{1}{3}$

5. $x \in \left(\pi, \frac{3\pi}{2}\right),$

$\tan x = \frac{12}{5} \Rightarrow$ **$\sin x \cdot \sec x \cdot \operatorname{cosec} x = ?$**

A) $-\frac{5}{12}$ B) $-\frac{12}{13}$ C) $-\frac{13}{5}$ D) $\frac{5}{13}$ E) $\frac{12}{13}$

6. $\cos 80° = a$

$$\Rightarrow \frac{\cos^2 350°}{\cos 80° + \sin 810°} = ?$$

A) $1 + a$ B) $1 - a$ C) $\frac{a}{a+1}$ D) $\frac{a}{a-1}$ E) $a - \frac{1}{a}$

7.

$$\frac{\dfrac{1}{\sin 190°} + \dfrac{1}{\cos 190°}}{\dfrac{1}{\sin 80°} + \dfrac{1}{\cos 80°}} = ?$$

A) 2 B) −1 C) $-\frac{1}{2}$ D) $\frac{1}{2}$ E) 1

8. $\tan 35° = a \Rightarrow \dfrac{\tan 215° + \tan 215°}{\tan 215° - \tan 315°} = ?$

A) $\frac{a^2}{a^2+1}$ B) $\frac{a-1}{a^2+1}$ C) $\frac{2a}{a+1}$ D) $1-\frac{1}{a^2}$ E) $\frac{1-a^2}{a^2+1}$

9. $\dfrac{\sin(-880°)\cdot\tan(-1650°)}{\cos(-430°)\cdot\cot(-510°)}=?$

A) $\sqrt{3}$ B) $\frac{1}{2}$ C) $\frac{1}{3}$ D) $\frac{2}{\sqrt{3}}$ E) $\frac{1}{6}$

10. $\dfrac{\tan 570°\cdot\cot 150°}{\sec 480°\cdot\operatorname{cosec} 150°}=?$

A) -1 B) $-\frac{1}{2}$ C) $\frac{1}{4}$ D) $\frac{1}{2}$ E) 1

11. $\dfrac{\sin 405°\cdot\tan(-20)°\cdot\tan 675°}{\cot 110°\cdot\cos 235°}=?$

A) $-\frac{\sqrt{3}}{2}$ B) $-\frac{1}{2}$ C) -1 D) 1 E) $\frac{1}{2}$

12. $\dfrac{1}{1-\sin 2x}-\dfrac{1}{1+\sin 2x}=\dfrac{4}{3}$

$x < 45° \Rightarrow$ **x = ?**

A) 10° B) 15° C) 22,5° D) 30° E) 40°

13. $x+y=\frac{\pi}{7}$

$\Rightarrow$ **tan(4x – 3y) + tan(3x + 10y) = ?**

A) –2 B) –1 C) 0 D) 1 E) 2

14. $x\in\left(0,\frac{\pi}{2}\right)$,

3sinx = 4cosx $\Rightarrow$ **cotx + sinx = ?**

A) $\frac{20}{31}$ B) $\frac{31}{20}$ C) $\frac{31}{10}$ D) $\frac{10}{31}$ E) $\frac{5}{31}$

15. $x\in\left(0,\frac{\pi}{2}\right)$,

4sinx = 3tanx $\Rightarrow$ **tanx = ?**

A) 1 B) $\frac{\sqrt{7}}{3}$ C) $\frac{\sqrt{5}}{3}$ D) $\frac{\sqrt{3}}{3}$ E) $\frac{1}{3}$

16. $\dfrac{\cot 1200°\cdot\sin 1800°}{\tan 2260°\cdot\cos 360°}=?$

A) 0 B) $\frac{1}{2}$ C) $\frac{\sqrt{3}}{2}$ D) $-\frac{\sqrt{2}}{2}$ E) $-\frac{\sqrt{3}}{2}$

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1	2	3	4	5	6	7	8
C	D	A	D	C	B	B	C
9	10	11	12	13	14	15	16
C	C	D	B	C	B	B	A

1. $\dfrac{\sin(-430°)\cdot\tan(-410°)}{\cos(-740°)\cdot\cot(-860°)}=?$

A) -1 B) $-\frac{1}{2}$ C) 0 D) $\frac{1}{2}$ E) 1

2. $\dfrac{1+\sin x}{1+\cos x}:\dfrac{1+\operatorname{cosec} x}{1+\sec x}=?$

A) 1 B) sinx C) cosx D) tanx E) cotx

3. $\dfrac{\sin^5x+\cos^2x\cdot\sin^3x}{\sec x\cdot\cot x}=?$

A) $\sin^2x$ B) $\sin^4x$ C) $\cos^2x$ D) $\cos^4x$ E) $\tan^4x$

4. $\dfrac{\cos^4x-\sin^4x}{\cos x+\sin x}:\dfrac{\cos^2x-\cos x\cdot\sin x}{\sin x}=?$

A) tanx B) cotx C) sinx D) cosx E) secx

5. $f(x)=\sin\left(x+\frac{3\pi}{2}\right)+\cos\left(\frac{3\pi}{2}-x\right)-\sin(x-7\pi)$

$\Rightarrow f\left(\frac{2\pi}{3}\right)=?$

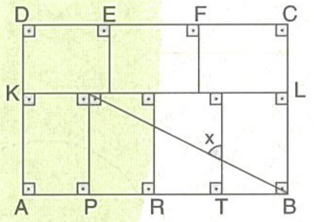
A) $-\frac{\sqrt{3}}{2}$ B) $-\frac{1}{2}$ C) $\frac{1}{2}$ D) $\frac{\sqrt{3}}{2}$ E) 1

6. $f(x)=\cos\left(x-\frac{\pi}{2}\right)+\sin(\pi-x)$

$\Rightarrow f\left(\frac{\pi}{6}\right)=?$

A) $-\sqrt{3}$ B) -1 C) 0 D) 1 E) $\sqrt{3}$

7.



$|DE| = |EF| = |FC| = |AK|$

$|AP| = |PR| = |RT| = |TB| = |DK| \Rightarrow$ **tanx = ?**

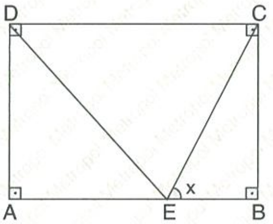
A) $\frac{3}{4}$ B) $\frac{5}{4}$ C) $\frac{7}{4}$ D) $\frac{9}{4}$ E) $\frac{11}{4}$

8. $x\in\left(0,\frac{\pi}{2}\right)$

$\sin x=\frac{3}{5}\ \Rightarrow\ \dfrac{2\tan x}{1-\dfrac{1}{\sec x}}=?$

A) $\frac{5}{2}$ B) $\frac{7}{2}$ C) $\frac{9}{2}$ D) $\frac{13}{2}$ E) $\frac{15}{2}$

9.



|AB| = 10 cm, |AD| = 4 cm,

$m(\widehat{ADE}) = m(\widehat{CEB}) = x \Rightarrow$ **tan x = ?**

A) 1 B) $\sqrt{2}$ C) $\sqrt{3}$ D) 2 E) $2\sqrt{2}$

10. $\dfrac{2-3\cos 2x}{4} = m \Rightarrow$ **? < m < ?**

A) $-\dfrac{1}{4} \le m \le \dfrac{5}{4}$ B) $-\dfrac{5}{4} \le m \le \dfrac{1}{4}$

C) $-2 \le m \le \dfrac{1}{4}$ D) $\dfrac{1}{2} \le m \le \dfrac{3}{2}$

E) $0 \le m \le 4$

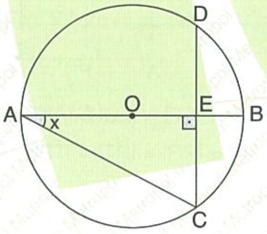
11. O merkez / O is center,

|AB| = 5 cm,

|DE| = 2 cm,

$m(\widehat{BAC}) = x$.

$\Rightarrow$ **sinx = ?**



A) $\dfrac{1}{5}$ B) $\dfrac{2}{5}$ C) $\dfrac{\sqrt{3}}{5}$ D) $\dfrac{4}{5}$ E) $\dfrac{\sqrt{5}}{5}$

12. $x + y \le 90°$,

$\sin x = \dfrac{1}{\sqrt{5}}, \sin y = \dfrac{1}{\sqrt{10}} \Rightarrow$ **x + y = ?**

A) 30° B) 45° C) 60° D) 75° E) 90°

13. $\cos(-6\pi) = 5^{3x-90°}$

$\Rightarrow$ **$\sin\alpha \cdot \cos(x+\alpha) - \sin(x+\alpha) \cdot \cos\alpha$ = ?**

A) $-\dfrac{1}{2}$ B) $-\dfrac{\sqrt{3}}{2}$ C) 0 D) $\dfrac{1}{2}$ E) $\dfrac{\sqrt{2}}{2}$

14. $\sin^2 \dfrac{3\pi}{8} - \cos^4 \dfrac{3\pi}{8} = ?$

A) $-\dfrac{\sqrt{3}}{2}$ B) $-\dfrac{\sqrt{2}}{2}$ C) $-\dfrac{1}{2}$ D) $\dfrac{1}{2}$ E) $\dfrac{\sqrt{2}}{2}$

15. [KF] ⊥ [AC],

[FE] ⊥ [BC],

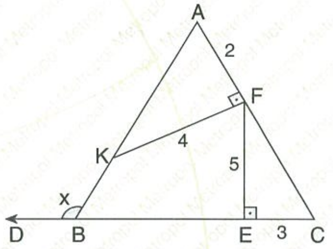
|AF| = 2 br (u),

|KF| = 4 br (u),

|FE| = 5 br (u),

|EC| = 3 br (u),

$m(\widehat{ABD}) = x \Rightarrow$ **tan x = ?**



A) $-\dfrac{11}{3}$ B) $-\dfrac{11}{5}$ C) $-\dfrac{11}{7}$ D) $-\dfrac{11}{9}$ E) – 1

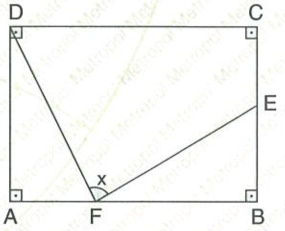
16. |EC| = 1 br (u),

|EB| = 2 br (u),

|AF| = 2 br (u),

|FB| = 4 br (u)

$\Rightarrow$ **tanx = ?**



A) $-\dfrac{2}{3}$ B) $-\dfrac{3}{4}$ C) $-\dfrac{4}{3}$ D) – 6 E) – 8

YANITLAR / ANSWERS

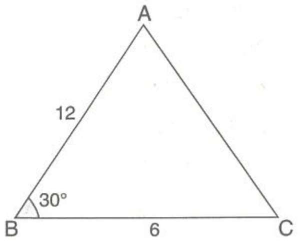
1	2	3	4	5	6	7	8
E	D	B	A	C	D	D	E
9	10	11	12	13	14	15	16
D	A	E	B	A	E	C	E

1. $m(\widehat{ABC}) = 30°$,

$|AB| = 12$ cm,

$|AC| = 6$ cm

$\Rightarrow$ **$A(\widehat{ABE}) = ?\ cm^2$**



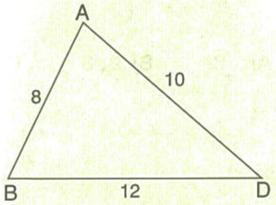
A) 72 B) 54 C) 48 D) 36 E) 18

2. $|AB| = 8$ cm,

$|AD| = 10$ cm,

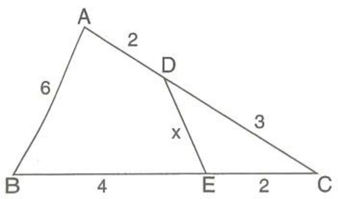
$|BD| = 12$ cm,

$\Rightarrow$ **$\cos\widehat{A} = ?$**



A) $\frac{1}{16}$ B) $\frac{1}{6}$ C) $\frac{1}{8}$ D) $\frac{1}{4}$ E) $\frac{1}{2}$

3.



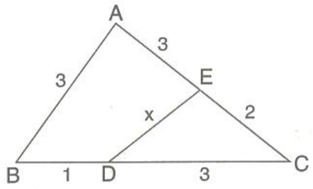
$D \in [AC]$, $E \in [BC]$,

$|AB| = 6$ cm, $|BE| = 4$ cm, $|AD| = |EC| = 2$ cm,

$|DC| = 3$ cm $\Rightarrow$ **$|DE| = x = ?$ cm**

A) 3 B) $2\sqrt{2}$ C) $3\sqrt{3}$ D) $5\sqrt{3}$ E) 8

4.



$|AB| = |AE| = |CD| = 3$ cm, $|CE| = 2$ cm, $|BD| = 1$ cm,

$|DE| = x \Rightarrow$ **$x^2 = ?$ cm**

A) 3 B) $\frac{17}{5}$ C) $\frac{19}{5}$ D) 4 E) 5

5. $[AC] \cap [BD] = \{E\}$,

$|AE| = 12$ cm,

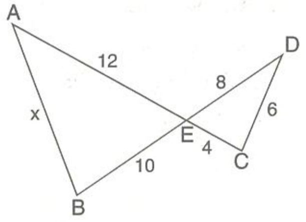
$|EB| = 10$ cm,

$|DE| = 8$ cm,

$|DC| = 6$ cm,

$|EC| = 4$ cm

$\Rightarrow$ **$|AB| = x = ?$**



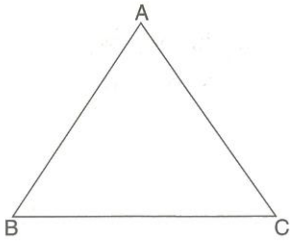
A) $\sqrt{79}$ B) $6\sqrt{2}$ C) $5\sqrt{3}$ D) $\sqrt{39}$ E) $\sqrt{15}$

6. $m(\widehat{B}) = 2 \cdot m(\widehat{C})$,

$|AC| = 15$ cm,

$|AB| = 12$ cm

$\Rightarrow$ **$\cos(\widehat{C}) = ?$**



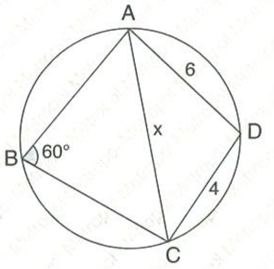
A) $\frac{3}{4}$ B) $\frac{5}{6}$ C) $\frac{5}{8}$ D) $\frac{4}{5}$ E) $\frac{3}{5}$

7. $m(\widehat{ABC}) = 60°$,

$|AD| = 6$ cm,

$|DC| = 4$ cm,

$\Rightarrow$ **$|AC| = x = ?$ cm**



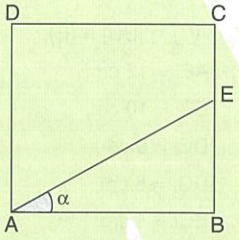
A) $2\sqrt{19}$ B) 8 C) $2\sqrt{7}$ D) $3\sqrt{6}$ E) $\sqrt{38}$

8. ABCD bir karedir.

ABCD is a square.

$A(AECD) = 9 \cdot A(\widehat{ABE})$

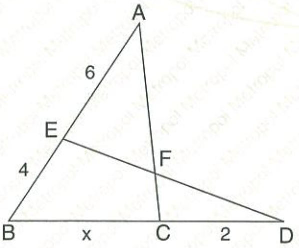
$\Rightarrow$ **$\cos\alpha = ?$**



A) $\frac{5}{13}$ B) $\frac{5}{\sqrt{13}}$ C) $\frac{4}{\sqrt{26}}$ D) $\frac{5}{\sqrt{26}}$ E) $\frac{1}{5}$

9. $A(\widehat{ABC}) = 2A(\widehat{BDE})$

$\Rightarrow$ **$|BC| = x = ?$ cm**



A) 1 B) 2 C) 3 D) 4 E) 8

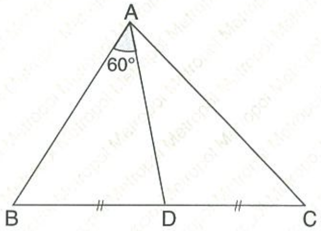
10. $|BD| = |DC|$,

$|AB| = \sqrt{18}$ cm,

$|AC| = \sqrt{27}$ cm,

$m(\widehat{BAD}) = 60°$

$\Rightarrow$ **$m(\widehat{BAC}) = ?$**



A) 90° B) 105° C) 120° D) 135° E) 150°

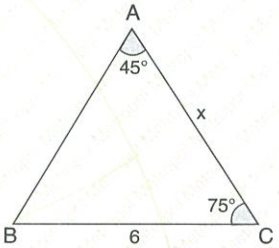
11. $m(\widehat{A}) = 45°$,

$m(\widehat{C}) = 75°$,

$|BC| = 6$ cm,

$|AC| = x$ cm

$\Rightarrow$ **$x = ?$**



A) $\sqrt{3}$ B) $2\sqrt{3}$ C) $3\sqrt{3}$ D) $2\sqrt{6}$ E) $3\sqrt{6}$

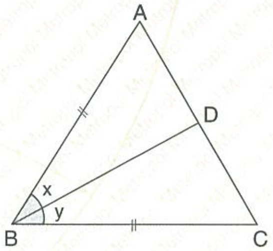
12. $m(\widehat{ABD}) = x$,

$m(\widehat{DBC}) = y$,

$|AB| = |BC|$,

$3 \cdot |AD| = 4 \cdot |DC|$

$\Rightarrow$ **$\frac{\sin x}{\sin y} = ?$**



A) $\frac{4}{7}$ B) $\frac{3}{5}$ C) $\frac{4}{5}$ D) $\frac{4}{3}$ E) 1

YANITLAR / ANSWERS

1	2	3	4	5	6	7	8	9	10	11	12
E	C	B	B	A	C	A	D	E	B	E	D

BÖLÜM CHAPTER 21

TARİHSEL NOT / HISTORICAL NOTE

SOFİA VASİLYEVNA KOVALEVSKAYA [1850 - 1891]

İlk büyük kadın Rus matematikçisidir. Analiz, diferansiyel denklemler ve mekanik alanlarına birçok orijinal katkıda bulunmuştur. Kuzey Avrupa'da ilk kez tam profesörlük alan kadındır. Ayrıca bilimsel bir dergide editör olarak çalışan ilk kadınlardandır.

The first Russian female mathematician. She made many original contributions to the analysis, differential equations and mechanical fields. She is the first woman to receive a full professorship in northern Europe. She is also one of the first women to work as an editor in a scientific journal.

TRİGONOMETRİ II / TRIGONOMETRY II

Bu bölüm, 183 test sorusu içermektedir.
This chapter includes 183 test questions.

BÖLÜM / CHAPTER 21

TRİGONOMETRİ II / TRIGONOMETRY II

- Trigonometri II / Trigonometry II .... 373 – 404

BÖLÜM

21

CHAPTER

TRİGONOMETRİ II
TRIGONOMETRY II

TOPLAM ve FARK FORMÜLLERİ / SUM and DIFFERENCE FORMULAS

- $\sin(a+b) = \sin a \cdot \cos b + \cos a \cdot \sin b$
- $\sin(a-b) = \sin a \cdot \cos b - \cos a \cdot \sin b$
- $\cos(a+b) = \cos a \cdot \cos b - \sin a \cdot \sin b$
- $\cos(a-b) = \cos a \cdot \cos b + \sin a \cdot \sin b$
- $\tan(a+b) = \dfrac{\tan a + \tan b}{1 - \tan a \cdot \tan b}$
- $\tan(a-b) = \dfrac{\tan a - \tan b}{1 + \tan a \cdot \tan b}$
- $\cot(a+b) = \dfrac{\cot a \cdot \cot b - 1}{\cot b + \cot a}$
- $\cot(a-b) = \dfrac{\cot a \cdot \cot b + 1}{\cot b - \cot a}$

YARIM AÇI FORMÜLLERİ / HALF ANGLE FORMULAS

- 1. toplam fark formülünde a = b alınırsa,

  If we take a = b in the sum and difference formula 1,

  $\sin(a+a) = \sin a \cdot \cos a + \cos a \cdot \sin a$

  $\boldsymbol{\sin 2a = 2 \cdot \sin a \cdot \cos a}$

- 3. toplam fark formulünde a = b alınırsa,

  If we take a = b in the sum and difference formula 3,

  $\cos(a+a) = \cos a \cdot \cos a - \sin a \cdot \sin a$

  $\boldsymbol{\cos 2a = \cos^2 a - \sin^2 a}$

- $\cos^2 a = 1 - 2\sin^2 a$ olduğundan, $\cos 2a = 1 - \sin^2 a - \sin^2 a$

  Since $\cos^2 a = 1 - \sin^2 a$ we get $\cos 2a = 1 - \sin^2 a - \sin^2 a = 1 - 2\sin^2 a$

  $\boldsymbol{\cos 2a = 1 - 2\sin^2 a}$

- $\sin^2 a = 1 - \cos^2 a$ olduğundan, $\cos 2a = \cos^2 a - (1 - \cos^2 a)$

  Since $\sin^2 a = 1 - \cos^2 a$, we get $\cos 2a = \cos^2 a - (1 - \cos^2 a) = 2\cos^2 a - 1$.

  $\boldsymbol{\cos 2a = 2\cos^2 a - 1}$

▼ 5. toplam fark formulünde a = b alınırsa,

If we take a = b in the sum and difference formula 5.

$$\tan(a+a)=\frac{\tan a+\tan a}{1-\tan a\cdot\tan a}$$

$$\boldsymbol{\tan 2a=\frac{2\cdot\tan a}{1-\tan^2 a}}$$

▼ 7. toplam fark formulünde a = b alınırsa,

If we take a = b in the sum and difference formula 7,

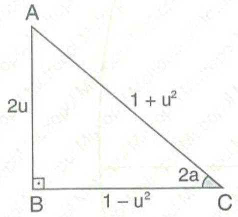
$$\cot(a+a)=\frac{\cot a\cdot\cot a-1}{\cot a+\cot a}$$

$$\boldsymbol{\cot 2a=\frac{\cot^2 a-1}{2\cdot\cot a}}$$

Önemli / Important

- $\tan 2a=\dfrac{2\cdot\tan a}{1-\tan^2 a}$

  $\tan a=u \Rightarrow \mathbf{\tan 2a}=\dfrac{2u}{1-u^2}$



$$\mathbf{\sin 2a}=\frac{2u}{1+u^2}\Rightarrow \sin 2a=\frac{2\cdot\tan a}{1+\tan^2 a}$$

$$\mathbf{\cos 2a}=\frac{1-u^2}{1+u^2}\Rightarrow \cos 2a=\frac{1-\tan^2 a}{1+\tan^2 a}$$

DÖNÜŞÜM FORMÜLLERİ / CONVERSION FORMULAS

▼ $\sin a+\sin b=2\cdot\sin(\frac{a+b}{2})\cdot\cos(\frac{a-b}{2})$

▼ $\sin a-\sin b=2\cdot\sin(\frac{a-b}{2})\cdot\cos(\frac{a+b}{2})$

▼ $\cos a+\cos b=2\cdot\cos(\frac{a+b}{2})\cdot\cos(\frac{a-b}{2})$

▼ $\cos a-\cos b=-2\cdot\sin(\frac{a+b}{2})\cdot\sin(\frac{a-b}{2})$

▼ $\tan a+\tan b=\dfrac{\sin(a+b)}{\cos a\cdot\cos b}$

▼ $\tan a - \tan b = \dfrac{\sin(a-b)}{\cos a \cdot \cos b}$

▼ $\cot a + \cot b = \dfrac{\sin(a+b)}{\sin a \cdot \sin b}$

▼ $\cot a - \cot b = \dfrac{-\sin(a-b)}{\sin a \cdot \sin b}$

Önemli / Important

- $\sin(x+y) = \sin x \cdot \cos y + \sin y \cdot \cos x$

$$\underline{+\ \sin(x-y) = \sin x \cdot \cos y - \sin y \cdot \cos x}$$

$\sin(x+y) + \sin(x-y) = 2\sin x \cdot \cos y$

$x + y = a$

$x - y = b$

$x = \dfrac{a+b}{2}$, $y = \dfrac{a-b}{2}$

$\Rightarrow \mathbf{\sin a + \sin b = 2 \cdot \sin\left(\dfrac{a+b}{2}\right) \cdot \cos\left(\dfrac{a-b}{2}\right)}$

Önemli / Important

- $\dfrac{x+y}{2} = z$

$\Rightarrow \dfrac{\sin x + \sin z + \sin y}{\cos x + \cos z + \cos y} = \tan z$

$\Rightarrow \dfrac{\cos x + \cos z + \cos y}{\sin x + \sin z + \sin y} = \cot z$

TERS DÖNÜŞÜM FORMÜLLERİ / INVERSE CONVERSION FORMULAS

▼ $\sin a \cdot \cos b = \dfrac{1}{2} \cdot [\sin(a+b) + \sin(a-b)]$

▼ $\cos a \cdot \cos b = \dfrac{1}{2} \cdot [\cos(a+b) + \cos(a-b)]$

▼ $\sin a \cdot \sin b = -\dfrac{1}{2} \cdot [\cos(a+b) - \cos(a-b)]$

▼ $\cos a \cdot \sin b = \dfrac{1}{2} \cdot [\sin(a+b) - \sin(a-b)]$

Önemli / Important

- $\cos(a+b) = \cos a \cdot \cos b - \sin a \cdot \sin b$

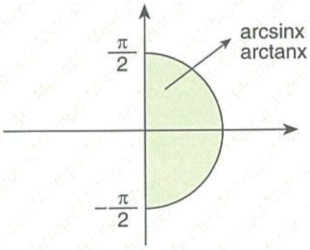
$$\begin{array}{l} \cos(a-b) = \cos a \cdot \cos b + \sin a \cdot \sin b \\ + \\ \hline \cos(a+b) + \cos(a-b) = 2\cos a \cdot \cos b \end{array}$$

$$\Rightarrow \mathbf{\cos a \cdot \cos b = \frac{1}{2} \cdot [\cos(a+b) + \cos(a-b)]}$$

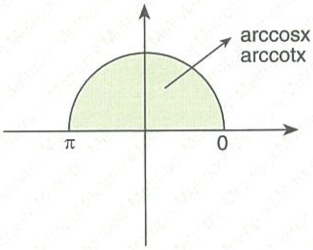
TERS TRİGONOMETRİK FONKSİYONLAR / INVERSE TRIGONOMETRIC FUNCTIONS

- ▼ $y = \arcsin x = \sin^{-1}x \Leftrightarrow x = \sin y$
  $x \in [-1, 1]$ ve/and $y \in [-\frac{\pi}{2}, \frac{\pi}{2}]$
- ▼ $y = \arccos x = \cos^{-1}x \Rightarrow x = \cos y$
  $x \in [-1, 1]$ ve/and $y \in [0, \pi]$
- ▼ $y = \arctan x = \tan^{-1}x \Leftrightarrow x = \tan y$
  $x \in R$ ve/and $y \in (-\frac{\pi}{2}, \frac{\pi}{2})$
- ▼ $y = \text{arccot}\, x = \cot^{-1}x \Rightarrow x = \cot y$
  $x \in R$ ve/and $y \in (0, \pi)$

▼



▼



Önemli / Important

- I. arcsin(sinx) = x
  II. arccos(cosx) = x
  III. arctan(tanx) = x
  IV. arccot(cotx) = x

- I. sin(arcsinx) = x
  II. cos(arccosx) = x
  III. tan(arctanx) = x
  IV. cot(arccotx) = x

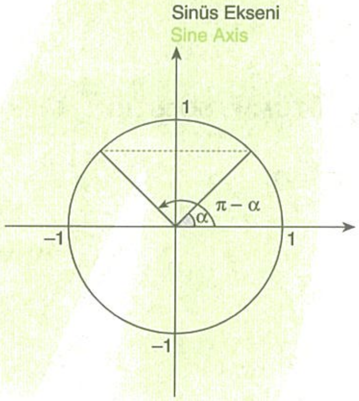
TRİGONOMETRİK DENKLEMLER / TRIGONOMETRIC EQUATIONS

sinx = sinα DENKLEMİNİN ÇÖZÜMÜ / SOLUTION of the EQUATION sinx = sinα

▼

$$\left.\begin{array}{l}\sin x = \sin\alpha \\ \sin x = \sin(\pi-\alpha)\end{array}\right\} \Rightarrow x_1 = \alpha,\ x_2 = \pi - \alpha$$

$$\sin x = \sin\alpha \Rightarrow \begin{cases} x_1 = \alpha + k\cdot 2\pi, & k \in Z \\ x_2 = (\pi - \alpha) + k\cdot 2\pi, & k \in Z \end{cases}$$

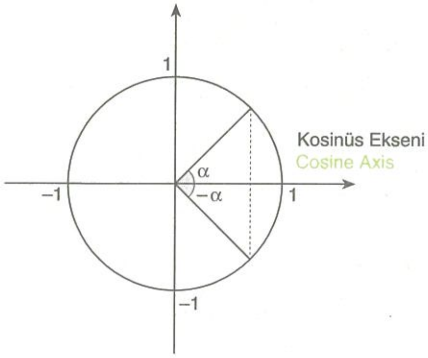


cosx = cosα DENKLEMİNİN ÇÖZÜMÜ / SOLUTION of the EQUATION cosx = cosα

▼

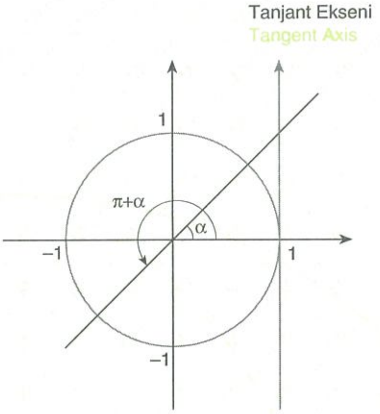
$$\left.\begin{array}{l}\cos x = \cos\alpha \\ \cos x = \cos(-\alpha)\end{array}\right\} \Rightarrow x_1 = \alpha,\ x_2 = -\alpha$$

$$\cos x = \cos\alpha \Rightarrow \begin{cases} x_1 = \alpha + k\cdot 2\pi, & k \in Z \\ x_2 = -\alpha + k\cdot 2\pi, & k \in Z \end{cases}$$



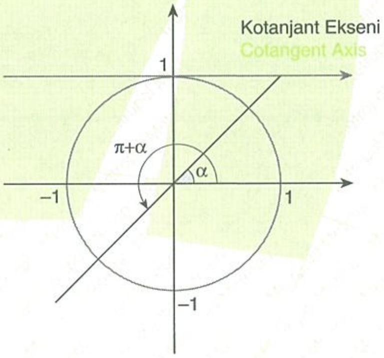
tanx = tanα DENKLEMİNİN ÇÖZÜMÜ / SOLUTION of the EQUATION tanx = tanα

▼ $\tan x = \tan\alpha \Rightarrow x = \alpha + k\cdot\pi, \ (k \in Z)$


Tanjant Ekseni
Tangent Axis
1
π+α
α
−1
1
−1

cotx = cotα DENKLEMİNİN ÇÖZÜMÜ / SOLUTION of the EQUATION cotx = cotα

▼ $\cot x = \cot\alpha \Rightarrow x = \alpha + k\cdot\pi, \ (k \in Z)$


Kotanjant Ekseni
Cotangent Axis
1
π+α
α
−1
1
−1

1. $\sin 105° = ?$

A) $\dfrac{\sqrt{3}-1}{2}$ B) $\sqrt{3}-2$ C) $\dfrac{\sqrt{6}-\sqrt{2}}{4}$

D) $2+\sqrt{3}$ E) $\dfrac{\sqrt{6}+\sqrt{2}}{4}$

2. $\cos 75° \cdot \cos 15° + \sin 75° \cdot \sin 15° = ?$

A) $-\dfrac{1}{2}$ B) -1 C) 0 D) $\dfrac{1}{2}$ E) 1

3. $\sin 35° \cdot \cos 85° + \cos 35° \cdot \sin 85° + \cos 510° = ?$

A) $-\dfrac{\sqrt{3}}{2}$ B) $-\dfrac{1}{2}$ C) 0 D) $\dfrac{1}{2}$ E) $\dfrac{\sqrt{3}}{2}$

4. $\cos 40° \cdot \sin 70° - \sin 20° \cdot \cos 50° = A \Rightarrow A = ?$

A) -1 B) 0 C) $\dfrac{1}{4}$ D) $\dfrac{1}{2}$ E) 1

5. $A - B = 60°$

$\Rightarrow (\sin A + \sin B)^2 + (\cos A + \cos B)^2 = ?$

A) 2 B) 3 C) 4 D) 5 E) 6

6. $x + y = \dfrac{4\pi}{3}$

$\Rightarrow (\sin x + \cos y)^2 + (\sin y + \cos x)^2 - 2 = ?$

A) $\dfrac{\sqrt{3}}{2}$ B) $\sqrt{2}$ C) $\sqrt{3}$ D) $-\sqrt{3}$ E) $-\sqrt{2}$

7. $\tan(\alpha - 45°) = \dfrac{1}{3} \Rightarrow \tan\alpha = ?$

A) $\dfrac{2}{3}$ B) $\dfrac{3}{4}$ C) 1 D) 2 E) 3

8. $\sin 10° + \tan 40° \cdot \cos 10° = ?$

A) $\dfrac{1}{4}$ B) $\dfrac{1}{3}$ C) $\dfrac{1}{2}$ D) 1 E) 2

9. $\sin(45°-x)\cdot\cos(45°+x)+\cos(45°-x)\cdot\sin(45°+x)=?$

A) 0 B) $\frac{1}{2}$ C) $\frac{\sqrt{2}}{2}$ D) $\sqrt{2}$ E) 1

10. $\cos 50° + \sin 50° \cdot \tan 25° = ?$

A) $\frac{1}{2}$ B) 1 C) 2 D) $\sin^2 50$ E) $\cos^2 50$

11. $0 < x < \frac{\pi}{2}$

$\frac{2\cdot\tan x}{1-\tan^2 x}=1 \Rightarrow$ **x = ?**

A) $\frac{\pi}{8}$ B) $\frac{\pi}{6}$ C) $\frac{\pi}{4}$ D) $\frac{\pi}{3}$ E) $\frac{5\pi}{12}$

12. $\sin x + \cos x = \frac{5}{4} \Rightarrow |\cos x - \sin x| = ?$

A) $\frac{3}{4}$ B) $\frac{4}{5}$ C) $\frac{\sqrt{7}}{2}$ D) $\frac{2}{7}$ E) $\frac{\sqrt{7}}{4}$

13. $x+y+z=\pi$

$$\left.\begin{array}{l}\sin y=\frac{3}{5}\\ \cos x=\frac{5}{13}\end{array}\right\} \Rightarrow \tan z = ?$$

A) $-\frac{63}{16}$ B) $-\frac{57}{16}$ C) $\frac{49}{16}$ D) $\frac{57}{16}$ E) $\frac{63}{16}$

14. $x+y+z=\pi$

$\Rightarrow$ **$\cos x\cdot\cos y-\sin x\cdot\sin y+\cos z=?$**

A) -1 B) $-\frac{1}{2}$ C) 0 D) $\frac{1}{2}$ E) 1

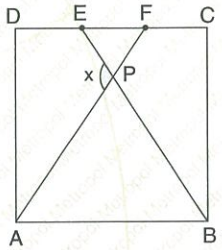
15. ABCD bir karedir.

ABCD is a square.

$|DE| = |EF| = |FC|$,

$m(\widehat{EPA}) = x$

$\Rightarrow$ **tanx = ?**



A) $\frac{-12}{5}$ B) $\frac{-9}{2}$ C) $\frac{5}{12}$ D) $\frac{2}{9}$ E) $\frac{5}{6}$

16. $[AB] \perp [BC]$,

$|AC| = 20$ br (u),

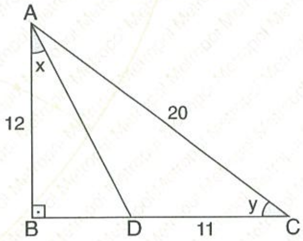
$|DC| = 11$ br (u),

$|AB| = 12$ br (u),

$m(\widehat{BAD}) = x$,

$m(\widehat{ACB}) = y$

$\Rightarrow \frac{\cot 2x}{\cos 2y} = ?$



A) $\frac{85}{24}$ B) $\frac{95}{24}$ C) $\frac{75}{16}$ D) $\frac{85}{16}$ E) $\frac{65}{12}$

YANITLAR / ANSWERS

1	2	3	4	5	6	7	8
E	D	C	D	B	D	D	D
9	10	11	12	13	14	15	16
E	B	A	E	E	C	A	A

1. $x = \frac{\pi}{12}$

$\Rightarrow$ **sin2x · cos2x · cos4x = ?**

A) $\frac{1}{8}$ B) $\frac{\sqrt{2}}{16}$ C) $\frac{\sqrt{2}}{8}$ D) $\frac{\sqrt{3}}{8}$ E) $-\frac{\sqrt{3}}{8}$

2. $\frac{\sin 4x}{2\sin 2x} = ?$

A) sin2x B) 2·sinx C) sin4x
D) cos2x E) cos4x

3. $\frac{\sin 40°}{\cos 70° \cdot \cos 20°} = ?$

A) –4 B) 4 C) 2sin20°
D) 2cos20° E) 2

4. $0 < \theta < \frac{\pi}{2}$

$\tan\theta = \frac{3}{4} \Rightarrow$ **$\sin 2\theta = ?$**

A) $\frac{9}{16}$ B) $\frac{16}{9}$ C) $\frac{24}{25}$ D) $\frac{25}{24}$ E) 1

5. $\frac{\sin 7{,}5° \cdot \cos 7{,}5° \cdot \cos 15°}{\cos 22{,}5° \cdot \sin 22{,}5°} = ?$

A) $\sqrt{2}$ B) $\frac{\sqrt{2}}{2}$ C) $\frac{\sqrt{2}}{3}$ D) $\frac{\sqrt{2}}{4}$ E) $\frac{\sqrt{2}}{6}$

6. $\frac{\sin 36° \cdot \cos 36° \cdot \cos 72°}{\cos 54°} = ?$

A) $\frac{3}{8}$ B) $\frac{2}{3}$ C) $\frac{1}{2}$ D) $\frac{1}{3}$ E) $\frac{1}{4}$

7. $\left.\begin{matrix}\sin 18° = x \\ \cos 18° = y\end{matrix}\right\} \Rightarrow$ **$\cos 54° = ?$**

A) $\frac{y}{x}$ B) 2xy C) x – y D) xy E) x

8. $\sin 8° = m \Rightarrow$ **$\sin 74° = ?$**

A) $1 + 2m^2$ B) $1 - 2m^2$ C) $m^2 + 1$
D) $2m^2 - 1$ E) $-2m^2 - 1$

9. $\cos\alpha - \sin\alpha = \frac{2}{3} \Rightarrow$ **$\sin 2\alpha = ?$**

A) $\frac{1}{4}$ B) $\frac{3}{4}$ C) $\frac{4}{7}$ D) $\frac{5}{9}$ E) $\frac{4}{5}$

10. $x \in \left(\pi, \frac{3\pi}{2}\right)$,

$\tan x = \frac{1}{2} \Rightarrow$ **$\sin 2x = ?$**

A) $\frac{3}{5}$ B) $\frac{3}{4}$ C) $\frac{4}{5}$ D) $\frac{8}{9}$ E) $\frac{9}{10}$

11. $\cos x = -\frac{1}{3} \Rightarrow$ **$\cos 2x = ?$**

A) $-\frac{7}{9}$ B) $-\frac{2}{9}$ C) 0 D) $\frac{1}{9}$ E) $\frac{2}{9}$

12. $\sin 2x = \frac{1}{5} \Rightarrow$ **$|\sin x - \cos x| = ?$**

A) $\frac{\sqrt{6}}{3}$ B) $\frac{\sqrt{6}}{5}$ C) $\frac{2\sqrt{5}}{5}$ D) $\frac{\sqrt{5}}{5}$ E) $\frac{2\sqrt{5}}{3}$

13. $\frac{\cos 42^\circ}{\cos x} - \frac{\sin 42^\circ}{\sin x} = \frac{1}{\sin 2x}$

$\Rightarrow x = ?$

A) 30° B) 36° C) 45° D) 60° E) 72°

14. $\cot 32^\circ = x \Rightarrow$ **$\frac{\cos 48^\circ}{\sin 74^\circ} + \frac{\sin 48^\circ}{\sin 16^\circ} = ?$**

A) $\sqrt{\frac{x^2+1}{2}}$ B) $2\cdot\sqrt{\frac{x+1}{2}}$ C) $3\cdot\sqrt{\frac{x^2+1}{2}}$ D) $\frac{4x}{\sqrt{x^2+1}}$ E) $8\cdot\sqrt{\frac{x+1}{2}}$

15. $\cot 24^\circ = x \Rightarrow$ **$\frac{\cos 48^\circ - 1}{\cos^2 24^\circ} = ?$**

A) $\frac{-2}{x^2}$ B) $\frac{-1}{x^2}$ C) $-x^2$ D) $\frac{1}{x^2}$ E) $\frac{2}{x^2}$

16. $(2\cos^2 12^\circ - 1)^2 - \cos^2 66^\circ = x$

$\Rightarrow \sin 24^\circ = ?$

A) $2x$ B) $2\sqrt{1-x}$ C) $2x\sqrt{1-x}$ D) $\sqrt{\frac{1-x}{2}}$ E) $2\sqrt{1-x^2}$

YANITLAR / ANSWERS

1	2	3	4	5	6	7	8
D	D	E	C	D	E	B	B
9	**10**	**11**	**12**	**13**	**14**	**15**	**16**
D	C	A	C	E	D	A	D

1. $A = \sin 15° \cdot \cos 15° \Rightarrow$ **A = ?**

A) $\frac{1}{2}$ B) $\frac{1}{4}$ C) $\frac{1}{8}$ D) $\frac{1}{16}$ E) $\frac{\sqrt{2}}{4}$

2. $\cos 26° = m \Rightarrow$ **$\sin 148° \cdot \cos 32° = ?$**

A) $\frac{m}{4}$ B) $\frac{m}{2}$ C) $\frac{m}{3}$ D) m E) $\frac{m^2}{2}$

3. $\sin\frac{\pi}{5}\cdot\cos\frac{2\pi}{15}+\cos\frac{\pi}{5}\cdot\sin\frac{2\pi}{15}=?$

A) $\frac{1}{4}$ B) $\frac{1}{2}$ C) $\frac{\sqrt{2}}{2}$ D) $\frac{\sqrt{3}}{2}$ E) 1

4. $\sin 18° = a \Rightarrow$ **$\sin 54° = ?$**

A) $1 - 2a^2$ B) $1 - a^2$ C) $1 + a^2$ D) $3 - a^2$ E) $2 - a^2$

5. $\frac{\sin 12x}{\sin 4x}-\frac{\cos 12x}{\cos 4x}=?$

A) 2 B) sin8x C) 1 D) –sinx E) –2

6. $a \in \left(0, \frac{\pi}{2}\right)$

$\sin a = k \Rightarrow$ **$\sin 2a = ?$**

A) $\frac{k}{\sqrt{1-k^2}}$ B) $\frac{-2k}{\sqrt{1+k^2}}$ C) $2k\sqrt{1-k^2}$ D) $k\sqrt{1+k^2}$ E) $\frac{\sqrt{1+k^2}}{2k}$

7. $\tan x = 3 \Rightarrow$ **$\cos 2x = ?$**

A) $-\frac{3}{5}$ B) $-\frac{4}{5}$ C) $\frac{1}{3}$ D) $\frac{4}{5}$ E) $\frac{3}{4}$

8. $\frac{2\cdot\cos^2 25°-1}{2\cdot\tan 20°\sin^2 70°}=?$

A) $\frac{1}{4}$ B) $\frac{1}{2}$ C) 1 D) $\frac{\sqrt{3}}{2}$ E) $\frac{\sqrt{3}}{4}$

9. cos22° = m,

$\Rightarrow \dfrac{\cos 33^\circ}{\cos 11^\circ} + \dfrac{\sin 33^\circ}{\sin 11^\circ} = ?$

A) m B) 2m C) 3m D) 4m E) 5m

10. $\dfrac{1}{\sin\frac{\pi}{8}} + \dfrac{1}{\cos\frac{\pi}{8}} = ?$

A) $\sqrt{\sqrt{2}-1}$ B) $\sqrt{2+\sqrt{2}}$ C) $2\sqrt{2-\sqrt{2}}$

D) $2\sqrt{1+\sqrt{2}}$ E) $2\sqrt{2+\sqrt{2}}$

11. $\cos^4x - \sin^4x = ?$

A) sinx B) cosx C) sinx + cosx

D) sin2x E) cos2x

12. $\cos^4 15^\circ - \sin^4 15^\circ = ?$

A) $\dfrac{\sqrt{3}}{2}$ B) $\dfrac{1}{2}$ C) $\dfrac{1}{\sqrt{3}}$ D) $\dfrac{1}{3}$ E) 1

13. $\sin 74^\circ = x \Rightarrow \dfrac{\sin 24^\circ}{\sin 8^\circ} + \dfrac{\cos 24^\circ}{\cos 8^\circ} = ?$

A) x + 2 B) x + 4 C) 2x

D) 3x E) 4x

14. $\dfrac{\pi}{2} < x < \pi$

sin2x = cosx ⇒ **x = ?**

A) $\dfrac{3\pi}{4}$ B) $\dfrac{2\pi}{3}$ C) $\dfrac{\pi}{6}$ D) $\dfrac{7\pi}{6}$ E) $\dfrac{5\pi}{6}$

15. $\dfrac{\cos^4 15^\circ - \sin^4 15^\circ}{\sin 60^\circ} = ?$

A) $\dfrac{1}{2}$ B) 1 C) $\dfrac{\sqrt{3}}{2}$ D) $\sqrt{3}$ E) $\dfrac{2\sqrt{3}}{3}$

16. $\pi < 2x < \dfrac{3\pi}{2}$,

sinx + cosx = $\dfrac{1}{2}$ ⇒ **tan2x = ?**

A) $\dfrac{1}{\sqrt{5}}$ B) $\dfrac{1}{\sqrt{6}}$ C) $\dfrac{3}{\sqrt{6}}$ D) $\dfrac{1}{\sqrt{7}}$ E) $\dfrac{3}{\sqrt{7}}$

YANITLAR / ANSWERS

1	2	3	4	5	6	7	8
B	B	D	A	A	C	B	C
9	**10**	**11**	**12**	**13**	**14**	**15**	**16**
D	E	E	A	E	E	B	E

1. $\sin x \cdot \cos x = m$

$\Rightarrow \cos^2\left(\frac{\pi}{4} - x\right) - \sin^2\left(\frac{\pi}{4} - x\right) = ?$

A) $\frac{m}{2}$ B) m C) $2m$ D) m^2 E) $2 + m$

2. $0 < x < \frac{\pi}{4}$,

$\sin 2x = \frac{1}{3} \Rightarrow \cot x = ?$

A) $3 - 2\sqrt{2}$ B) $3 + 2\sqrt{2}$ C) $2 + \sqrt{3}$ D) $2 - \sqrt{3}$ E) $3 + \sqrt{3}$

3. $4 \cdot \sin^2 \frac{\pi}{8} = ?$

A) $1 - \sqrt{2}$ B) $1 + \sqrt{2}$ C) $2\sqrt{2} - 1$ D) $2 - \sqrt{2}$ E) $\sqrt{2} + 2$

4. $\frac{2 \cdot \sin^2 15 - 1}{\sin 75 \cdot \cos 75} = ?$

A) $-\frac{2\sqrt{3}}{3}$ B) $-\frac{\sqrt{3}}{3}$ C) $-\sqrt{3}$ D) $\frac{\sqrt{3}}{3}$ E) $-2\sqrt{3}$

5. $x \in \left(0, \frac{\pi}{2}\right)$,

$2\cot x \cdot \sin 2x = 4 - \sin 2x \Rightarrow \tan 2x = ?$

A) $\frac{5}{3}$ B) $\frac{4}{3}$ C) $\frac{3}{4}$ D) $\frac{3}{5}$ E) $\frac{2}{3}$

6. $\frac{1 - \sin^2 a - \cos^2 a + \sin 2a}{\cos 2a + 1} = ?$

A) cota B) tana C) 1 − cota D) $\frac{1}{2}\sin a$ E) $\frac{1}{2}\cot a$

7. $\frac{1}{\tan\theta} - \frac{1}{\cot\theta} = ?$

A) $2\cos 2\theta$ B) $\sin 2\theta$ C) $2\tan\theta$ D) $2\cot 2\theta$ E) $\tan 2\theta$

8. $\frac{\sin 6x}{\sin 2x} + \frac{\cos 6x}{\cos 2x} = 1 \Rightarrow \cos^2 2x = ?$

A) $\frac{1}{2}$ B) $\frac{1}{3}$ C) $\frac{3}{4}$ D) $\frac{3}{8}$ E) $\frac{5}{8}$

9. $5x=\pi \Rightarrow \mathbf{\dfrac{\sin 3x}{\sin 2x}-\dfrac{\cos 3x}{\cos 2x}=?}$

A) $\frac{1}{3}$ B) $\frac{1}{2}$ C) 1 D) 2 E) 3

10. $\left(\dfrac{\sin 5x}{\sin 3x}+\dfrac{\cos 5x}{\cos 3x}\right)\cdot\left(\dfrac{\sin 4x}{\sin 2x}+\dfrac{\cos 4x}{\cos 2x}\right)=4$

$\Rightarrow \mathbf{\cos 4x = ?}$

A) $\frac{1}{8}$ B) $\frac{1}{6}$ C) $\frac{1}{4}$ D) $\frac{1}{3}$ E) $\frac{1}{2}$

11. $x\in\left(\dfrac{\pi}{2},\pi\right)$,

$\dfrac{2\sin x-\sin 2x}{2\sin x+\sin 2x}=3 \Rightarrow \mathbf{\tan x=?}$

A) – 1 B) $-\frac{\sqrt{3}}{3}$ C) $-\sqrt{3}$ D) 1 E) $\sqrt{3}$

12. $B=\left(\dfrac{\cos^4 x-\sin^4 x}{\sin 4x}\right)^2+\sec 4x$,

$x=\dfrac{\pi}{12} \Rightarrow \mathbf{B=?}$

A) 5 B) 3 C) $\frac{7}{3}$ D) 2 E) $\frac{5}{3}$

13. $\cos 10° = a \Rightarrow \mathbf{\cos 40° - \sin 40° = ?}$

A) $\sqrt{a}$ B) $\sqrt{1-a}$ C) $\frac{a^2+1}{2}$

D) $\frac{a^2}{a+1}$ E) $\sqrt{a+1}$

14. $\sin 20° = m \Rightarrow \mathbf{\cos 10° - \sin 10° = ?}$

A) m + 1 B) $\sqrt{1-m}$ C) m – 1

D) $\frac{3}{m}$ E) $\frac{3}{2}$

15. $\dfrac{\sec x+\sin x}{\operatorname{cosec} x+\cos x}=2 \Rightarrow \mathbf{\tan 2x=?}$

A) $-\frac{4}{3}$ B) $-\frac{3}{4}$ C) $\frac{3}{4}$ D) $\frac{4}{3}$ E) 4

16. $\dfrac{1}{\cos 80°}-\dfrac{\sqrt{3}}{\sin 80°}=A \Rightarrow \mathbf{A=?}$

A) 4 B) 3 C) 2 D) 1 E) $\frac{1}{4}$

YANITLAR / ANSWERS

1	2	3	4	5	6	7	8
C	B	D	E	B	B	D	E
9	**10**	**11**	**12**	**13**	**14**	**15**	**16**
D	E	C	B	B	B	A	A

1. $\cos 18° = a \Rightarrow \dfrac{\sin 27°}{\sin 9°} + \dfrac{\cos 27°}{\cos 9°} = ?$

A) 1 B) a C) 2a D) 4a E) 9a

2. $\sin 7° = a \Rightarrow \cos 166° = ?$

A) 2a B) 1 – 2a C) $1 - a^2$ D) $1 - 2a^2$ E) $2a^2 - 1$

3. $2\cdot(\operatorname{cosec} 2x + \cot 2x) = ?$

A) cot2x B) tan2x C) $2\tan\dfrac{x}{2}$ D) 2cotx E) tanx

4. $0 < x < \dfrac{\pi}{2}$

$\Rightarrow \dfrac{2\cdot\sqrt{1-\cos 2x}}{\sin 2x} = ?$

A) $\sqrt{2}\cdot\sec x$ B) $\sqrt{2}\cdot\operatorname{cosec} x$ C) $2\sqrt{2}\cdot\sec x$ D) $-\sqrt{2}\cdot\sec x$ E) $-\sqrt{2}\cdot\operatorname{cosec} x$

5. $0 < x < \dfrac{\pi}{4} \Rightarrow \sqrt{1-\sin 2x} = ?$

A) sinx + cosx B) sinx – cosx C) cosx – sinx D) 1 – sinx E) –sinx – cosx

6. $\left(\dfrac{3+\cos 2x+4\cos x}{2\cdot\sin^2 x}\right)\cdot(1-\cos x) = ?$

A) sinx B) cosx C) 1 + sinx D) 1 + cosx E) 1 – cosx

7. $\sin x - \cos x = \dfrac{1}{3} \Rightarrow \sin^4 x + \cos^4 x = ?$

A) $\dfrac{16}{81}$ B) $\dfrac{25}{81}$ C) $\dfrac{36}{81}$ D) $\dfrac{49}{81}$ E) $\dfrac{64}{81}$

8. $\dfrac{1}{\sin 10°} - \dfrac{\sqrt{3}}{\cos 10°} = ?$

A) 1 B) 2 C) 3 D) 4 E) 5

9. $\frac{\sin 2a}{1-\cos 2a}\cdot\tan a-2\sin^2 a=?$

A) sin2a B) cos2a C) tan2a
D) cot2a E) sec2a

10. $x\in\left(\frac{\pi}{2},\frac{3\pi}{4}\right)$,

$\tan x-\frac{2}{\sin 2x}=\frac{1}{2}\Rightarrow \tan 2x=?$

A) $\frac{4}{3}$ B) $\frac{3}{4}$ C) $\frac{8}{3}$ D) $\frac{3}{8}$ E) 2

11. $x\in\left(0,\frac{\pi}{2}\right)$,

$2\tan x-\frac{4}{\sin 2x}=1\Rightarrow \tan x=?$

A) $-\frac{4}{3}$ B) $-\frac{4}{5}$ C) -2 D) 2 E) 3

12. $x=\frac{\pi}{12}\Rightarrow \sin^6 x+\cos^6 x=?$

A) $\frac{13}{16}$ B) $\frac{11}{16}$ C) $\frac{7}{8}$ D) $\frac{3}{4}$ E) $\frac{2}{3}$

13. $\theta=\frac{\pi}{12}\Rightarrow \cos^6\theta-\cos^2\theta\cdot\sin^2\theta+\sin^6\theta=?$

A) $\frac{3}{8}$ B) $\frac{1}{2}$ C) $\frac{7}{8}$ D) 1 E) $\frac{3}{2}$

14. $\tan 4=\frac{1}{3}$

$\Rightarrow \frac{\cos 8}{1+\cos 8}\cdot\frac{\sin 16}{1+\cos 16}=?$

A) $\frac{1}{3}$ B) $\frac{2}{3}$ C) $\frac{3}{2}$ D) 3 E) 6

15. $\frac{\cos(a+b)+\cos(a-b)+1}{\sin(a+b)+\sin(a-b)}=\cot a+\frac{1}{\cos b}$

$\Rightarrow \sin a=?$

A) 0 B) $\frac{\sqrt{3}}{2}$ C) $-\frac{\sqrt{3}}{2}$ D) $\frac{\sqrt{2}}{2}$ E) $\frac{1}{2}$

16. $\sin\frac{\pi}{8}\cdot\left(1-2\cdot\cos^2\frac{\pi}{16}\right)=?$

A) $-\frac{\sqrt{2}}{4}$ B) $-\frac{\sqrt{2}}{2}$ C) $-\sqrt{2}$
D) $\frac{\sqrt{2}}{2}$ E) $\frac{\sqrt{2}}{4}$

YANITLAR / ANSWERS

1	2	3	4	5	6	7	8
D	E	D	A	C	D	D	D
9	10	11	12	13	14	15	16
B	A	C	A	C	A	E	A

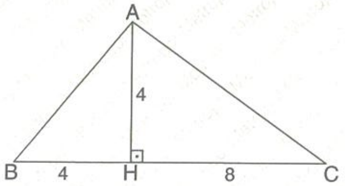
1. $[AH] \perp [BC]$,

$|BH| = |HA| = 4$ cm,

$|HC| = 8$ cm,

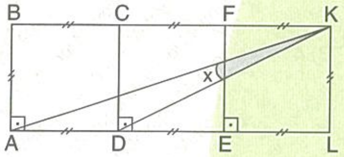
$m(\widehat{BAC}) = x$

$\Rightarrow$ **tanx = ?**



A) –3 B) –2 C) –1 D) 2 E) 3

2.



$m(\widehat{AKD}) = x \Rightarrow$ **tan x = ?**

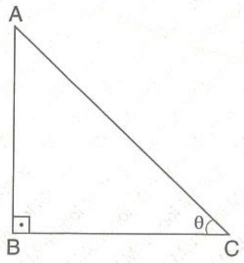
A) $\frac{1}{3}$ B) $\frac{1}{4}$ C) $\frac{1}{5}$ D) $\frac{1}{6}$ E) $\frac{1}{7}$

3. $4 \cdot |AB| = 3 \cdot |BC|$,

$[AB] \perp [BC]$,

$m(\widehat{ACB}) = \theta$

$\Rightarrow$ $\mathbf{\tan\frac{\theta}{2} = ?}$



A) $\frac{3}{8}$ B) $\frac{3}{4}$ C) $\frac{1}{2}$ D) $\frac{1}{3}$ E) $\frac{2}{\sqrt{3}}$

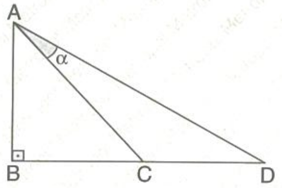
4. $m(\widehat{ABD}) = 90°$,

$|AB| = 3$ cm,

$|BC| = 4$ cm,

$|CD| = 2$ cm,

$\Rightarrow$ **cotα = ?**



A) $\frac{2}{3}$ B) $\frac{2}{11}$ C) $\frac{4}{7}$ D) $\frac{1}{2}$ E) $\frac{11}{2}$

5. $|AB| = |BC|$,

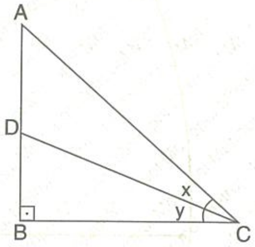
$[AB] \perp [BC]$,

$m(\widehat{ACD}) = x$,

$m(\widehat{DCB}) = y$,

$\sin y = \frac{3}{5}$

$\Rightarrow$ **tanx = ?**



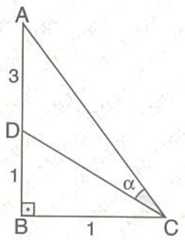
A) 1 B) $\frac{1}{2}$ C) $\frac{1}{4}$ D) $\frac{1}{7}$ E) $\frac{2}{9}$

6. $|BD| = |BC| = 1$ cm,

$|DA| = 3$ cm,

$m(\widehat{DCA}) = \alpha$

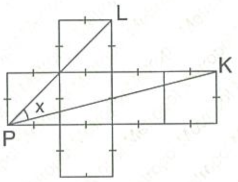
$\Rightarrow$ $\mathbf{\tan\alpha = ?}$



A) $\frac{5}{3}$ B) $\frac{3}{4}$ C) $\frac{3}{5}$ D) 4 E) $\frac{4}{5}$

7. $m(\widehat{KPL}) = x$

$\Rightarrow$ **$\tan x = ?$**



A) $\frac{3}{4}$ B) $\frac{3}{5}$ C) $\frac{\sqrt{3}}{4}$ D) $\frac{\sqrt{5}}{6}$ E) 2

8. ABCD bir dikdörtgendir.

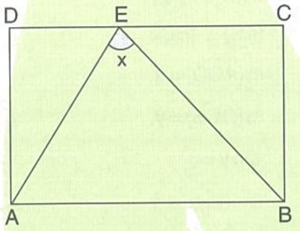
ABCD is a rectangle.

$|CE| = 3 \cdot |DE|$,

$|AB| = 3 \cdot |BC|$,

$m(\widehat{AEB}) = x$

$\Rightarrow$ **$\tan x = ?$**



A) $-\frac{48}{11}$ B) -3 C) $-\frac{15}{2}$ D) $-\frac{17}{8}$ E) $-\frac{7}{16}$

9. ABCD bir dikdörtgendir.

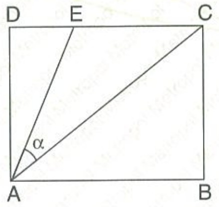
ABCD is a rectangle.

$3 \cdot |AB| = 4 \cdot |BC|$,

$|EC| = |BC|$,

$m(\widehat{EAC}) = \alpha$

$\Rightarrow$ **$\tan \alpha = ?$**



A) $\frac{9}{13}$ B) $\frac{13}{9}$ C) $\frac{5}{9}$ D) $\frac{2}{5}$ E) $\frac{9}{10}$

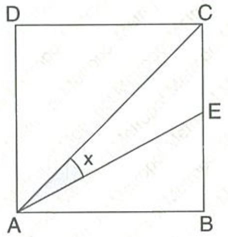
10. ABCD bir karedir.

ABCD is a square.

$|BE| = 2 \cdot |EC|$,

$m(\widehat{CAE}) = x$

$\Rightarrow$ **$\tan x = ?$**



A) 1 B) 2 C) $\frac{1}{3}$ D) $\frac{1}{4}$ E) $\frac{1}{5}$

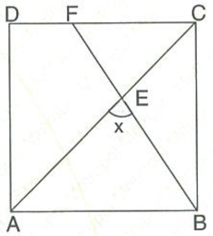
11. ABCD bir karedir.

ABCD is a square.

$|FC| = 2 \cdot |FD|$,

$m(\widehat{AEB}) = x$

$\Rightarrow$ **$\tan x = ?$**



A) 3 B) 4 C) 5 D) 6 E) 7

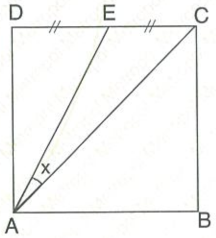
12. ABCD bir karedir.

ABCD is a square.

$|DE| = |EC|$,

$m(\widehat{EAC}) = x$

$\Rightarrow$ **$\tan x = ?$**



A) 1 B) 2 C) 3 D) $\frac{1}{2}$ E) $\frac{1}{3}$

YANITLAR / ANSWERS

1	2	3	4	5	6	7	8	9	10	11	12
A	E	D	E	D	C	B	A	A	E	C	E

1. ABCD bir karedir.

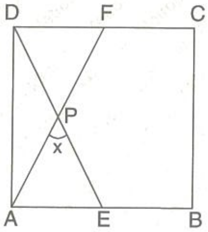
ABCD is a square.

$|AE| = |EB|$,

$|DF| = |FC|$,

$m(\widehat{APE}) = x$

$\Rightarrow$ **sinx = ?**



A) $\frac{3}{4}$ B) $\frac{3}{5}$ C) $\frac{4}{5}$ D) $\frac{2}{5}$ E) $\frac{2}{3}$

2. ABCD bir karedir.

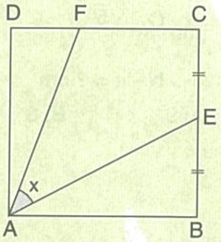
ABCD is a square.

$|BE| = |EC|$,

$|FC| = 3 \cdot |DF|$,

$m(\widehat{FAE}) = x$

$\Rightarrow$ **sinx = ?**



A) $\frac{3}{2}$ B) $\frac{6}{\sqrt{85}}$ C) $\frac{6}{7}$ D) $\frac{7}{6}$ E) $\frac{7}{\sqrt{85}}$

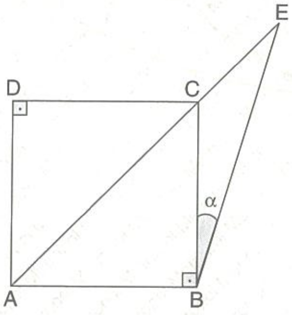
3. ABCD bir karedir.

ABCD is a square.

$|AC| = 4 \cdot |CE|$

$m(\widehat{EBC}) = \alpha$

$\Rightarrow$ **tanα = ?**



A) $\frac{1}{5}$ B) $\frac{3}{5}$ C) $\frac{4}{5}$ D) $\frac{5}{3}$ E) 5

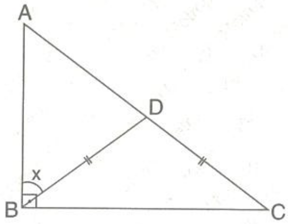
4. $|BD| = |DC|$,

$\frac{|BC|}{|AC|} = \frac{\sqrt{5}}{3}$,

$[AB] \perp [BC]$,

$m(\widehat{ABD}) = x$

$\Rightarrow$ **cotx = ?**



A) $\sqrt{5}$ B) $\frac{\sqrt{5}}{5}$ C) $\frac{2\sqrt{5}}{5}$ D) $\frac{1}{5}$ E) $\frac{2}{3}$

5. ABCD bir karedir.

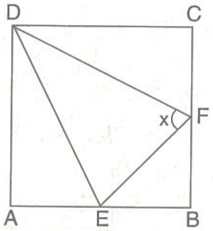
ABCD is a square.

$|AE| = |EB|$,

$|BF| = |FC|$,

$m(\widehat{DFE}) = x$

$\Rightarrow$ **sinx = ?**



A) $\frac{2\sqrt{5}}{5}$ B) $\frac{3\sqrt{10}}{10}$ C) $\frac{2\sqrt{10}}{5}$ D) $\frac{3\sqrt{6}}{4}$ E) $\frac{3\sqrt{15}}{15}$

6. $m(\widehat{BAD}) = m(\widehat{DAC})$,

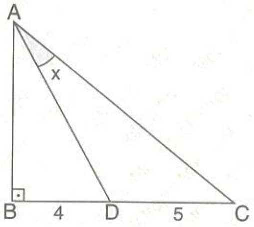
$m(\widehat{DAC}) = x$,

$[AB] \perp [BC]$,

$|BD| = 4$ cm,

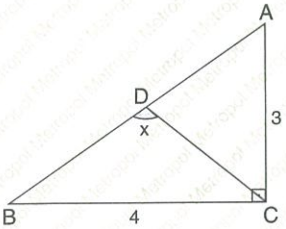
$|DC| = 5$ cm

$\Rightarrow$ **tanx = ?**



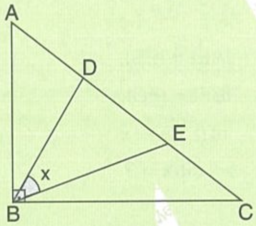
A) $\frac{1}{2}$ B) $\frac{1}{3}$ C) $\frac{1}{4}$ D) $\frac{20}{73}$ E) $\frac{4}{11}$

7. [AC] ⊥ [BC],
|AC| = 3 cm,
|BC| = 4 cm,
|AD| = |DB|,
$m(\widehat{BDC}) = x$
⇒ **sinx = ?**



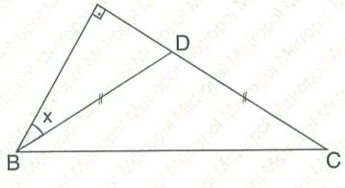
A) $-\frac{7}{24}$ B) $-\frac{24}{7}$ C) $\frac{7}{24}$ D) $\frac{24}{7}$ E) $\frac{24}{25}$

8. [AB] ⊥ [BC],
|AB| = |BC|,
|AD| = |DE| = |EC|,
$m(\widehat{DBE}) = x$
⇒ **tanx = ?**



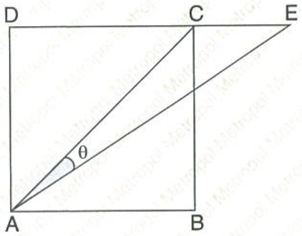
A) $\frac{2}{3}$ B) $\frac{3}{4}$ C) $\frac{4}{5}$ D) $\frac{5}{6}$ E) $\frac{4}{3}$

9. [BA] ⊥ [AC],
|BD| = |DC|,
4·|AB| = 3·|AC|,
$m(\widehat{ABD}) = x$
⇒ **cosx = ?**



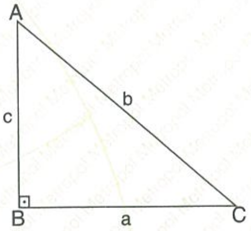
A) $\frac{3}{10}$ B) $\frac{3}{8}$ C) $\frac{4}{5}$ D) $\frac{16}{25}$ E) $\frac{24}{25}$

10. ABCD bir karedir.
ABCD is a square.
|AB| = 3·|CE|, $m(\widehat{CAE}) = \theta$
⇒ **tanθ = ?**



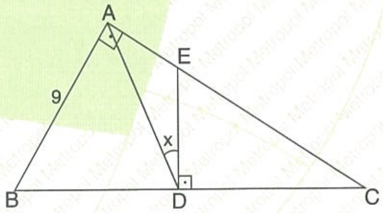
A) $\frac{1}{7}$ B) $\frac{1}{5}$ C) 1 D) $\frac{1}{8}$ E) $\frac{1}{9}$

11. [AB] ⊥ [BC],
|BC| = a,
|AC| = b,
|AB| = c,
a + b = 30 cm,
$\tan\frac{C}{2} = \frac{\sqrt{5}}{5}$
⇒ **b − a = ? cm**



A) 9 B) 8 C) 7 D) 6 E) 4

12.



[BA] ⊥ [AC], [ED] ⊥ [BC], |BD| = |DC|, |AB| = 9 cm,
|BC| = 15 cm, $m(\widehat{ADE}) = x$ ⇒ **sinx = ?**

A) $\frac{24}{25}$ B) $\frac{3}{5}$ C) $\frac{14}{25}$ D) $\frac{7}{25}$ E) $\frac{7}{15}$

YANITLAR / ANSWERS

1	2	3	4	5	6	7	8	9	10	11	12
C	E	A	C	B	B	E	B	E	A	D	D

1. $\dfrac{\sin 40^\circ + \cos 70^\circ}{\cos 10^\circ} = ?$

A) $\frac{\sqrt{3}}{3}$ B) $\frac{\sqrt{3}}{2}$ C) $\frac{\sqrt{2}}{2}$ D) $\frac{1}{2}$ E) 1

2. $x = \dfrac{\pi}{17} \Rightarrow \dfrac{\cos 14x + \cos 12x}{\cos 3x + \cos 5x} = ?$

A) –2 B) $-\frac{3}{2}$ C) –1 D) 0 E) 1

3. $\dfrac{\sin 132^\circ + \cos 282^\circ}{\cos 312^\circ + \sin 102^\circ} = ?$

A) 2 B) $2\sqrt{3}$ C) $\frac{\sqrt{3}}{2}$ D) $\frac{\sqrt{3}}{3}$ E) $\sqrt{3}$

4. $\dfrac{4\sqrt{3}}{\sin 15^\circ + \cos 15^\circ} = ?$

A) 2 B) $2\sqrt{3}$ C) $3\sqrt{2}$ D) $4\sqrt{2}$ E) $6\sqrt{3}$

5. $\dfrac{\cos x + \cos y}{\sin x - \sin y} = ?$

A) sinx B) $\cot\frac{x-y}{2}$ C) $\tan\frac{x-y}{2}$

D) 0 E) 1

6. $\dfrac{\cos 78^\circ + \cos 42^\circ}{\sin 78^\circ + \sin 42^\circ} = ?$

A) $\frac{1}{2}$ B) $\frac{\sqrt{3}}{2}$ C) $\frac{\sqrt{3}}{3}$ D) $\sqrt{3}$ E) $2\sqrt{3}$

7. $x = \dfrac{\pi}{8} \Rightarrow \dfrac{\cos x - \cos 3x}{\sin 3x - \sin x} = ?$

A) $-\frac{\sqrt{3}}{3}$ B) $-\sqrt{3}$ C) $\frac{\sqrt{3}}{3}$ D) $\sqrt{3}$ E) 1

8. cos130° + cos110° + cos10° = ?

A) 2cos10° B) cos10° C) 0

D) 1 E) 2

9. **$\sin 185° + \sin 115° - \sin 235° = ?$**

A) 1 B) $\frac{\sqrt{3}}{2}$ C) cos35°

D) 2sin55° E) –2cos35°

10. $a = \frac{\pi}{13}$

$\Rightarrow \dfrac{\cos 8a \cdot \cos 10a}{\cos 2a + \cos 8a} = ?$

A) $-\frac{1}{4}$ B) $-\frac{1}{2}$ C) 0 D) $\frac{1}{4}$ E) $\frac{1}{2}$

11. $\dfrac{\cos\frac{13\pi}{12} + \sin\frac{13\pi}{12}}{\cos\frac{5\pi}{12} + \sin\frac{5\pi}{12}} = ?$

A) –1 B) 0 C) 1 D) $\frac{\sqrt{2}}{2}$ E) $\sqrt{3}$

12. $16x = \pi \Rightarrow \dfrac{\sin 12x - \sin 8x}{\cos 16x + \cos 4x} = ?$

A) –2 B) $-\frac{1}{2}$ C) $\frac{1}{2}$ D) 1 E) 2

13. $16x = \pi \Rightarrow \dfrac{\sin 2x + 2\sin 4x + \sin 6x}{\cos 2x + 2\cos 4x + \cos 6x} = ?$

A) $\sqrt{3}$ B) $-\sqrt{3}$ C) $\frac{\sqrt{3}}{3}$ D) 1 E) $-\frac{\sqrt{3}}{3}$

14. $A = \dfrac{\sin 3x + \sin 4x + \sin 5x}{\cos 3x + \cos 4x + \cos 5x}$,

$x = \frac{\pi}{12} \Rightarrow$ **A = ?**

A) 1 B) $\sqrt{3}$ C) $\frac{\sqrt{3}}{3}$ D) –1 E) $-\frac{\sqrt{3}}{3}$

15. $x = \frac{\pi}{13} \Rightarrow \dfrac{\cos x \cdot \cos 10x}{\cos 2x + \cos 4x} = ?$

A) 2 B) 1 C) 0 D) $-\frac{1}{2}$ E) –1

16. $\dfrac{\cos 7x + a\cos 5x + \cos 3x}{\sin 7x + a\sin 5x + \sin 3x} = ?$

A) tan3x B) cot5x C) tan2x

D) sec3x E) cosec5x

17. $\dfrac{\sin 230°}{\cos^2 40° - \cos^2 10°} = ?$

A) $\frac{\sqrt{2}}{2}$ B) $\sqrt{2}$ C) 2 D) $\frac{2\sqrt{3}}{3}$ E) $\frac{\sqrt{3}}{3}$

YANITLAR / ANSWERS

1	2	3	4	5	6	7	8	9
E	C	D	D	B	C	E	C	D
10	11	12	13	14	15	16	17	
E	A	D	D	B	D	B	C	

1. $\cos\frac{3\pi}{8}\cdot\cos\frac{\pi}{8}=?$

A) $-\frac{1}{2}$ B) 0 C) $\frac{1}{4}$ D) $\frac{\sqrt{2}}{4}$ E) $\frac{\sqrt{3}}{4}$

2. $x=\frac{\pi}{12} \Rightarrow \frac{\sin 3x+\sin 11x}{\sin 5x\cdot\cos 4x}=?$

A) –2 B) –1 C) 1 D) $\frac{1}{2}$ E) 2

3. $\sin\frac{5\pi}{12}\cdot\sin\frac{11\pi}{12}=?$

A) $\frac{1}{2}$ B) $\frac{1}{4}$ C) $\frac{\sqrt{2}}{2}$ D) $\frac{\sqrt{2}}{4}$ E) $\frac{\sqrt{3}}{4}$

4. $A=\frac{\pi}{17} \Rightarrow \frac{\cos 15A+\cos 11A}{4\cos 4A\cdot\cos 2A}=?$

A) $\frac{\sqrt{3}}{2}$ B) $\frac{1}{2}$ C) – 1 D) $-\frac{1}{3}$ E) $-\frac{1}{2}$

5. $x+y=\frac{5\pi}{2} \Rightarrow \frac{\cos x-\cos y}{\sin x-\sin y}=?$

A) $-\frac{\sqrt{3}}{2}$ B) –1 C) $-\sqrt{3}$ D) 1 E) $\frac{1}{2}$

6. sin78° – sin18° + cos132° = ?

A) 2sin48° B) 2cos48° C) 1 D) $\frac{1}{2}$ E) 0

7. $\frac{\sin 200^\circ+\sin 100^\circ}{\cos 280^\circ-\cos 340^\circ}=?$

A) –sin40° B) cot50° C) –tan40° D) $-\frac{1}{2}$ E) –1

8. cos105° + sin75° = ?

A) $\frac{1}{2}$ B) $\frac{\sqrt{3}}{2}$ C) $\frac{\sqrt{2}}{2}$ D) $\sqrt{2}$ E) 1

9. $\dfrac{\sin\dfrac{10\pi}{7}\cdot\cos\dfrac{13\pi}{16}}{\sin\dfrac{4\pi}{7}\cdot\cos\dfrac{3\pi}{16}}=?$

A) –1 B) $-\frac{1}{2}$ C) $\frac{1}{4}$ D) $\frac{1}{2}$ E) 1

10. **cos105° · cos45° = ?**

A) $1-\sqrt{2}$ B) $\frac{1+\sqrt{2}}{4}$ C) $\frac{1+\sqrt{3}}{4}$ D) $\frac{1-\sqrt{3}}{4}$ E) $\frac{\sqrt{2}}{2}$

11. $A=\frac{\pi}{10}$ ⇒ **cos 8A + cos 4A = ?**

A) 1 B) $\frac{1}{2}$ C) $\frac{1}{4}$ D) $-\frac{1}{2}$ E) $-\frac{1}{4}$

12. sec 72° – cosec 54° = A ⇒ **A = ?**

A) $\frac{3}{4}$ B) 1 C) $\frac{4}{3}$ D) $\frac{3}{2}$ E) 2

13. $\cos\frac{\pi}{5}+\cos\frac{3\pi}{5}=?$

A) $\frac{\sqrt{2}}{2}$ B) $\frac{\sqrt{3}}{2}$ C) $\frac{1}{2}$ D) $\frac{3}{4}$ E) $\frac{1}{8}$

14. $\sin\frac{3\pi}{10}-\sin\frac{\pi}{10}=?$

A) $\frac{1}{8}$ B) $\frac{1}{4}$ C) $\frac{1}{2}$ D) $\frac{3}{8}$ E) $\frac{5}{16}$

15. A = sin10° · sin50° · sin60° · sin70° ⇒ **A = ?**

A) $\frac{1}{16}$ B) $\frac{1}{8}$ C) $\frac{\sqrt{6}}{8}$ D) $\frac{\sqrt{2}}{4}$ E) $\frac{\sqrt{3}}{16}$

16. $\left(\cos\frac{3\pi}{5}+\cos\frac{\pi}{5}\right)\cdot\sin\frac{7\pi}{18}\cdot\sin\frac{5\pi}{18}\cdot\sin\frac{\pi}{18}=?$

A) $\frac{3}{4}$ B) $\frac{3}{8}$ C) $\frac{3}{16}$ D) $\frac{1}{8}$ E) $\frac{1}{16}$

17. A = cos10° · cos30° · cos50° · cos70°

⇒ **A = ?**

A) $\frac{1}{8}$ B) $\frac{\sqrt{3}}{8}$ C) $\frac{3}{8}$ D) $\frac{\sqrt{3}}{16}$ E) $\frac{3}{16}$

YANITLAR / ANSWERS

1	2	3	4	5	6	7	8	9
D	E	B	E	B	E	C	C	E
10	11	12	13	14	15	16	17	
D	D	E	C	C	E	E	E	

1. $\sin\left(\arcsin\frac{5}{13}\right)=?$

A) $\frac{5}{12}$ B) $\frac{12}{13}$ C) $\frac{5}{11}$ D) $\frac{7}{13}$ E) $\frac{5}{13}$

2. $\sin(\arccos\frac{1}{2})=?$

A) $-\frac{1}{2}$ B) -1 C) $\frac{\sqrt{3}}{2}$ D) $\frac{1}{2}$ E) 1

3. $f:(0,\pi)\rightarrow[-1,1]$

$f(x)=\arccos\left(\frac{2x-1}{3}\right) \Rightarrow f^{-1}(x)=?$

A) $\frac{2\cdot\cos x-1}{3}$ B) $\frac{2\cdot\cos x+1}{3}$ C) $\frac{3\cdot\cos x+1}{2}$

D) $\frac{3\cdot\cos x-1}{2}$ E $\frac{3\cdot\cos x+1}{3}$

4. $\arcsin\frac{\sqrt{3}}{2}-\arctan\frac{\sqrt{3}}{3}=?$

A) $\frac{2\pi}{3}$ B) $\frac{\pi}{3}$ C) $\frac{5\pi}{6}$ D) $\frac{\pi}{6}$ E) $\frac{\pi}{12}$

5. $\sin(\arctan\sqrt{3}+\arccos\frac{1}{2})=?$

A) $-\frac{1}{2}$ B) 1 C) $\frac{1}{2}$ D) $\frac{\sqrt{3}}{2}$ E) $\frac{\sqrt{3}}{3}$

6. $\tan\left(5\arctan\frac{\sqrt{3}}{3}-\frac{1}{2}\arcsin\frac{\sqrt{3}}{2}\right)=?$

A) $-\frac{\sqrt{3}}{3}$ B) $-\sqrt{3}$ C) -1 D) 1 E) $\sqrt{3}$

7. $\sin(3\arctan\sqrt{3}+2\arcsin\frac{\sqrt{3}}{2})=?$

A) $-\frac{\sqrt{3}}{2}$ B) $-\frac{\sqrt{2}}{2}$ C) $\frac{1}{2}$ D) $\frac{\sqrt{3}}{2}$ E) $\frac{\sqrt{2}}{2}$

8. $\cot\left(3\arccos\frac{1}{2}+\arcsin\left(-\frac{\sqrt{3}}{2}\right)\right)=?$

A) $\frac{1}{2}$ B) $\sqrt{3}$ C) $\frac{\sqrt{3}}{3}$ D) $-\frac{\sqrt{3}}{3}$ E) $-\sqrt{3}$

9. $\sin(3\arctan\sqrt{3} + 2\arccos\frac{1}{2}) = ?$

A) $-\frac{1}{2}$ B) $-\frac{\sqrt{3}}{2}$ C) $-\frac{\sqrt{2}}{2}$ D) $\frac{1}{2}$ E) $\frac{\sqrt{3}}{2}$

10. $A = \cos(2\arctan\frac{2}{3}) \Rightarrow A = ?$

A) $\frac{5}{13}$ B) $\frac{4}{13}$ C) $\frac{3}{5}$ D) $-\frac{3}{5}$ E) $-\frac{4}{13}$

11. $\cos(2\arctan\frac{4}{3}) = ?$

A) $\frac{12}{5}$ B) $\frac{7}{3}$ C) $\frac{5}{3}$ D) $\frac{7}{25}$ E) $-\frac{7}{25}$

12. $\arccos\frac{3}{5} - \arcsin\frac{4}{5} = ?$

A) –3 B) –2 C) $-\frac{3}{5}$ D) 0 E) 30

13. $4\arctan(x^2 - 3x - 3) - \pi = 0 \Rightarrow x_1 + x_2 = ?$

A) –2 B) 0 C) 3 D) 4 E) $2\sqrt{3}$

14. $\sin\left(\arccos\frac{3}{5} + \arcsin\frac{5}{13}\right) = ?$

A) $\frac{27}{43}$ B) $\frac{41}{49}$ C) $\frac{63}{65}$ D) $\frac{65}{76}$ E) 1

15. $\sin\left[\frac{1}{2}\arcsin\left(-\frac{2\sqrt{2}}{3}\right)\right] = A \Rightarrow A = ?$

A) $-\sqrt{3}$ B) $-\frac{\sqrt{3}}{3}$ C) $\sqrt{3}$ D) $\frac{\sqrt{3}}{3}$ E) 1

16. $\arctan(3 + 2\sqrt{2}) - \arctan\frac{\sqrt{2}}{2} = ?$

A) $\frac{\pi}{4}$ B) $\frac{\pi}{3}$ C) $\frac{\pi}{6}$ D) $\frac{2\pi}{3}$ E) $\frac{4\pi}{3}$

YANITLAR / ANSWERS

1	2	3	4	5	6	7	8
E	C	C	D	D	B	A	D
9	10	11	12	13	14	15	16
B	A	E	D	C	C	D	A

1. $\sin 4x = \frac{1}{2} \Rightarrow$ **ÇK(SS) = ?**

A) $\left\{x \mid x = \frac{\pi}{6} + 2k\pi \vee x = \frac{5\pi}{6} + 2k\pi, k \in Z\right\}$

B) $\left\{x \mid x = \frac{\pi}{24} + 2k\pi \vee x = \frac{5\pi}{24} + 2k\pi, k \in Z\right\}$

C) $\left\{x \mid x = \frac{\pi}{24} + \frac{k\pi}{2} \vee x = \frac{5\pi}{24} + \frac{k\pi}{2}, k \in Z\right\}$

D) $\left\{x \mid x = \frac{\pi}{3} + 2k\pi \vee x = \frac{2\pi}{3} + 2k\pi, k \in Z\right\}$

E) $\left\{x \mid x = \frac{\pi}{12} + \frac{k\pi}{2} \vee x = \frac{\pi}{6} + \frac{k\pi}{2}, k \in Z\right\}$

2. $\tan 2x = 1 \Rightarrow$ **ÇK(SS) = ?**

A) $\left\{x \mid x = \frac{\pi}{6} + 2k\pi, k \in Z\right\}$

B) $\left\{x \mid x = \frac{\pi}{4} + 2k\pi, k \in Z\right\}$

C) $\left\{x \mid x = \frac{\pi}{3} + k\pi, k \in Z\right\}$

D) $\left\{x \mid x = \frac{\pi}{8} + k\pi, k \in Z\right\}$

E) $\left\{x \mid x = \frac{\pi}{8} + \frac{k\pi}{2}, k \in Z\right\}$

3. $0 < x < 180°$,

$\sin 2x = \frac{1}{2} \Rightarrow$ **ÇK(SS) = ?**

A) {30° , 45°}

B) {45° , 75°}

C) {30° , 45°}

D) {15° , 60°}

E) {15° , 75°}

4. $\tan(x - \pi) = \cot(3x) \Rightarrow$ **ÇK(SS) = ?**

A) $\left\{x \mid x = \frac{3\pi}{8} + k\pi, k \in Z\right\}$

B) $\left\{x \mid x = \frac{3\pi}{8} + \frac{k\pi}{2}, k \in Z\right\}$

C) $\left\{x \mid x = \frac{3\pi}{8} + \frac{k\pi}{4}, k \in Z\right\}$

D) $\left\{x \mid x = \frac{3\pi}{4} + \frac{k\pi}{2}, k \in Z\right\}$

E) $\left\{x \mid x = \frac{3\pi}{4} + k\pi, k \in Z\right\}$

5. $x \in \left(\frac{3\pi}{2}, 2\pi\right)$,

$2\cos^2 x + 3\cos x - 2 = 0 \Rightarrow$ **x = ?**

A) $\frac{2\pi}{3}$ B) $\frac{4\pi}{3}$ C) $\frac{5\pi}{6}$ D) $\frac{5\pi}{3}$ E) $\frac{7\pi}{3}$

6. $\frac{\pi}{2} < x < \pi$,

$\frac{1}{1 - \sin x} + \frac{1}{1 + \sin x} = 8 \Rightarrow$ **x = ?**

A) $\frac{2\pi}{3}$ B) $\frac{3\pi}{4}$ C) $\frac{5\pi}{6}$ D) $\frac{7\pi}{12}$ E) $\frac{8\pi}{9}$

7. $x \in \left(0, \frac{\pi}{4}\right)$,

$\frac{1}{\sin^2 x} + \frac{1}{\cos^2 x} = \frac{16}{3} \Rightarrow$ **x = ?**

A) $\frac{\pi}{12}$ B) $\frac{\pi}{8}$ C) $\frac{\pi}{6}$ D) $\frac{\pi}{4}$ E) $\frac{\pi}{3}$

8. $0 < x < \frac{\pi}{4}$,

$\frac{\sin x}{\cos x} + \frac{\cos x}{\sin x} = 4 \Rightarrow$ **x = ?**

A) $\frac{\pi}{24}$ B) $\frac{\pi}{12}$ C) $\frac{\pi}{7}$ D) $\frac{\pi}{6}$ E) $\frac{\pi}{5}$

9. $\frac{2\tan x}{\tan^2 x + 1} = 1 \Rightarrow$ **ÇK(SS) = ?**

A) $\{x \mid x = \frac{\pi}{6} + k\pi,\ k \in Z\}$

B) $\{x \mid x = \frac{\pi}{3} + k\pi,\ k \in Z\}$

C) $\{x \mid x = \frac{\pi}{12} + k\pi,\ k \in Z\}$

D) $\{x \mid x = \frac{5\pi}{6} + k\pi,\ k \in Z\}$

E) $\{x \mid x = \frac{\pi}{4} + k\pi,\ k \in Z\}$

10. $\cos 6x - \sin 6x = \frac{\sqrt{2}}{2} \Rightarrow$ **ÇK(SS) = ?**

A) $\{x \mid x = \frac{\pi}{6} + 2k\pi \ \vee\ x = \frac{5\pi}{6} + 2k\pi,\ k \in Z\}$

B) $\{x \mid x = \frac{\pi}{3} + 2k\pi \ \vee\ x = \frac{2\pi}{3} + 2k\pi,\ k \in Z\}$

C) $\{x \mid x = \frac{\pi}{72} + 2k\pi \ \vee\ x = \frac{5\pi}{72} + 2k\pi,\ k \in Z\}$

D) $\{x \mid x = \frac{\pi}{72} + \frac{k\pi}{6} \ \vee\ x = \frac{5\pi}{72} + \frac{k\pi}{6},\ k \in Z\}$

E) $\{x \mid x = \frac{\pi}{36} + \frac{k\pi}{6} \ \vee\ x = \frac{\pi}{9} + \frac{k\pi}{6},\ k \in Z\}$

11. $x \in [0, 2\pi]$

$\cos^2 x + 2\sin x - 2 = 0 \Rightarrow$ **ÇK(SS) = ?**

A) $\left\{0, \frac{\pi}{2}, \pi\right\}$ B) $\left\{\frac{\pi}{2}, \pi\right\}$ C) $\left\{\frac{\pi}{2}\right\}$

D) $\{0, \pi\}$ E) $\left\{0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}\right\}$

12. $x \in (0, 2\pi)$

$\cos 2x = 2\sin^2 x + 2\sin x + 1 \Rightarrow$ **ÇK(SS) = ?**

A) $\left\{\frac{\pi}{6}, \frac{5\pi}{6}\right\}$ B) $\left\{\frac{5\pi}{6}, \frac{7\pi}{6}, \pi\right\}$ C) $\left\{\pi, \frac{7\pi}{6}, \frac{11\pi}{6}\right\}$

D) $\left\{\frac{4\pi}{2}, \frac{5\pi}{3}\right\}$ E) $\varnothing$

13. $x \in (0, 2\pi)$

$2\sin 3x - \cos 6x - 3 = 0 \Rightarrow$ **x = ?**

A) $\left\{\frac{\pi}{6}, \frac{5\pi}{6}, \frac{3\pi}{2}\right\}$ B) $\left\{\frac{\pi}{3}, \frac{5\pi}{3}\right\}$

C) $\left\{\frac{\pi}{4}, \frac{5\pi}{6}, \frac{7\pi}{4}\right\}$ D) $\left\{\frac{\pi}{2}, \frac{3\pi}{2}\right\}$

E) $\left\{\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}\right\}$

YANITLAR / ANSWERS

1	2	3	4	5	6	7	8	9	10	11	12	13
C	E	E	C	D	A	C	B	E	D	C	C	A

1. $-2\tan\left[\arcsin\frac{3}{5}+\arccos\frac{1}{\sqrt{5}}\right]=?$

A) $\frac{-7}{4}$ B) $\frac{-11}{4}$ C) 11 D) $\frac{4}{11}$ E) $\frac{7}{11}$

2. $\sin\left[\pi-\arccos\left(-\frac{3}{5}\right)\right]=?$

A) $-\frac{3}{4}$ B) $-\frac{4}{5}$ C) $-\frac{1}{2}$ D) $\frac{3}{4}$ E) $\frac{4}{5}$

3. $\cos\left(\arctan\frac{3}{4}+\frac{3\pi}{2}\right)=?$

A) $\frac{3}{5}$ B) $\frac{4}{5}$ C) $\frac{3}{4}$ D) $\frac{5}{13}$ E) $\frac{12}{13}$

4. $\sin(\arcsin x + \arccos y) = 1 \Rightarrow x = ?$

A) $1-y$ B) $1-y^2$ C) y D) $\frac{y}{2}$ E) $\frac{1}{y}$

5. $\cos\left(\arcsin\frac{5}{13}+\arcsin\frac{3}{5}\right)=?$

A) $\frac{12}{65}$ B) $\frac{27}{65}$ C) $\frac{31}{65}$ D) $\frac{33}{65}$ E) $\frac{63}{65}$

6. $\sin\left[\frac{3\pi}{2}+\operatorname{arccot}\frac{5}{12}\right]=?$

A) $-\frac{12}{13}$ B) $-\frac{5}{13}$ C) $\frac{5}{13}$ D) $\frac{12}{13}$ E) $\frac{8}{13}$

7. $x \in (0,2\pi)$,

$\cos 2x + \sin x + 2 = 0 \Rightarrow x = ?$

A) $\frac{\pi}{3}$ B) $\frac{\pi}{2}$ C) π D) $\frac{3\pi}{2}$ E) $\frac{5\pi}{3}$

8. $x \in (0,2\pi)$,

$\sin x + \sqrt{3}\cos x - 1 = 0 \Rightarrow x = ?$

A) $\frac{\pi}{8}$ B) $\frac{\pi}{6}$ C) $\frac{\pi}{4}$ D) $\frac{\pi}{3}$ E) $\frac{\pi}{2}$

9. $0 < x < \pi$,

$\sqrt{3}\cdot\tan^2x + (1-\sqrt{3})\tan x = 1 \Rightarrow$ **ÇK(SS) = ?**

A) $\left\{\frac{\pi}{4}, \frac{5\pi}{6}\right\}$ B) $\left\{\frac{\pi}{4}, \frac{2\pi}{3}\right\}$ C) $\left\{\frac{\pi}{4}, \frac{\pi}{6}\right\}$

D) $\left\{\frac{\pi}{3}, \frac{\pi}{6}\right\}$ E) $\left\{\frac{\pi}{6}, \frac{3\pi}{4}\right\}$

10. $\pi < x < \frac{3\pi}{2}$

$3\sin x + \cos 2x + 1 = 0 \Rightarrow$ **ÇK(SS) = ?**

A) $\left\{\frac{5\pi}{6}\right\}$ B) $\left\{\frac{7\pi}{6}\right\}$ C) $\left\{\frac{5\pi}{6}, \frac{7\pi}{6}\right\}$

D) $\left\{\frac{7\pi}{6}, \frac{9\pi}{8}\right\}$ E) $\left\{\frac{5\pi}{6}, \frac{9\pi}{8}\right\}$

11. $0 \le x \le 2\pi$,

$\cos\frac{3x}{2} = \cos\left(\frac{x}{2} + \frac{\pi}{2}\right)$

$\Rightarrow \sum x = ?$

A) $\frac{3\pi}{2}$ B) $\frac{7\pi}{4}$ C) 2π D) $\frac{9\pi}{4}$ E) $\frac{5\pi}{4}$

12. $\left(\frac{2}{7}\right)^{\tan(4x-10^\circ)} \cdot \left(\frac{7}{2}\right)^{\tan(3x+40^\circ)} = 1$

$0 \le x \le 360^\circ \Rightarrow \sum x = ?$

A) 180° B) 240° C) 260° D) 280° E) 300°

13. $6\arcsin(x^2 - 6x + 8{,}5) = \pi \Rightarrow x_1 \cdot x_2 = ?$

A) –12 B) –8 C) 0 D) 6 E) 8

14. $x = \arctan y \Rightarrow$ $\cot^2x + \tan^2x = ?$

A) $\frac{y^4+1}{y^2}$ B) $\frac{y^4-1}{y^2}$ C) $\frac{y^2+1}{y^2}$

D) $\frac{y^2-1}{y^2}$ E) $\frac{y^4+1}{y}$

15. $\arctan(3x - 6) = y$,

$\tan y = 84^\circ$

$\Rightarrow 2\sin x + 2\sqrt{3}\cos x = ?$

A) 1 B) 2 C) 3 D) 4 E) 5

16. $\sqrt{3}\arcsin 1 = \frac{\pi^2}{4} \cdot x\sqrt{3} \Rightarrow x = ?$

A) $\frac{1}{\pi}$ B) $\frac{2}{\pi}$ C) $\frac{3}{\pi}$ D) $\frac{4}{\pi}$ E) $\frac{5}{\pi}$

YANITLAR / ANSWERS

1	2	3	4	5	6	7	8
C	E	A	C	D	B	D	E
9	**10**	**11**	**12**	**13**	**14**	**15**	**16**
A	B	D	D	E	A	D	B

BÖLÜM
CHAPTER
22

TARİHSEL NOT / HISTORICAL NOTE

RENÉ DESCARTES [1596 - 1650]

Fransız filozof, matematikçi ve yazar. 1637'de René Descartes karesi negatif sayı olan karmaşık sayıların varlığına dikkati çekmiştir. Uzaydaki bir noktayı bir numaralar seti olarak işaretleyebilmeyi ve cebirsel denklemleri iki boyutlu koordinat sisteminde geometrik şekiller olarak göstermeyi (ve tam tersini) sağlayan Kartezyen koordinat sistemi, ismini Descartes'tan alır. Analitik geometrinin de temellerini Descartes atmıştır.

French philosopher, mathematician and writer. In 1637, René Descartes drew attention to the existence of complex numbers whose squares are negative numbers. The Cartesian coordinate system, which can mark a point in space as a set of numbers and display algebraic equations as geometric shapes in the two-dimensional coordinate system, takes its name from Descartes. Descartes also laid the foundations of analytical geometry.

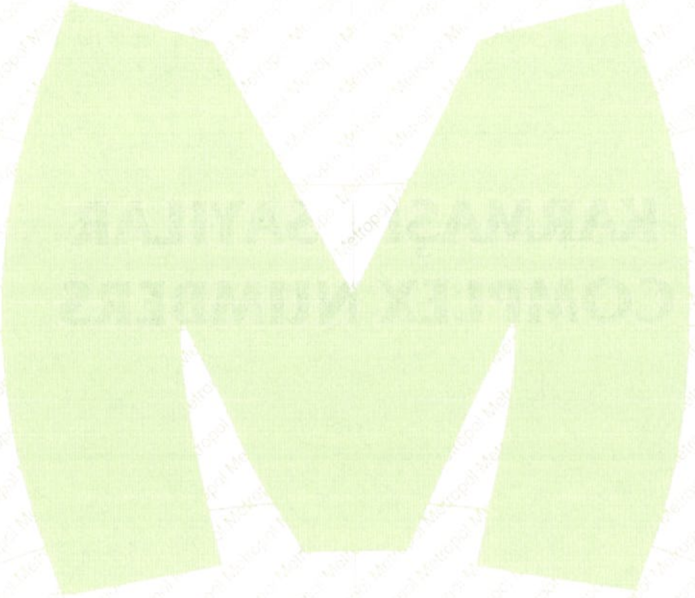
KARMAŞIK SAYILAR
COMPLEX NUMBERS

Bu bölüm, 111 test sorusu içermektedir.
This chapter includes 111 test questions.

BÖLÜM / CHAPTER 22

KARMAŞIK SAYILAR / COMPLEX NUMBERS

- Karmaşık Sayılar / Complex Numbers .... 405 – 426



BÖLÜM

22

CHAPTER

KARMAŞIK SAYILAR
COMPLEX NUMBERS

KARMAŞIK SAYILAR / COMPLEX NUMBERS

▼ Reel (gerçel) sayılar kümesinde "$x^2 + 1 = 0$" ikinci dereceden denkleminde "$\Delta = -4 < 0$" olduğundan reel (gerçel) kök yoktur. Bu denklemden hareketle, $x^2 = -1 \Rightarrow |x| = \sqrt{-1}$ sayısına ***"i karmaşık sayısı"*** denmiş ve karmaşık sayılar kümesi oluşturulmuştur.

▼ The quadratic equation "$x^2 + 1 = 0$" has no root in real numbers set as "$\Delta = -4 < 0$" using this equation, $x^2 = -1 \Rightarrow |x| = \sqrt{-1}$ is called "i complex number" and the set of complex numbers has created.

Tanım: Sanal sayı birimi $i = \sqrt{-1}$ ile tanımlanır.

Definition: The imaginary unit is defined by $i = \sqrt{-1}$.

Tanım: a, b ∈ R olmak üzere, z = a + bi şeklinde yazılan sayılara ***karmaşık sayılar***, bu sayıların oluşturduğu kümeye de ***karmaşık sayılar kümesi*** denir ve ℂ ile gösterilir.

Definition: *A complex number* is a number of the form a + bi where a and b are real numbers. *The set of complex numbers* consists of complex numbers and this set is denoted by ℂ.

$$\mathbb{C} = \{a + bi \mid a,b \in R,\ i = \sqrt{-1}\}$$

Tanım: z = a + bi karmaşık sayısında, a reel sayısına karmaşık sayının ***reel kısmı*** denir ve

Re(z) ile gösterilir.

Definition: In complex number z = a + bi, the real number a is called the real part of the complex number and that is denoted by Re(z) = a.

Önemli / Important

- $N^+ \subset N \subset Z \subset Q \subset R \subset \mathbb{C}$

  N^+ : Pozitif doğal sayılar

  Positive natural numbers

  N : Doğal sayılar

  Natural numbers

  Z : Tam sayılar

  Integers

  Q : Rasyonel sayılar

  Rational numbers

  R : Reel(Gerçel) sayılar

  Real numbers

  ℂ : Karmaşık sayılar

  Complex numbers

i'nin TAM SAYI KUVVETLERİ / INTEGER POWERS of i

▼ $i = \sqrt{-1}$

$i^1 = i$

$i^2 = -1$

$i^3 = -i$

$\mathbf{i^4 = 1}$

$i^5 = i$

$i^6 = -1$

⋮

▼ n ∈ Z,

$$i^n = \begin{cases} 1, & n \equiv 0 \pmod 4 \\ i, & n \equiv 1 \pmod 4 \\ -1, & n \equiv 2 \pmod 4 \\ -i, & n \equiv 3 \pmod 4 \end{cases}$$

▼ $i^m \cdot i^n = i^{m+n}$

$$\frac{i^m}{i^n} = i^{m-n}$$

İKİ KARMAŞIK SAYININ EŞİTLİĞİ / EQUALITY of TWO COMPLEX NUMBERS

▼ $z_1 = a + bi$, $z_2 = c + di$,

$z_1 = z_2 \Rightarrow \boldsymbol{a = c,\ b = d}$

KARMAŞIK SAYILARIN EŞLENİĞİ / CONJUGATE of COMPLEX NUMBERS

▼ $z = a + bi$ karmaşık sayısının eşleniği $\boldsymbol{\overline{z} = a - bi}$ dir.

The conjugate of $z = a + bi$ is denoted by $\overline{z} = a - bi$.

Özellik / Property

★ $\overline{(\overline{z})} = z$

★ $\overline{z_1 \pm z_2} = \overline{z_1} \pm \overline{z_2}$

★ $\overline{z_1 \cdot z_2} = \overline{z_1} \cdot \overline{z_2}$

★ $\overline{\left(\frac{z_1}{z_2}\right)} = \frac{\overline{z_1}}{\overline{z_2}}$, $(z_2 \neq 0)$

★ $z = a + bi \Rightarrow z \cdot \overline{z} = (a + bi) \cdot (a - bi)$

$$= a^2 - (bi)^2$$

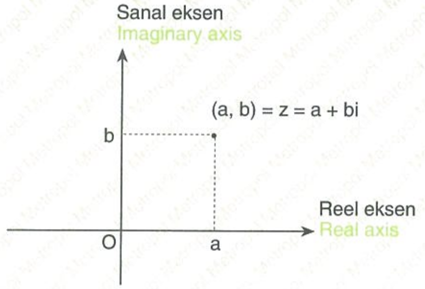
$$= a^2 - (-b^2)$$

$$= a^2 + b^2$$

$$\Rightarrow \boldsymbol{z \cdot \overline{z} = a^2 + b^2}$$

KARMAŞIK SAYILARIN ANALİTİK DÜZLEMDE GÖSTERİLMESİ
INTERPRETATION of COMPLEX NUMBERS on ANALYTIC PLANE

▼

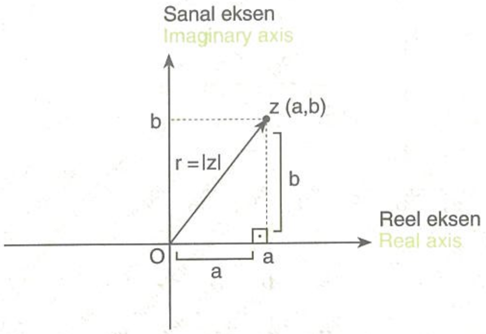


BİR KARMAŞIK SAYININ MUTLAK DEĞERİ (MODÜLÜ) / ABSOLUTE VALUE (MODULUS) of a COMPLEX NUMBER

▼ Bir karmaşık sayının orijine olan uzaklığına o ***karmaşık sayının mutlak değeri (modülü)*** denir. **IzI** biçiminde gösterilir.

▼ The distance from a complex number z to the origin is called the ***absolute value or modulus of z***. It is shown by **IzI**.

$$|z| = \sqrt{a^2+b^2}$$



Özellik / Property

★ $|z| = |\bar{z}| = |-z| = |-\bar{z}|$

★ $z \cdot \bar{z} = |z|^2$

$\left[z \cdot \bar{z} = a^2+b^2,\ |z| = \sqrt{a^2+b^2}\right]$

★ $|z_1 \cdot z_2| = |z_1| \cdot |z_2|$

★ $\left|\frac{z_1}{z_2}\right| = \frac{|z_1|}{|z_2|} \quad (z_2 \neq 0)$

★ $|z^n| = |z|^n$

★ $\left|\frac{1}{z}\right| = \frac{1}{|z|}$

KARMAŞIK SAYILARDA DÖRT İŞLEM / FOUR OPERATIONS IN COMPLEX NUMBERS

$z_1 = a + bi, \quad z_2 = c + di$

▼ $z_1 + z_2 = (a + c) + (b + d)i$

▼ $z_1 - z_2 = (a - c) + (b - d)i$

▼ $z_1 \cdot z_2 = (a + bi) \cdot (c + di)$

▼ $\frac{z_1}{z_2} = \underset{(c-di)}{\frac{a+bi}{c+di}}$

Önemli / Important

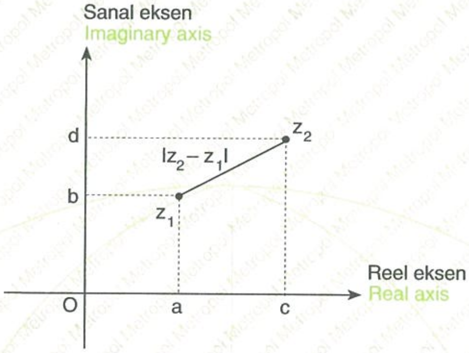
- z nin çarpma işlemine göre tersi:
- Multiplication inverse of z:

$$z^{-1} = \frac{1}{z} = \frac{1}{a+bi}$$

$$= \frac{a-bi}{(a+bi)\cdot(a-bi)}$$

$$= \frac{a-bi}{a^2+b^2} = \frac{a}{a^2+b^2} - \frac{b}{a^2+b^2} \cdot i$$

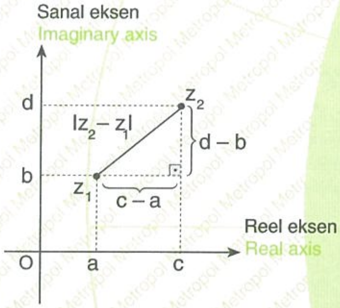
KARMAŞIK DÜZLEMDE İKİ NOKTA ARASINDAKI UZAKLIK
THE DISTANCE BETWEEN TWO POINTS on the COMPLEX PLANE


Sanal eksen
Imaginary axis
d
b
z_2
$|z_2 - z_1|$
z_1
Reel eksen
Real axis
O
a
c

$z_1 = a + bi, \quad z_2 = c + di$

$$\Rightarrow |z_2 - z_1| = \sqrt{(d-b)^2 + (c-a)^2}$$

Önemli / Important

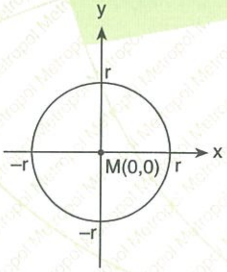

Sanal eksen
Imaginary axis
d
b
z_2
$|z_2 - z_1|$
z_1
d – b
c – a
Reel eksen
Real axis
O
a
c

$(d-b)^2 + (c-a)^2 = |z_2 - z_1|^2$

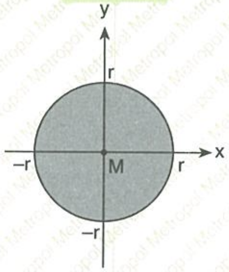
[Pisagor / Pythagorean]

$\Rightarrow |z_2 - z_1| = \sqrt{(d-b)^2 + (c-a)^2}$

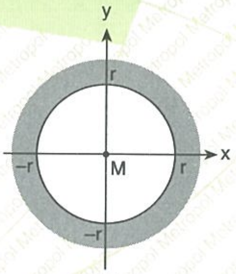
SONUÇLAR / CONCLUSIONS



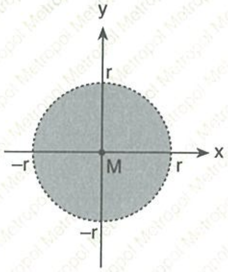
$\Rightarrow |z| = r$



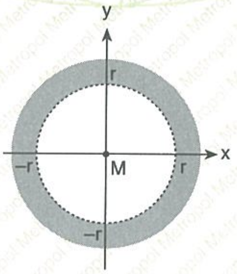
$\Rightarrow |z| \le r$



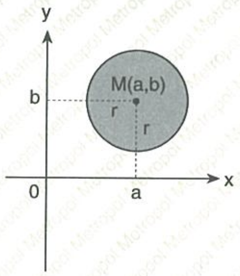
$\Rightarrow |z| \ge r$



$\Rightarrow |z| < r$



$\Rightarrow |z| > r$


y
x
M(a,b)
b
r
r
0
a

$\Rightarrow |z - (a+bi)| \le r$

Önemli / Important

$z = x + yi$, $z_1 = a + bi$ olmak üzere,

- $|z - z_1| = r \Rightarrow$ M(a,b) merkezli r yarıçaplı bir çember belirtir.
- $|z - z_1| < r \Rightarrow$ Çemberin içini belirtir.
- $|z - z_1| > r \Rightarrow$ Çemberin dışını belirtir.

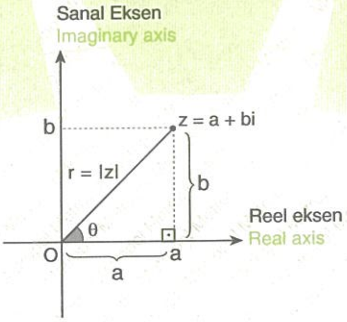
Let $z = x + yi$, $z_1 = a + bi$,

- $|z - z_1| = r$ represents a circle with the center M(a,b) and radius of r.
- $|z - z_1| < r$ represents interior of the circle.
- $|z - z_1| > r$ represents exterior of the circle.

KARMAŞIK SAYILARIN KUTUPSAL FORMDA YAZILIŞI / POLAR (TRIGONOMETRIC) FORM OF COMPLEX NUMBERS

▼ $z = a + bi$, $\cos\theta = \frac{a}{|z|} \Rightarrow a = |z| \cdot \cos\theta$

$\sin\theta = \frac{b}{|z|} \Rightarrow b = |z| \cdot \sin\theta$



▼ Bu değerler $z = a + bi$ ifadesinde yerine yazılırsa aşağıdaki denklem bulunur.

▼ Substituting these values in $z = a + bi$, we get

$z = |z| \cdot \cos\theta + i \cdot |z| \cdot \sin\theta$

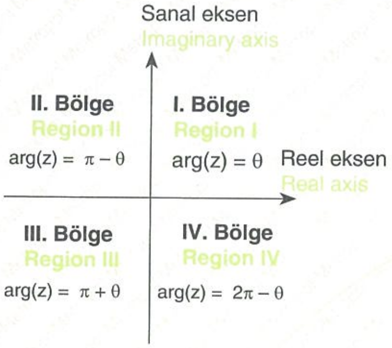
$\boldsymbol{z = |z| \cdot (\cos\theta + i \cdot \sin\theta) = r \cdot cis\theta}$

▼ Karmaşık sayıların bu şekilde yazılışına ***kutupsal (trigonometrik) biçimde yazılış*** denir. Ox ekseni ile pozitif yönde yapılan θ gerçel (reel) sayısına ***z nin argümenti*** denir ve ***arg(z)*** ile gösterilir.

▼ This form of a complex number is called the polar or trigonometric form of a complex number. The angle θ is called the argument of z, shown by arg(z).

Önemli / Important

- $z = a + bi$,


Sanal eksen
Imaginary axis
II. Bölge
Region II
arg(z) = π − θ
I. Bölge
Region I
arg(z) = θ
Reel eksen
Real axis
III. Bölge
Region III
arg(z) = π + θ
IV. Bölge
Region IV
arg(z) = 2π − θ

Özellik / Property

$z_1 = r_1 \cdot (\cos\theta_1 + i \cdot \sin\theta_1) = r_1 \cdot cis\theta_1$

$z_2 = r_2 \cdot (\cos\theta_2 + i \cdot \sin\theta_2) = r_2 \cdot cis\theta_2$

★ $z_1 \cdot z_2 = r_1 \cdot r_2 (\cos(\theta_1 + \theta_2) + i \cdot \sin(\theta_1 + \theta_2)) = \mathbf{r_1 \cdot r_2 \cdot cis(\theta_1 + \theta_2)}$

★ $\frac{z_1}{z_2} = \frac{r_1}{r_2} \cdot (\cos(\theta_1 - \theta_2) + i \cdot \sin(\theta_1 - \theta_2)) = \mathbf{\frac{r_1}{r_2} \cdot cis(\theta_1 - \theta_2)}$

★ $z = r \cdot (\cos\theta + i \cdot \sin\theta) \Rightarrow z^n = r^n \cdot (\cos(n \cdot \theta) + i \cdot \sin(n \cdot \theta)) = \boldsymbol{r^n \cdot cis(n \cdot \theta)}$

BİR KARMAŞIK SAYININ KÖKLERİ / ROOTS of a COMPLEX NUMBER

▼ z bir karmaşık sayı ise,

▼ If z is a complex number,

$z = r \cdot (\cos(\theta + k \cdot 2\pi) + i \cdot \sin(\theta + k \cdot 2\pi))$.

$w_k = \sqrt[n]{z} = r^{\frac{1}{n}} \cdot \left(\cos\frac{1}{n} \cdot (\theta + k \cdot 2\pi) + i \cdot \sin\frac{1}{n} \cdot (\theta + k \cdot 2\pi)\right)$

($k \in \{0, 1, 2, 3, \ldots\ldots, n-1\}$) ifadesine ***z nin n. kuvvetten kökleri*** denir.

($k \in \{0, 1, 2, 3, \ldots\ldots, n-1\}$) the expression above is called the n^{th} roots of z.

▼ z karmaşık sayısının kareköklerinden biri w_0 ise ikinci kök $w_1 = -w_0$ dır.

▼ If one of the square roots of z is w_0, then other root is $w_1 = -w_0$

▼ $z = a \pm bi$ karmaşık sayısının karekökleri aşağıdaki formül ile bulunabilir.

▼ The square roots of $z = a \pm bi$ can be found also by the following formula.

$$w_{0,1} = \pm\left(\sqrt{\frac{|z|+a}{2}} \pm \left(\sqrt{\frac{|z|-a}{2}}\right)i\right)$$

1. $\sqrt{-3}\cdot\sqrt{-6}\cdot\sqrt{2}=?$

A) –6 B) –6i C) 6 D) 6i E) $\sqrt{6}\cdot i$

2. $i=\sqrt{-1} \Rightarrow \sqrt{-4}\cdot\sqrt{-9}=?$

A) –6 B) 6 C) 6i D) –6i E) i

3. $i^2=-1$

$$\left.\begin{array}{l} z_1=3+4i \\ z_2=5-i \end{array}\right\} \Rightarrow z_1+z_2=?$$

A) 4 + 3i B) 8 + 6i C) 8 – 3i D) 8 + 2i E) 8 + 3i

4. $(3-4i)\cdot(1+5i)=?$

A) 11i + 12 B) 11i + 23 C) 11i – 17 D) 19i + 23 E) 19i – 17

5. $i^{23}+i^{32}+i^{73}=?$

A) 2i + 1 B) 1 – 2i C) 2 + i D) –i E) 1

6. $i^2=-1$

$\Rightarrow i^{2011}+i^{2012}+i^{2013}=?$

A) –1 B) –i C) i D) 1 E) 0

7. $i^2=-1$

$z_1=3x+4(1+y)i$

$z_2=2x-3+3(3+y)i$

$z_1=z_2 \Rightarrow x+y=?$

A) 8 B) 6 C) 5 D) 2 E) 0

8. $a, b \in R$,

$$\left.\begin{array}{l} z_1=3-a+i \\ z_2=1+(a-b)i \end{array}\right\} \text{ ve / and } z_1=z_2 \Rightarrow a+b=?$$

A) 3 B) 4 C) 5 D) 6 E) 7

9. $P(x) = 2x^{2016} - 3x^{2015} + 5x^{2014}$

$\Rightarrow \mathbf{P(i) = ?}$

A) $7 + 3i$ B) $-3 - 3i$ C) $-3 + 3i$ D) $7 - 3i$ E) $-3 + 2i$

10. $x, y \in R$,

$(3x - 2y) + (3x + 2y)i = 5 + 13i \Rightarrow \mathbf{x + y = ?}$

A) 2 B) 3 C) 5 D) 6 E) 8

11. $\mathbf{i + i^2 + i^3 + ... + i^{94} = ?}$

A) $-1 - i$ B) $-1 + i$ C) -1 D) $-i$ E) 0

12. $i^2 = -1$

$z = \frac{4-5i}{1-i} \Rightarrow \mathbf{Re(z) = ?}$

A) $-\frac{1}{2}$ B) $-\frac{9}{2}$ C) 1 D) $\frac{1}{2}$ E) $\frac{9}{2}$

13. $\mathbf{(1 + i)^{24} = ?}$

A) 2^{12} B) $2^{12}i$ C) $-2^{12}i$ D) $2^{24}i$ E) 2^{24}

14. $x > 0$,

$z = x - 2i$,

$z \cdot \overline{z} = 40 \Rightarrow \mathbf{x = ?}$

A) 7 B) 6 C) 5 D) 4 E) 3

15. $\mathbf{(1 - 2i)^2 \cdot (1 + 2i)^3 = ?}$

A) $10 - 25i$ B) $8 - 16i$ C) $20 + 25i$ D) $30 - 15i$ E) $25 + 50i$

16. $(3 + i) \cdot z = 9 - 7i \Rightarrow \mathbf{z = ?}$

A) $2 + 3i$ B) $3 + 2i$ C) $3 - 2i$ D) $2 - 3i$ E) $-3 - 2i$

17. $x, y \in R$

$\frac{3x-i}{3y+i} = 1 - i \Rightarrow \mathbf{x + y = ?}$

A) $\frac{1}{3}$ B) $\frac{2}{3}$ C) 1 D) $\frac{4}{3}$ E) $\frac{5}{3}$

YANITLAR / ANSWERS

1	2	3	4	5	6	7	8	9
A	A	E	B	E	D	D	A	C
10	11	12	13	14	15	16	17	
C	B	E	A	B	E	D	E	

1. $z = 4i + 3 \Rightarrow \mathbf{Im(z^{-1}) = ?}$

A) $-\frac{4}{25}$ B) $-\frac{3}{25}$ C) $-\frac{1}{25}$ D) $\frac{3}{25}$ E) $\frac{4}{25}$

2. $n \in N,\ i^2 = -1$

$$\Rightarrow \frac{i^{n+3} + i^{n+2} + i^{n+1}}{i^{n} + i^{n-1} + i^{n-2}} = ?$$

A) –1 B) 1 C) i D) – i E) 1 – i

3. $4 - i = 2\bar{z} + 3i \Rightarrow \mathrm{Im}(z) = ?$

A) –2 B) –1 C) $\frac{1}{2}$ D) 1 E) 2

4. $x, y \in R,\ z = x + iy$,

$(2 + i)\, z = 4i + 7 - i\bar{z} \Rightarrow \mathbf{x + y = ?}$

A) 1 B) 2 C) 3 D) 4 E) 5

5. $n \in N$

$$\Rightarrow \frac{i^{12n+8} - i^{8n-2}}{i^{3-16n} - i^{8n+5}} = ?$$

A) –2 B) –2i C) –i D) i E) 1

6. $\left(\frac{1-i}{1+i}\right)^{10} = ?$

A) –2i B) i C) –1 D) 1 E) 4i

7. $i^2 = -1 \Rightarrow \left(\frac{1-i}{1+i}\right)^{30} = ?$

A) 1 B) –1 C) i D) –i E) 2i

8. $z \cdot (1 + i) = 2 + i + \bar{z} \Rightarrow \mathbf{Re(z) + Im(z) = ?}$

A) 1 B) 2 C) 3 D) 4 E) 5

9. $(1 - 3i) \cdot \bar{z} = z + 6 \Rightarrow$ **$\text{Im}(z) = ?$**

A) -2 B) $-\frac{4}{3}$ C) -1 D) 1 E) $\frac{4}{3}$

10. $\left(\frac{1+i}{1-i}\right)^{2016} = ?$

A) 2^{2008} B) 2^{2004} C) $2^{2004}i$ D) $-2^{2004}i$ E) 1

11. $z = i + i^2 + i^3 + ... + i^{103} = ?$

A) $2i - 1$ B) $1 + i$ C) i D) 1 E) -1

12. $(z + 1)\cdot(1 - i) = (1 + i) \Rightarrow$ **$z = ?$**

A) $1 + 2i$ B) $1 - 2i$ C) $2 - i$ D) $1 + i$ E) $-1 + i$

13. $\bar{z}\cdot(1 - i) = z + 1 \Rightarrow$ **$z = ?$**

A) $2 - i$ B) $2 + i$ C) $1 - i$ D) $1 + i$ E) $-1 + 2i$

14. $i^2 = -1 \Rightarrow$ **$(1 - i)^{40} = ?$**

A) -2^{20} B) $2^{20}i$ C) 2^{20} D) $-2^{20}i$ E) 2^{10}

15. $\frac{(1-i)^8 + (1+i)^8}{(1+i)^4 + (1-i)^4} = ?$

A) -4 B) -2 C) 2 D) 4 E) 16

16. $\bar{z} - iz = z + 4 + i \Rightarrow$ **$z = ?$**

A) $-9 + 4i$ B) $7 - 3i$ C) $6 + i$ D) $3 - 3i$ E) $2 + 5i$

17. $f(x,y) = 2x - 3i\cdot y + xy$

$g(x, y) = x - y + ixy,$

$f(i, a) = g(b, i) \Rightarrow$ **$a = ?$**

A) -1 B) $\frac{3}{2}$ C) -3 D) $\frac{i}{2}$ E) i

YANITLAR / ANSWERS

1	2	3	4	5	6	7	8	9
A	D	E	B	D	C	B	C	A
10	11	12	13	14	15	16	17	
E	E	E	A	C	A	A	B	

1. $z_1 = 3 + 6i,\ z_2 = -5 + 12i$

$\Rightarrow |z_1 - z_2| = ?$

A) $\sqrt{6}$ B) $2\sqrt{2}$ C) 10 D) $10\sqrt{2}$ E) $\sqrt{13}$

2. $\left.\begin{array}{l} z_1 = 4 - 2i \\ z_2 = 1 + 3i \end{array}\right\} \Rightarrow |z_1 - z_2| = ?$

A) $\sqrt{41}$ B) $\sqrt{34}$ C) 5 D) $\sqrt{21}$ E) 4

3. $z_1 = 7 - \sqrt{3}i,$

$z_2 = 1 + 5i,$

$\Rightarrow \left|\dfrac{z_1}{z_2}\right| = ?$

A) 1 B) $\sqrt{2}$ C) $\sqrt{3}$ D) 2 E) $\sqrt{5}$

4. $z_1 = 4 + 3i,$

$z_2 = 2 - i,$

$z_3 = 1 - 2i$

$\Rightarrow \left|\dfrac{z_1 \cdot z_2^2}{z_3^2}\right| = ?$

A) 1 B) $\sqrt{5}$ C) 5 D) $2\sqrt{2}$ E) $3\sqrt{2}$

5. $\left.\begin{array}{l} z_1 = 3 + 4i \\ z_2 = 5 - 12i \end{array}\right\} \Rightarrow |z_1 \cdot z_2| = ?$

A) 65 B) 63 C) 52 D) 48 E) 42

6. $\left.\begin{array}{l} z = 2 + 3i \\ w = 1 + 2i \end{array}\right\} \Rightarrow \dfrac{|z^4 \cdot w^6|}{65} = ?$

A) 335 B) 325 C) 320 D) 215 E) 210

7. $\left.\begin{array}{l} z_1 = 3 - i \\ z_2 = 5 - \sqrt{5}i \\ z_3 = 7 - i \end{array}\right\} \Rightarrow \left|\dfrac{z_1^3 \cdot z_2}{z_3^2}\right| = ?$

A) 2 B) $2\sqrt{2}$ C) $2\sqrt{3}$ D) $3\sqrt{3}$ E) $3\sqrt{2}$

8. $\left.\begin{array}{l} z_1 = 2 - i \\ z_2 = 1 + 3i \end{array}\right\} \Rightarrow \left|\dfrac{z_1 - z_2 + 5i}{z_2 - z_1 + 3}\right| = ?$

A) $\sqrt{5}$ B) $2\sqrt{5}$ C) $\sqrt{10}$ D) $\dfrac{\sqrt{10}}{10}$ E) $2\sqrt{13}$

9. $z=\dfrac{(1-i)^4\cdot(3+4i)^5}{i^5\cdot(-\sqrt{3}+\sqrt{2}i)^8} \Rightarrow |z|=?$

A) $20\sqrt{5}$ B) $10\sqrt{5}$ C) 20

D) 10 E) 4

10. $z=3+4i \Rightarrow \left|\overline{|z|+z}\right|=?$

A) $2\sqrt{5}$ B) 5 C) $4\sqrt{5}$ D) 9 E) 10

11. $z=\dfrac{(2-i)(1+3i)}{(1+2i)^2} \Rightarrow |\bar{z}|=?$

A) 5 B) $\sqrt{5}$ C) $\sqrt{3}$ D) $\sqrt{2}$ E) 1

12. $(\bar{z}-i)i=2z+3 \Rightarrow |z|=?$

A) $\dfrac{2\sqrt{5}}{3}$ B) $\sqrt{10}$ C) $2\sqrt{5}$ D) $\dfrac{3}{2}$ E) $\sqrt{5}$

13. $a\in R$,

$z=a-(a+2)i \Rightarrow |z+i\cdot\bar{z}|=?$

A) $2\sqrt{2}$ B) $\sqrt{3}$ C) $\sqrt{2a+1}$

D) $3\sqrt{a}$ E) $\sqrt{5}$

14. $z=(1+\sqrt{3}\cdot i)^4\cdot\left(\dfrac{\sqrt{3}}{2}-\dfrac{1}{2}i\right)^6 \Rightarrow |z|=?$

A) $\dfrac{1}{4}$ B) 8 C) 16 D) 32 E) 81

15. $P(x)=mx^{88}+2x^{70}-4x^{32}+x+1$

$$\begin{array}{r|l} P(x) & x-i \\ \cline{2-2} - & \\ \hline 1+i & \end{array} \Rightarrow m=?$$

A) –2 B) –1 C) 1 D) 2 E) 6

16. $f(x)=x^2+bx+c$

$f(1-2i)=0 \Rightarrow c=?$

A) 2 B) $\sqrt{2}$ C) 5 D) $\sqrt{5}$ E) $\sqrt{7}$

17. $|m|=3$, $|n|=2$,

$z=\dfrac{-4+mi}{n-i} \Rightarrow |z|=?$

A) 1 B) 2 C) $\sqrt{5}$ D) $\sqrt{7}$ E) 5

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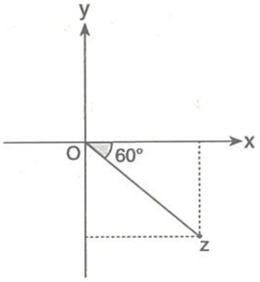
1	2	3	4	5	6	7	8	9
C	A	B	C	B	B	C	D	C
10	11	12	13	14	15	16	17	
C	D	A	A	C	E	C	C	

1. $|z| = 6 \Rightarrow \mathbf{Im(z) = ?}$



A) $2\sqrt{3}$ B) $-3\sqrt{3}$ C) $-3\sqrt{2}$ D) $3\sqrt{3}$ E) $3\sqrt{2}$

2. $z = 6\left(\cos\frac{4\pi}{3} + \sin\frac{4\pi}{3}\right) \Rightarrow \mathbf{z = ?}$

A) $3 - 3\sqrt{3}i$ B) $-3 + 3\sqrt{3}i$ C) $-3 - 3\sqrt{3}i$ D) $-3\sqrt{3}i$ E) $3\sqrt{3} + 3i$

3. $z = 2 - 2\sqrt{3}i \Rightarrow \mathbf{z = ?}$

A) $3\cdot\left(\cos\frac{5\pi}{6} + i\sin\frac{5\pi}{6}\right)$

B) $4\cdot\left(\cos\frac{5\pi}{3} + i\sin\frac{5\pi}{3}\right)$

C) $2\cdot\left(\cos\frac{5\pi}{6} + i\sin\frac{5\pi}{6}\right)$

D) $4\cdot\left(\cos\frac{5\pi}{6} + i\sin\frac{5\pi}{6}\right)$

E) $4\cdot\left(\cos\frac{11\pi}{6} + i\sin\frac{11\pi}{6}\right)$

4. $z = 2 - 2i \Rightarrow \mathbf{z = ?}$

A) $2\sqrt{2}\cdot \text{cis}\frac{7\pi}{4}$ B) $2\sqrt{2}\cdot \text{cis}\frac{\pi}{4}$ C) $\sqrt{2}\cdot \text{cis}\frac{\pi}{4}$ D) $\sqrt{2}\cdot \text{cis}\frac{7\pi}{4}$ E) $\text{cis}\frac{5\pi}{4}$

5. $z_1 = 2\cdot\left(\cos\frac{5\pi}{3} + i\sin\frac{5\pi}{3}\right)$

$z_2 = 4\cdot\left(\cos\frac{4\pi}{3} + i\sin\frac{4\pi}{3}\right)$

$\Rightarrow \mathbf{z_1 \cdot z_2 = ?}$

A) −8 B) −8i C) 8 + 8i D) 8 E) 8i

6. $\dfrac{\mathbf{12\cdot cis\,62^\circ}}{\mathbf{3\cdot cis\,32^\circ}} = ?$

A) $\sqrt{3} + 1$ B) $2(\sqrt{3} + i)$ C) $4(\sqrt{3} + i)$ D) $1 + \sqrt{3}i$ E) $2(1 + \sqrt{3}i)$

7. $z_1 = 6\ \text{cis}125^\circ$, $z_2 = 3\ \text{cis}55^\circ$

$\Rightarrow \mathbf{z_1 \cdot z_2 = ?}$

A) 18 B) −18i C) −9i D) −18 E) 18i

8. $z=\dfrac{2\left[\cos 128^\circ + i\sin 128^\circ\right]}{\cos 83^\circ + i\sin 83^\circ}=?$

A) $\frac{\sqrt{2}}{2}(1+i)$ B) $\frac{\sqrt{2}}{2}(1-i)$ C) $\frac{\sqrt{3}}{2}(1+i)$

D) $\sqrt{2}(1+i)$ E) $\sqrt{3}(1+i)$

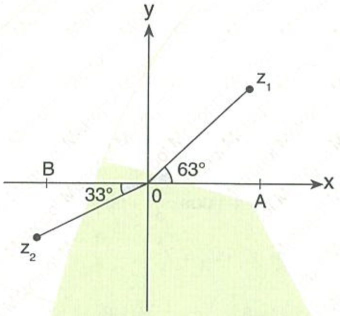
9. $|z_1| = 8$

$|z_2| = 2$

$m(\widehat{z_1OA}) = 63^\circ$

$m(\widehat{BOz_2}) = 33^\circ$

$\Rightarrow \dfrac{z_1}{z_2} = ?$



A) $\frac{1}{2}-\frac{\sqrt{3}}{2}i$ B) $\frac{1}{2}+\frac{\sqrt{3}}{2}i$ C) $-2-2\sqrt{3}i$

D) $-2\sqrt{3}+2i$ E) $2-2\sqrt{3}i$

10. $z = 4(\cos 10^\circ + i\cdot\sin 10^\circ)$

$\Rightarrow z^3 = ?$

A) $32-32\sqrt{3}i$ B) $32+32\sqrt{3}i$ C) $64+64\sqrt{3}i$

D) $64-64\sqrt{3}i$ E) $32\sqrt{3}+32i$

11. $z = 2(\cos 30^\circ + i\cdot\sin 30^\circ) \Rightarrow z^6 = ?$

A) – 32 B) – 32i C) –64

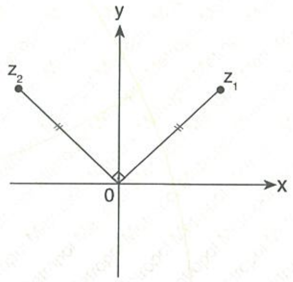
D) –64i E) 64(–1 + i)

12. $z = \frac{\sqrt{3}}{2}-\frac{1}{2}i \Rightarrow z^{23} = ?$

A) $-\frac{\sqrt{3}}{2}-\frac{1}{2}i$ B) $\frac{\sqrt{3}}{2}-\frac{1}{2}i$ C) $-\frac{\sqrt{3}}{2}+\frac{1}{2}i$

D) $\frac{\sqrt{3}}{2}+\frac{1}{2}i$ E) 1 + i

13. $\dfrac{z_1}{z_2} = ?$



A) –i B) –1 C) 1 – i D) 1 E) i

14. $|z| + z = 1 - 3i \Rightarrow z = ?$

A) 1 + 3i B) –2 – 3i C) 2 – 3i

D) 1 + 2i E) –4 – 3i

YANITLAR / ANSWERS

1	2	3	4	5	6	7
B	C	B	A	A	B	D
8	**9**	**10**	**11**	**12**	**13**	**14**
D	C	E	C	D	A	E

1. $z_1 = 3 \cdot \text{cis}20°$

$z_2 = 5 \cdot \text{cis}140° \Rightarrow |z_1 - z_2| = ?$

A) $\sqrt{19}$ B) $\sqrt{39}$ C) 7 D) $\sqrt{53}$ E) $\sqrt{59}$

2. $z = \dfrac{-2\sqrt{2} - 2\sqrt{2}i}{\text{cis}\frac{5\pi}{12}} \Rightarrow \text{Im}(z) = ?$

A) $-2\sqrt{3}$ B) $-2\sqrt{2}$ C) -2 D) 2 E) $2\sqrt{2}$

3. $|z + 6 - 8i| = 2$

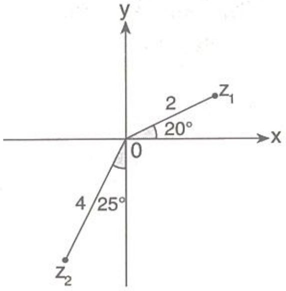
$\Rightarrow \min|z| = ?$

A) 6 B) 8 C) 10 D) 12 E) 14

4. $z = \dfrac{\sqrt{2} + \sqrt{3}i}{2 - \sqrt{3}i} \cdot (1 + \sqrt{6}i) \Rightarrow |z^2| = ?$

A) 5 B) 6 C) 8 D) 10 E) 12

5. $\dfrac{z_2}{z_1} = ?$



A) $1 + i$ B) $1 - i$ C) $-1 - i$

D) $-\sqrt{2}(1+i)$ E) $\sqrt{2}(1-i)$

6. $z = 1 + i - (\sin x + i \cdot \cos x)$

$|z| = \sqrt{2} - 1 \Rightarrow x = ?$

A) 15° B) 30° C) 45° D) 60° E) 75°

7. $x - y = \dfrac{5\pi}{3}$

$z = \cos x + \cos y + i \cdot \sin x + i \cdot \sin y \Rightarrow |z| = ?$

A) 1 B) $\sqrt{2}$ C) $\sqrt{3}$ D) 2 E) $2\sqrt{3}$

8. $0 < x < \dfrac{\pi}{2}$

$z = 1 + i \cdot \sin x - \cos x \Rightarrow |z^2| = ?$

A) $2\cos x$ B) $2(1 - \cos x)$ C) $2(1 + \cos x)$

D) $2\cos\frac{x}{2}$ E) $\sin x$

9. $|z| = 4, \ |z - 2i| = 12$

$\Rightarrow$ **$\text{Im}(z) = ?$**

A) –31 B) –16 C) 4 D) 16 E) 31

10. $z = \dfrac{(\sqrt{3}+i)^3 \cdot (2-\sqrt{5}i)}{(2+2\sqrt{3}i)^2}$

$\Rightarrow$ **$|z| = ?$**

A) $\frac{3}{2}$ B) $\frac{5}{2}$ C) $\frac{7}{2}$ D) 4 E) 5

11. $z_1 = 4\cdot[\cos 25° + i\sin 25°]$

$z_2 = \cos 35° + i\sin 35°$

$\Rightarrow$ **$\sqrt{z_1 \cdot z_2} = ?$**

A) $1-\sqrt{3}i$ B) $i-\sqrt{3}$ C) $\sqrt{3}+i$

D) $2+\sqrt{3}i$ E) $2-\sqrt{3}i$

12. $z = i\cdot\cos 310° - \sin 230°$

$\Rightarrow$ **$\text{Arg}(z) = ?$**

A) 20° B) 40° C) 60° D) 80° E) 100°

13. $z = \cos\dfrac{\pi}{6} + i \cdot \sin\dfrac{\pi}{6}$

$\Rightarrow$ **$(z + 1) \cdot (z^2 - z + 1) = ?$**

A) $-1 - i$ B) $1 - i$ C) $1 + i$

D) $i + 2$ E) 1

14. $z = \dfrac{1-i}{2-i} - \dfrac{2+i}{1+i} \Rightarrow$ **$\text{Re}(z) = ?$**

A) $-\frac{9}{10}$ B) $-\frac{3}{10}$ C) 0 D) $\frac{3}{10}$ E) $\frac{9}{10}$

15. $z = \dfrac{\sqrt[3]{-8} - \sqrt{-9}}{\sqrt[5]{-32} + \sqrt{-25}} \Rightarrow$ **$\text{Im}(z) = ?$**

A) $\frac{16}{27}$ B) $\frac{16}{29}$ C) $\frac{14}{27}$ D) $\frac{14}{29}$ E) $\frac{15}{31}$

16. $z = 1 - 4\sqrt{5}i$

$\Rightarrow$ **$9 \cdot \left|z^{-\frac{1}{2}}\right| = ?$**

A) 1 B) $\frac{3}{2}$ C) 2 D) $\frac{5}{2}$ E) 3

YANITLAR / ANSWERS

1	2	3	4	5	6	7	8
C	D	B	A	D	C	C	B
9	**10**	**11**	**12**	**13**	**14**	**15**	**16**
A	A	C	B	C	A	B	E

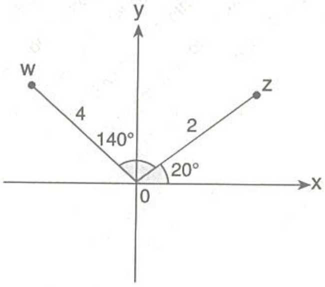
1. $|z| = 2$

$|w| = 4$

$m(\widehat{woz}) = 140^\circ$

$m(\widehat{zox}) = 20^\circ$

$\Rightarrow \mathbf{z \cdot w = ?}$



A) 8 B) –8 C) 8i D) –8i E) 8 + 8i

2. $\left.\begin{aligned} z_1 &= 3\cdot \text{cis}\frac{\pi}{3} \\ z_2 &= 9\cdot \text{cis}\frac{\pi}{6} \end{aligned}\right\} \Rightarrow z_1^3 \cdot z_2^3 = ?$

A) -3^9 B) -3^9i C) 3^9i D) 3^9 E) 3^6i

3. $z = 2\sqrt{2}\,(\cos 15^\circ + i\cdot\sin 15^\circ) \Rightarrow \mathbf{z^4 = ?}$

A) $16 + 16\sqrt{3}\,i$ B) $16 - 16\sqrt{3}\,i$

C) $32 - 32\sqrt{3}\,i$ D) $32 + 32\sqrt{3}\,i$

E) $32 + 32i$

4. $z = \dfrac{1 - i\cdot\tan x}{1 + i\cdot\cot x} \Rightarrow \mathbf{|z| = ?}$

A) 1 B) $|\sin x|$ C) $|\cos x|$

D) $|\tan x|$ E) $|\cot x|$

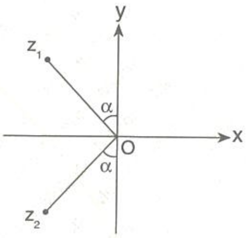
5. $z = \sin 640^\circ + i\cdot\cos 100^\circ \Rightarrow \mathbf{z^{18} = ?}$

A) –1 B) –i C) 0 D) i E) 1

6. $|z_1| = 3,$

$|z_2| = 4$

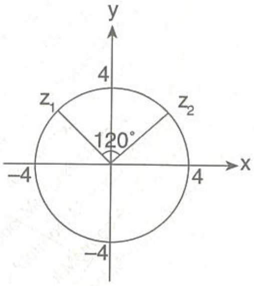
$\Rightarrow \mathbf{z_1 \cdot z_2 = ?}$



A) 12 B) –12 C) 12i D) –12i E) 5i

7. $m(\widehat{z_1Oz_2}) = 120^\circ$

$\Rightarrow \dfrac{\mathbf{2\cdot z_1}}{\mathbf{z_2}} = ?$



A) $-1-\sqrt{3}\,i$ B) $-1+\sqrt{3}\,i$ C) $\sqrt{3}+i$

D) $-2+2\sqrt{3}\,i$ E) $2-2\sqrt{3}\,i$

8. $z = \dfrac{\cos 35^\circ - i\sin 35^\circ}{\cos 5^\circ - i\sin 5^\circ} \Rightarrow \mathbf{|z^4| = ?}$

A) 1 B) $\sqrt{2}$ C) $\sqrt{3}$ D) $\sqrt{7}$ E) $\sqrt{5}$

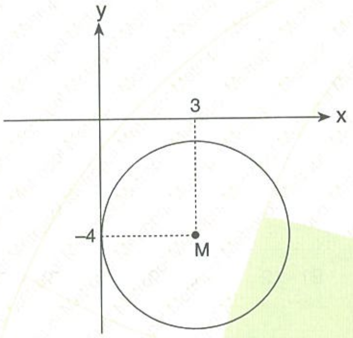
9. $f(x) = x^2 + bx + c$

$f(x_1) = f(x_2) = 0$

$x_1 = 2 + i \Rightarrow$ **f(x) = ?**

A) $x^2 - 4x - 5$
B) $x^2 - 4x + 3$
C) $x^2 - 4x - 3$
D) $x^2 + 4x + 3$
E) $x^2 - 4x + 5$

10.

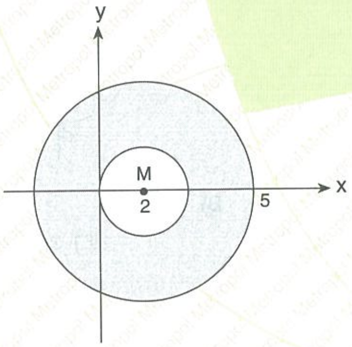


Şekildeki çemberin denklemi aşağıdakilerden hangisidir?

What is the equation of the circle in the figure?

A) $|z - (4 + 3i)| = 16$
B) $|z - (4 + 3i)| = 3$
C) $|z + (4 + 3i)| = 16$
D) $|z + (-3 + 4i)| = 9$
E) $|z + (-3 + 4i)| = 3$

11.



Taralı bölgeyi veren bağıntı aşağıdakilerden hangisidir?

What is the relation of the shaded area?

A) $2^2 < (x - 2)^2 + y^2 < 3^2$
B) $2^2 \leq (x - 2)^2 + y^2 \leq 3^2$
C) $2^2 \leq (x - 2)^2 + y^2 \leq 3$
D) $2^2 \leq x^2 + (y - 2)^2 \leq 3$
E) $2^2 \leq x^2 + (y - 2)^2 \leq 4^2$

12. $z = 1 + \cos 10° + i \cdot \sin 10° \Rightarrow$ **|z| = ?**

A) $2\cos 5°$
B) $\sqrt{2}\cos 5°$
C) $2\cos 10°$
D) $\sqrt{2}\cos 10°$
E) $\cos 10°$

13. $|z| \leq 1$,

$A = |z+3 - 4i| \Rightarrow$ **max(A) + min(A) = ?**

A) 4
B) 5
C) 6
D) 8
E) 10

14. $|z| \leq 3 \Rightarrow$ **max(|z – 12 – 9i|) = ?**

A) 12
B) 14
C) 16
D) 18
E) 21

15. $i^2 = -1$,

$|z| \leq 10$

$\Rightarrow$ **min(|z – 2 + $\sqrt{5}$i|) = ?**

A) 7
B) 13
C) $10\sqrt{2}$
D) 14
E) 19

YANITLAR / ANSWERS

1	2	3	4	5	6	7	8
B	B	D	D	A	A	B	A
9	**10**	**11**	**12**	**13**	**14**	**15**	
E	E	B	A	E	D	A	

1. $\dfrac{i^{1975}+i^{1955}+i^{2008}}{i^{1938}+2\cdot i^{1881}}=?$

A) –i B) –1 C) 0 D) 1 E) i

2. $z=\left(\dfrac{1-i}{1+i}\right)^{63}+\left(\dfrac{1+i}{1-i}\right)^{64} \Rightarrow z=?$

A) i B) i – 1 C) 1 – i D) –1 – i E) 1 + i

3. $z_1 = a + 3 - 2i$

$z_2 = 1 + 3bi - 5i$

$z_1 = z_2 \Rightarrow \mathbf{Re(z_1) + Im(z_1) = ?}$

A) –4 B) –3 C) –2 D) –1 E) 0

4. $z=\dfrac{2-i}{2+3i} \Rightarrow \mathrm{Im}(z^{-1})=?$

A) $\frac{5}{8}$ B) $\frac{3}{8}$ C) $\frac{8}{3}$ D) 2 E) $\frac{8}{5}$

5. $x \neq 0,$

$z = x - 3 + 3\sqrt{3}i$

$|z| = 6 \Rightarrow x = ?$

A) 3 B) 5 C) 6 D) 9 E) 12

6. $z_1=\sqrt{3}-\sqrt{7}i,\ z_2=4+\sqrt{14}i$

$z_3=2\sqrt{5}+\sqrt{7}i$

$\Rightarrow \left|\dfrac{z_1\cdot z_2}{z_3}\right|=?$

A) $\frac{10}{3}$ B) $\frac{14}{3}$ C) $\frac{17}{3}$ D) $\frac{19}{3}$ E) $\frac{20}{3}$

7. $ax^2 + bx + c = 0$, ÇK(SS) = $\{x_1, x_2\}$

$x_1 = 1 - 2i \Rightarrow$ **$a - b + c = ?$**

A) 4 B) 5 C) 6 D) 7 E) 8

8. $z_1 \Delta z_2 = z_1 + z_2 - |z_1^2 \cdot z_2^2|$

$\Rightarrow \mathbf{(2 + i)\ \Delta\ (3 - i) = ?}$

A) –45 B) –40 C) –35 D) 35 E) 40

9. $zi \cdot 2 + 3 = \dfrac{\overline{z}}{i} + 5i$

$\Rightarrow$ **Im(z) + Re(z) = ?**

A) $\dfrac{10}{3}$ B) $\dfrac{11}{3}$ C) 4 D) $\dfrac{13}{3}$ E) $\dfrac{14}{3}$

10. $z = \dfrac{(3 + xi) \cdot (\sqrt{3} + i)^4}{\sqrt{7} - 3i}$

$|z| = 24,\ x \in R^+ \Rightarrow$ **x = ?**

A) $2\sqrt{3}$ B) $3\sqrt{3}$ C) $4\sqrt{3}$ D) $5\sqrt{3}$ E) $6\sqrt{3}$

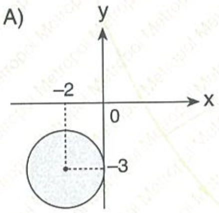
11. $x^2 + (3m - 1)x + 5n + 3 = 0$

ÇK(SS) = $\{x_1, x_2\}$, $x_1 = 8 - 3i \Rightarrow$ **m + n = ?**

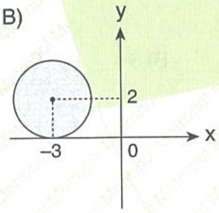
A) 9 B) 10 C) 11 D) 12 E) 13

12. $|z - 3 + 2i| \le 2 \Rightarrow$ **ÇK(SS) = ?**

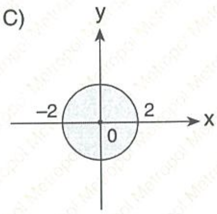
A)



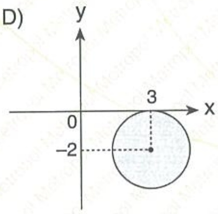
B)



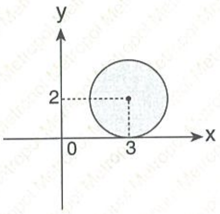
C)



D)



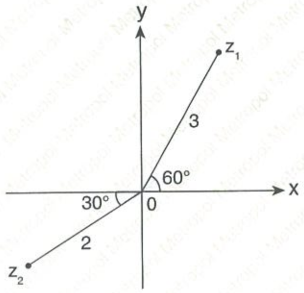
E)



13. $|z_1| = 3$ br(u),

$|z_2| = 2$ br(u)

$\Rightarrow$ **$|z_1 - z_2|$ = ? br (u)**


y
z1
3
60°
x
30°
0
2
z2

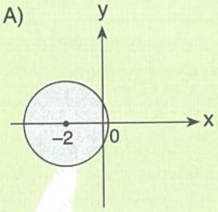
A) $\sqrt{13 - 6\sqrt{3}}$ B) $\sqrt{13 + 6\sqrt{3}}$ C) $\sqrt{13 - 2\sqrt{3}}$ D) $\sqrt{13 + 2\sqrt{3}}$ E) $\sqrt{19}$

14. $|z - 2 + 5i| = 1 \Rightarrow$ **max(|z + 3 – 7i|) = ?**

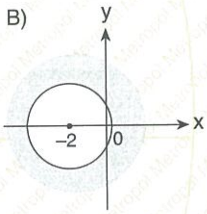
A) 12 B) 13 C) 14 D) 15 E) 16

15. $2 \cdot |z| \ge |z + 6| \Rightarrow$ **ÇK(SS) = ?**

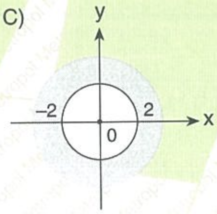
A)



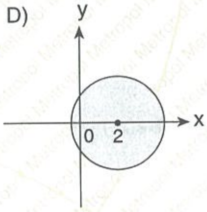
B)



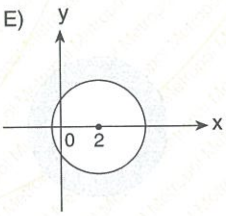
C)



D)



E)



YANITLAR / ANSWERS

1	2	3	4	5	6	7	8
B	E	D	E	C	A	E	A
9	10	11	12	13	14	15	
E	B	A	D	A	C	E	

BÖLÜM CHAPTER 23

TARİHSEL NOT / HISTORICAL NOTE

JOHN NAPİER [1550 - 1617]

Merchiston Baronu ve İskoçyalı bir matematikçi olan Napier, logaritmanın bulucusu olarak bilinir. 1, 2, 3, ... şeklindeki aritmetik dizi ile, buna karşılık gelen 10, 100, 1000, ... biçimindeki geometrik dizi arasındaki ilişkiyi gördü. 1614 yılında yazdığı "Logaritma Kurallarının Tanımı" adlı eserinde, aritmetik dizi ile geometrik dizinin karşılaştırılmasından, matematiğe logaritma kavramını getirdi. Günümüzdekilerden farklı olarak kurulan bu diziler, logaritmayı, sayısının azalan bir fonksiyonu olarak tanımlıyordu. Buradaki aritmetik dizi, geometrik dizinin logaritmasıdır.

The Baron of Merchiston and a Scottish mathematician. He is known as the finder of the logarithm. He saw the relationship between the arithmetic sequence 1, 2, 3, ... and the corresponding geometric sequence 10, 100, 1000, ... In his work titled "Definition of the Logarithm Rules" published in 1614, he introduced the concept of mathematical logarithm derived from the comparison of the arithmetic sequence with the geometric sequence. These sequences, which were established differently then, defined the logarithm as a decreasing function of the numbers. The arithmetic sequence here is the logarithm of the geometric sequence.

LOGARİTMA / LOGARITHM

Bu bölüm, 134 test sorusu içermektedir.
This chapter includes 134 test questions.

BÖLÜM / CHAPTER 23

LOGARİTMA / LOGARITHM

- Logaritma / Logarithm .... 427 – 448

BÖLÜM

23

CHAPTER

LOGARİTMA
LOGARITHM

LOGARİTMİK FONKSİYONLAR / LOGARITHMIC FUNCTIONS

Tanım: $a \in R$, $a > 0$, $a \neq 1$ ve $x > 0$ olmak üzere, $f(x) = a^x$ fonksiyonunun tersi ***x'in a tabanında logaritması*** olarak adlandırılır. **$y = \log_a x$** ile gösterilir.

Definition: The inverse of $f(x) = a^x$ is called **the logarithm of x to the base a**, where $a \in R$, $a > 0$, $a \neq 1$ and $x > 0$. It is denoted by $y = \log_a x$.

Önemli / Important

- $a > 0$, $a \neq 1$, $f(x) = a^x$ ise f(x) bir üstel fonksiyondur.
- If $a > 0$, $a \neq 1$, $f(x) = a^x$, then f(x) is an exponential function.

Özellik / Property

★ 1'in herhangi a tabanındaki logaritması 0'dır.

★ The logarithm of 1 to any base, is zero.

$$\log_a 1 = 0$$

Özellik / Property

★ Sayının, aynı tabanındaki logaritması 1'dir.

★ The logarithm of a number to the same number as its base is equal to 1.

$$\log_a a = 1$$

Özellik / Property

★ Bir sayının kuvvetinin logaritması, sayının logaritması ile kuvvetinin çarpımına eşittir.

★ The logarithm of a number raised to a power is the power multiplied by the logarithm of the number.

$$\log_a x^n = n \cdot \log_a x$$

Özellik / Property

★ $\log_{a^n} x^m = \frac{m}{n} \cdot \log_a x$

Özellik / Property

★ $a^{\log_a x} = x$ $\left[y = \log_a x,\ a^y = x \Rightarrow a^{\log_a x} = x\right]$

Özellik / Property

★ İki sayının çarpımının logaritması, sayıların logaritmalarının toplamıdır.

★ The logarithm of the product of two numbers is the sum of the logarithms of the two numbers.

$$\log_a(x \cdot y) = \log_a x + \log_a y$$

Özellik / Property

★ İki sayının bölümünün logaritması, bölünenin logaritmasından bölenin logaritması çıkarılarak bulunur.

★ The logarithm of the division of two numbers is the logarithm of the dividend minus the logarithm of the divisor.

$$\log_a\left(\frac{x}{y}\right) = \log_a x - \log_a y$$

Özellik / Property

★ Taban Değiştirme :

★ Changing the Base :

$$\log_a x = \frac{\log_b x}{\log_b a}$$

Özellik / Property

★ $\log_a x = \frac{1}{\log_x a}$ $\left[\log_a x = \frac{\log_x x}{\log_x a} = \frac{1}{\log_x a}\right]$

Özellik / Property

★ $\log_{10} x = \log x$

Özellik / Property

★ Doğal logaritma / Natural logarithm :

$\log_e x = \ln x$, $e \cong 2,71\ldots$

Özellik / Property

★ $\log_a b \cdot \log_b c \cdot \log_c d = \log_a d$, $\left[\frac{\log b}{\log a} \cdot \frac{\log c}{\log b} \cdot \frac{\log d}{\log c} = \frac{\log d}{\log a} = \log_a d\right]$

LOGARİTMA FONKSİYONUNUN TERSİ / INVERSE of a LOGARITHM FUNCTION

- $f(x) = \log_a x \Leftrightarrow f^{-1}(x) = a^x$

LOGARİTMİK DENKLEMLER / LOGARITHMIC EQUATIONS

- Logaritma terimi içeren ve logaritma özellikleri kullanılarak çözülen denklemlere logaritmik denklemler denir.

  Bir logaritmik denklemi çözmek için, denklem üstel biçimde yazılır ve değişken için çözüm yapılır.
- The equations involving a logarithmic term and solved by using the properties of logarithm are called *logarithmic equations.*

  To solve a logarithmic equation, rewrite the equation in exponential form and solve for the variable.
- **Logaritma fonksiyonunun en geniş tanım kümesi:**
- **Domain of a logarithmic function:**

  $f: R^+ \rightarrow R, \; f(x) = \log_a x$

  ★ $a > 0$

  ★ $a \neq 1$

  ★ $x > 0$

LOGARİTMİK EŞİTSİZLİKLER / LOGARITHMIC INEQUALITIES

- $a \in R, \; a > 0$

  $a > 1, \; \log_a f(x) < \log_a g(x) \Rightarrow 0 < f(x) < g(x)$

  $0 < a < 1, \; \log_a f(x) < \log_a g(x) \Rightarrow f(x) > g(x) > 0$

1. $\log_6 2 + \log_6 3 = ?$

A) 2 B) 1 C) 0 D) -1 E) -2

2. $\log_9 27 = ?$

A) $\frac{1}{3}$ B) 3 C) 6 D) $\frac{2}{3}$ E) $\frac{3}{2}$

3. $y = \log_7 \frac{1}{x}, \quad x = 7^5$

$\Rightarrow$ **y = ?**

A) 1 B) 0 C) -5 D) -7 E) -49

4. $\log_3 5 = a \quad \Rightarrow \quad \log_5 15 = ?$

A) $a + 1$ B) $a - 1$ C) $1 + \frac{1}{a}$ D) $\frac{a-1}{a}$ E) $\frac{1}{a}$

5. $\log_{\frac{1}{\sqrt{2}}} 8 = ?$

A) 0 B) -2 C) -4 D) -6 E) -8

6. $\log_{\sqrt{8}} b = \frac{10}{3} \Rightarrow$ **b = ?**

A) 8 B) 16 C) 32 D) 64 E) 128

7. $\log_{\frac{1}{x}} 4 = -2 \Rightarrow$ **x = ?**

A) 1 B) 2 C) 3 D) 4 E) 5

8. $\log_x 4 = -\frac{1}{3} \Rightarrow$ **x = ?**

A) $\frac{1}{4}$ B) $\frac{1}{16}$ C) $\frac{1}{24}$ D) $\frac{1}{64}$ E) -1

9. $\left.\begin{array}{l}\log_3 2 = a\\ \log_3 5 = b\end{array}\right\} \Rightarrow \mathbf{\log 30 = ?}$

A) $1+\frac{1}{a+b}$ B) $1-\frac{1}{a+b}$ C) $\frac{a}{b+1}$

D) $a-b+1$ E) $1-\frac{a}{b}$

10. $\log_3 [\log_2 (\log_4 (x-1))] = 0$

$\Rightarrow \mathbf{x = ?}$

A) 17 B) 18 C) 19 D) 20 E) 21

11. $(\log_x 8)^{\log_5 125} = 27 \Rightarrow \mathbf{x = ?}$

A) 5 B) 4 C) 3 D) 2 E) 1

12. $\log_2 m = \log_{\frac{1}{2}} n$,

$m + n = 5 \Rightarrow \mathbf{m^2 + n^2 = ?}$

A) 27 B) 26 C) 25 D) 24 E) 23

13. $\left.\begin{array}{l}\log(xy) = 2\\ \log\left(\frac{x}{y}\right) = -2\end{array}\right\} \Rightarrow \mathbf{y = ?}$

A) 1 B) 10 C) 100 D) 1000 E) $\frac{1}{10}$

14. $\log 2 = a \Rightarrow \mathbf{\log 25 = ?}$

A) $1-a$ B) $2-a$ C) $1+a$

D) $2+a$ E) $2-2a$

15. $\mathbf{\log_{\sqrt{2}} 16 + \log_3 \sqrt{27} + \log_{25} 5 = ?}$

A) 10 B) 9 C) 8 D) 7 E) 6

16. $\log_7(\log_2 16) = \frac{1}{\log_x 49} \Rightarrow \mathbf{x = ?}$

A) 64 B) 16 C) 8 D) 4 E) 2

17. $\log_3 12 = a \Rightarrow \mathbf{\log_3 18 = ?}$

A) $\frac{a+1}{2}$ B) $\frac{a+2}{2}$ C) $\frac{a+3}{2}$

D) $\frac{a-1}{2}$ E) $\frac{a-2}{2}$

YANITLAR / ANSWERS

1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
B	E	C	C	D	C	B	D	A
10	**11**	**12**	**13**	**14**	**15**	**16**	**17**	
A	D	E	C	E	A	B	C	

1. $\log_2(\log_2(x-1)) = 1 \Rightarrow$ **$x = ?$**

A) 9 B) 8 C) 7 D) 6 E) 5

2. $\left.\begin{array}{l} f(x)=3^{x+2} \\ g(x)=\log_3 x \end{array}\right\} \Rightarrow$ **$(gof)(2)=?$**

A) 2 B) 3 C) 4 D) 6 E) 9

3. $a^3 = b^5 \Rightarrow$ **$\log_a b = ?$**

A) $\frac{5}{3}$ B) $\frac{4}{3}$ C) 1 D) $\frac{3}{5}$ E) $\frac{6}{5}$

4. **$5^{\log_2 3} - 3^{\log_2 5} = ?$**

A) –2 B) –1 C) 0 D) 1 E) 2

5. $\log 5 = a \Rightarrow$ **$\frac{1}{\log 8} = ?$**

A) $3-3a$ B) $\frac{1}{3-3a}$ C) $\frac{1}{1-a}$ D) $1-a$ E) $\frac{1}{3+3a}$

6. $\left.\begin{array}{l} \log_2 5 = x \\ \log_2 3 = y \end{array}\right\} \Rightarrow$ **$\log 75 = ?$**

A) $\frac{2x+y}{1-x}$ B) $\frac{x+y}{1+x}$ C) $\frac{x+2y}{1+x}$ D) $\frac{2x+y}{1+x}$ E) $\frac{2x-y}{1+x}$

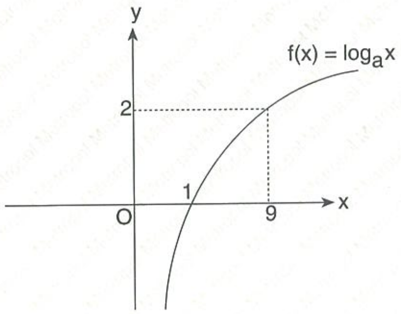
7. $\log_2 3 = x \Rightarrow$ **$\log_9 8 = ?$**

A) $\frac{3x}{2}$ B) $\frac{2x}{3}$ C) $\frac{2}{3x}$ D) $\frac{3}{2x}$ E) x

8. $\log_a a \cdot b = 3 \Rightarrow$ **$\log_{a \cdot b} b = ?$**

A) $\frac{2}{3}$ B) 1 C) $\frac{3}{2}$ D) $\frac{4}{3}$ E) $\frac{5}{3}$

9.



$\Rightarrow$ **f(27) = ?**

A) $\frac{3}{2}$ B) 1 C) 3 D) 2 E) $\frac{5}{2}$

10. $\log_3 18 = a \Rightarrow \boldsymbol{\log_{24} 3 = ?}$

A) $\frac{1}{3a-6}$ B) $\frac{1}{3a-4}$ C) $\frac{1}{3a-3}$ D) $\frac{1}{3a-1}$ E) $\frac{1}{3a-5}$

11. $2^{4+\log_4 9} = ?$

A) 24 B) 48 C) 64 D) 72 E) 96

12. $\left.\begin{array}{l}\log_2 9 = a\\ \log_3 2 = b\end{array}\right\} \Rightarrow \boldsymbol{a \cdot b = ?}$

A) 2 B) 3 C) 4 D) 5 E) 6

13. $3^{\log_2(1-\frac{1}{2})} \cdot 3^{\log_2(1-\frac{1}{3})} \cdot \ldots \cdot 3^{\log_2(1-\frac{1}{32})} = ?$

A) 3^{-1} B) 3^{-2} C) 3^{-3} D) 3^{-4} E) 3^{-5}

14. $\frac{1}{\log_2 13!} + \frac{1}{\log_3 13!} + \frac{1}{\log_4 13!} + \ldots + \frac{1}{\log_{13} 13!} = ?$

A) $\frac{1}{13}$ B) $\frac{1}{2}$ C) 1 D) 2 E) 13

15. $\log_3(\log_2 8) = \log_9(x-1) \Rightarrow \boldsymbol{x = ?}$

A) 7 B) 8 C) 9 D) 10 E) 11

16. $3^x = 5^y \Rightarrow \frac{x}{x+y} = ?$

A) $\log_3 5$ B) $\log_5 3$ C) $\log_5 15$ D) $\log_3 15$ E) $\log_{15} 5$

YANITLAR / ANSWERS

1	2	3	4	5	6	7	8
E	C	D	C	B	D	D	A
9	**10**	**11**	**12**	**13**	**14**	**15**	**16**
C	E	B	A	E	C	D	E

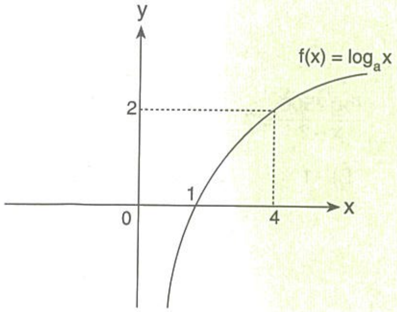
1. $\log_9(\log_4 16) = \log_3 x \Rightarrow$ **x = ?**

A) 4 B) $\sqrt{3}$ C) 2 D) $\sqrt{2}$ E) 1

2. $\log 5 = a \Rightarrow$ **log2 = ?**

A) a – 1 B) a + 1 C) 2a D) 1 – a E) $\frac{1}{a}$

3.



$\Rightarrow$ **f(8) + f⁻¹(1) = ?**

A) 4 B) 5 C) 6 D) 7 E) 8

4. $\dfrac{1}{\log_5 90} + \dfrac{1}{\log_3 90} + \dfrac{1}{\log_6 90} = ?$

A) $\frac{1}{2}$ B) 1 C) 2 D) $\frac{2}{3}$ E) 5

5. $\left.\begin{aligned}\log_3(a+b)=2\\ \log_3(a-b)=1\end{aligned}\right\} \Rightarrow$ **b = ?**

A) 3 B) 4 C) 5 D) 6 E) 8

6. $2^x = 27^{\log_9 8} \Rightarrow$ **x = ?**

A) 9 B) $\frac{15}{2}$ C) 6 D) 5 E) $\frac{9}{2}$

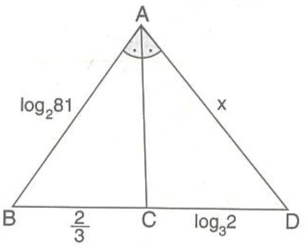
7. $|AB| = (\log_2 81)$ cm,

$|BC) = \frac{2}{3}$ cm,

$|DC| = (\log_3 2)$ cm,

$m(\widehat{BAC}) = m(\widehat{DAC})$

$\Rightarrow$ **|AD| = x = ? cm**



A) 8 B) 6 C) 5 D) 4 E) 3

8. $x = \log_3 \sqrt[3]{\sqrt[4]{27}} \Rightarrow$ **x = ?**

A) $\frac{1}{12}$ B) $\frac{1}{8}$ C) $\frac{1}{6}$ D) $\frac{1}{4}$ E) $\frac{1}{3}$

9. $\log 29! = a \Rightarrow \mathbf{\log 30! = ?}$

A) a B) 30a C) 30 + a D) log3 + a + 1 E) a + 3

10. $\log_3\left(1-\frac{1}{2}\right)+\log_3\left(1-\frac{1}{3}\right)+\ldots+\log_3\left(1-\frac{1}{27}\right)=?$

A) –3 B) –2 C) –1 D) $\frac{1}{2}$ E) $\frac{1}{3}$

11. $\left.\begin{array}{l}\log_a b = x\\ \log_c b = y\end{array}\right\} \Rightarrow \mathbf{\log_{a\cdot c} b = ?}$

A) $\frac{xy}{x+y}$ B) $\frac{x+y}{xy}$ C) $\frac{x}{x+y}$ D) $\frac{y}{x+y}$ E) $\frac{x+y}{x}$

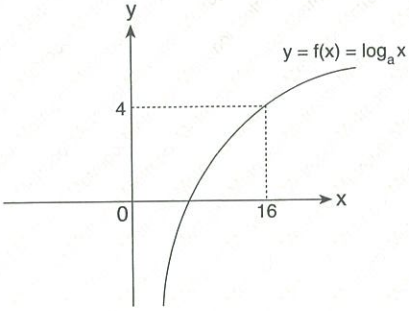
12. $x = \frac{\log 9 + 2}{\log 30} \Rightarrow \mathbf{x = ?}$

A) 3 B) 2 C) 1 D) – 1 E) – 2

13. $\ln(a-1) = x + \ln x \Rightarrow \mathbf{a = ?}$

A) $e^{2x}+1$ B) $x\cdot e^x - 1$ C) $x\cdot e^x + 1$ D) $e^{2x}-1$ E) $e^x + 1$

14.



$\Rightarrow \mathbf{f(32) = ?}$

A) 1 B) 2 C) 3 D) 4 E) 5

15. $(\log_2 x)^2 - 2\log_2 x + 1 = 0 \Rightarrow \mathbf{x = ?}$

A) 0 B) 1 C) 2 D) 3 E) 4

16. $\log 2 = x \Rightarrow \mathbf{\frac{\log 2500}{x-2} = ?}$

A) $-\frac{1}{2}$ B) –1 C) –2 D) –3 E) –4

17. $f(x) = 2 - \log_3(2x-1)$

$\Rightarrow \mathbf{f(5) + f^{-1}(2) = ?}$

A) 2 B) 1 C) 0 D) –2 E) –4

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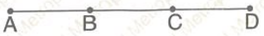
1	2	3	4	5	6	7	8	9
D	D	B	B	A	E	B	D	D
10	11	12	13	14	15	16	17	
A	A	B	C	E	C	C	B	

1.



$|AB| = \log(2x+1)$, $|CD| = \log x$

$|AC| = \log 8$, $|BD| = 3 \cdot \log\left[\frac{1}{3} \cdot \sqrt[3]{54}\right]$

$\Rightarrow$ **$x = ?$**

A) $\frac{1}{2}$ B) 1 C) $\frac{3}{2}$ D) 2 E) $\frac{5}{2}$

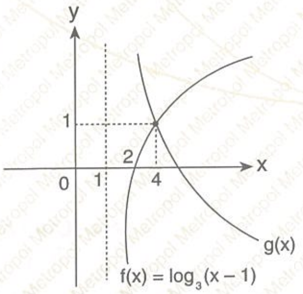
2. $\log x = 72 \Rightarrow$ **$\log\sqrt[4]{x^3\sqrt[3]{x\sqrt{x}}} = ?$**

A) 7 B) 8 C) 14 D) 16 E) 27

3. $e^x + 4e^{-x} = 5 \Rightarrow$ **ÇK(SS) = ?**

A) $\{0\}$ B) $\{0, e\}$ C) $\{\ln 4\}$ D) $\{0, \ln 4\}$ E) $\{\varnothing\}$

4. **$f(10) + (f^{-1} \circ g)(4) = ?$**



A) 5 B) 6 C) 8 D) 9 E) 10

5. $(\ln x^2 - 3)^2 + \ln^2 x - 26 = 0$

$\Rightarrow$ **ÇK(SS) = ?**

A) $\{e^{-1}\}$ B) $\{e^{\frac{7}{5}}\}$ C) $\{e^{\frac{17}{5}}\}$ D) $\{e^{\frac{7}{5}}, e^{\frac{17}{5}}\}$ E) $\{e^{-1}, e^{\frac{17}{5}}\}$

6. $\log x = t \Rightarrow$ **$\log x^2 = ?$**

A) t B) 2t C) 3t D) 4t E) 5t

7. $\left.\begin{array}{l}\log_3 x = a \\ \log_3 y = b\end{array}\right\} \Rightarrow$ **$\log_3(x^3 \cdot y^2) = ?$**

A) 2a – b B) 3a + 2b C) 3a – 2b D) 2a + b E) 2a – 3b

8. $0 < x < \pi$

$\frac{1}{e^{\ln(\sin x)}} + \log_2 4^{\cot x} = 0 \Rightarrow$ **$x = ?$**

A) $\frac{\pi}{3}$ B) $\frac{\pi}{4}$ C) $\frac{\pi}{6}$ D) $\frac{2\pi}{3}$ E) $\frac{5\pi}{6}$

9. $\left.\begin{matrix}\log 2 = x\\ \log y = 4\end{matrix}\right\} \Rightarrow \mathbf{\frac{\log 5}{\sqrt{y}} = ?}$

A) 1 B) $1 - x$ C) $\frac{1-x}{10}$

D) $\frac{1-x}{100}$ E) $x + 1$

10. $\log x = 1,2 \Rightarrow \mathbf{x^{10} = ?}$

A) 10^6 B) 10^8 C) 10^{10}

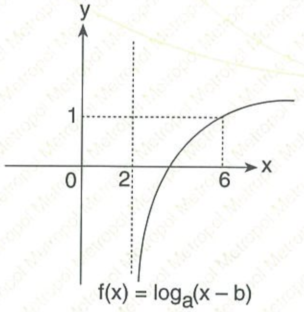
D) 10^{12} E) 10^{14}

11. $\log_5 9 = x,\ \log_8 5 = y \Rightarrow \mathbf{\log 3 = ?}$

A) $\frac{2xy}{6y+2}$ B) $\frac{3xy}{6y+2}$ C) $\frac{3xy}{4y+2}$

D) $\frac{2xy}{6y-2}$ E) $\frac{2xy}{4y-3}$

12. $\mathbf{f(3) + f^{-1}(2) + a + b = ?}$



A) 18 B) 20 C) 22 D) 24 E) 26

13. $f(x) = \log_{(x-2)}(36 - x^2)$

$x \in Z \Rightarrow \mathbf{\sum x = ?}$

A) 9 B) 12 C) 15 D) 16 E) 18

14. $\ln x = 4$

$x^{\log_a 7} + 7^{\log_a x} = 2e^4 \Rightarrow \mathbf{a = ?}$

A) 4 B) 5 C) 6 D) 7 E) 9

15. $\ln(y + 2) = x + x \cdot \ln x \Rightarrow \mathbf{y = ?}$

A) $x \cdot e^x + 2$ B) $x^x \cdot e^x + 2$ C) $x^2 \cdot e^x + 2$

D) $x^2 \cdot e^x - 2$ E) $x^x \cdot e^x - 2$

16. $\log_{15} 21 = a,\ \log_3 49 = b$

$\Rightarrow \mathbf{\log_3 5 = ?}$

A) $\frac{b+2}{2a}$ B) $\frac{b+2-2a}{2a}$ C) $\frac{b+2-2a}{a}$

D) $\frac{b+2-2a}{a^2}$ E) $\frac{b-a+2}{2a}$

YANITLAR / ANSWERS

1	2	3	4	5	6	7	8
A	E	D	B	E	B	B	D
9	**10**	**11**	**12**	**13**	**14**	**15**	**16**
D	D	B	D	A	D	E	B

1. $7^{\log_7(\log_3 x)} = 6 \Rightarrow \mathbf{\log_9 x = ?}$

A) 6 B) 5 C) 4 D) 3 E) 2

2. $\left.\begin{array}{l} \log 2 = a \\ \log 3 = b \\ \log 7 = c \end{array}\right\} \Rightarrow \mathbf{\log 4536 = ?}$

A) $2a + 3b + c$ B) $3a + 4b + c$ C) $2a + 4b + c$ D) $2a + 5b + 2c$ E) $a + 4b + 3c$

3. $\log\left(\frac{2z^2}{x^2}\right) + \log\left(\frac{25x}{y^2}\right) + \log\left(\frac{20xy^2}{z^2}\right) = ?$

A) $\frac{5}{2}$ B) 3 C) 5 D) 10 E) 20

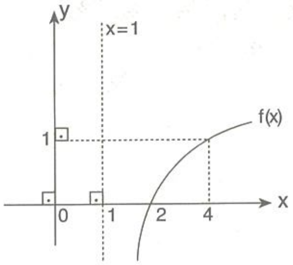
4. $\log_{\frac{1}{2}} x = \log_4 y^3 \Rightarrow \mathbf{\log_{x^2} y^3 = ?}$

A) –3 B) –2 C) –1 D) $\frac{1}{2}$ E) $\frac{3}{2}$

5. $\frac{2^{x+1}}{\log_3 16} = \frac{\log_2 9}{2^{2x-1}} \Rightarrow \mathbf{x = ?}$

A) $\frac{3}{4}$ B) 1 C) $1\frac{1}{4}$ D) $1\frac{2}{3}$ E) 2

6. **f(x) = ?**



A) $\log_2(x-2)$ B) $\log_2(6-x)$ C) $\log_3(x-1)$ D) $\log_5(x+1)$ E) $\log_4 x$

7. $f: \left(-\frac{4}{7}, \infty\right) \longrightarrow \mathbb{R}$,

$f(x) = \log_5(7x + 4)$,

$f^{-1}(a) = 3 \Rightarrow \mathbf{a = ?}$

A) 1 B) 2 C) 3 D) 4 E) 5

8. $\log_{25} \sqrt[3]{5\sqrt[3]{5\sqrt[3]{5\ldots}}} = ?$

A) $\sqrt{5}$ B) $\sqrt[3]{5}$ C) 4 D) 1 E) $\frac{1}{4}$

9. $a = \log_2 6 \Rightarrow \mathbf{\log_3 2 + \log_6 1024 = ?}$

A) $\frac{a^2}{10a+1}$ B) $\frac{a^2}{11a-1}$ C) $\frac{11a-10}{a^2-1}$ D) $\frac{10a-1}{a^2-1}$ E) $\frac{11a-1}{a^2-a}$

10. $\log\sqrt{a} = b \Rightarrow \mathbf{\ln a^3 = ?}$

A) $\ln a - b$ B) $a \cdot \ln b$ C) $b \cdot \ln a$ D) $6b \cdot \ln 10$ E) $\log a + b$

11. $\log_3 21 = a \Rightarrow \mathbf{\log_{441} 189 = ?}$

A) $\frac{a+2}{a}$ B) $\frac{a-1}{2a}$ C) $\frac{a+2}{2a}$ D) $\frac{a}{a+2}$ E) $\frac{a}{a-2}$

12. $f(x) = 3^x$

$g(x) = \log_4(x-2) \Rightarrow \mathbf{(gof^{-1})^{-1}(2) = ?}$

A) 3^{16} B) 3^{17} C) 3^{18} D) 3^{19} E) 3^{20}

13. $\log_5 7 = a \Rightarrow \mathbf{\log_{25} 1715 = ?}$

A) $3a$ B) $\frac{6a+1}{2}$ C) $\frac{1+3a}{2}$ D) $\frac{2+3a}{2}$ E) $2 + 3a$

14. $\left.\begin{array}{l}\log_a 343 = 6 \\ \log_7 a = b\end{array}\right\} \Rightarrow \mathbf{a \cdot b = ?}$

A) 7 B) $\sqrt{7}$ C) $\frac{\sqrt{7}}{2}$ D) $\frac{\sqrt{7}}{3}$ E) $\frac{\sqrt{7}}{4}$

15. $a = \sqrt{3^{2+\frac{1}{2}\log_3 81}} \Rightarrow \mathbf{\log_{27} a = ?}$

A) $\frac{1}{2}$ B) $\frac{3}{4}$ C) $\frac{1}{3}$ D) $\frac{5}{8}$ E) $\frac{2}{3}$

16. $0 < x < \frac{\pi}{2}$

$\log_2(\tan x) + \log_2(\cos x) = -1 \Rightarrow \mathbf{x = ?}$

A) $\frac{\pi}{12}$ B) $\frac{\pi}{6}$ C) $\frac{\pi}{4}$ D) $\frac{\pi}{3}$ E) $\frac{\pi}{2}$

17. $e^{2x} - 3e^x - 4 = 0 \Rightarrow \mathbf{x = ?}$

A) 0 B) 1 C) ln2 D) ln4 E) ln8

18. $f(x) = 3\log_2^2 x - \log_2 x - 2$

$f(x_1) = f(x_2) = 0 \Rightarrow \mathbf{x_1 \cdot x_2 = ?}$

A) $\sqrt{2}$ B) 2 C) $\frac{\sqrt{2}}{2}$ D) $\sqrt[3]{2}$ E) $\frac{\sqrt[3]{2}}{2}$

YANITLAR / ANSWERS

1	2	3	4	5	6	7	8	9
D	B	B	C	B	C	B	E	C
10	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**
D	C	C	C	C	E	B	D	D

1. $\log_7[6+\log_3(\log_2 x)] = 1 \Rightarrow$ **$x = ?$**

A) 7 B) 8 C) 9 D) 10 E) 12

2. $f:\left(\frac{2}{5},\infty\right) \longrightarrow R$

$f(x) = \log_3(5x-2) \Rightarrow$ **$f^{-1}(x) = ?$**

A) $\frac{3^x+2}{5}$ B) $\frac{3^x-2}{5}$ C) $\frac{5^x+2}{3}$

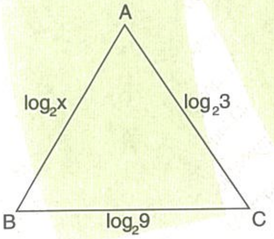
D) $\frac{5^x-2}{3}$ E) $\frac{3^x+5}{2}$

3. $|AB| = \log_2 x$,

$|BC| = \log_2 9$,

$|AC| = \log_2 3$

$\Rightarrow ? < x < ?$



A) $3 < x < 9$ B) $3 < x < 18$ C) $3 < x < 27$

D) $9 < x < 18$ E) $9 < x < 27$

4. $f(x) = \log_{(5-x)}(x+1)$,

$x \in Z \Rightarrow$ **$\sum x = ?$**

A) 6 B) 7 C) 8 D) 9 E) 10

5. $\log\{\log_2[\log_3(\ln x)]\} = 0$

$\Rightarrow x = ?$

A) e B) e^9 C) e^{10} D) e^{81} E) e^{100}

6. $\log_8 \cos 22{,}5° + \log_8 \sin 22{,}5° = x \Rightarrow$ **$x = ?$**

A) -1 B) $-\frac{1}{2}$ C) $-\frac{1}{4}$ D) $\frac{1}{2}$ E) $\frac{1}{4}$

7. $\log_5 2 \cdot \log_6 5 \cdot \log_7 6 \cdot \ldots \cdot \log_{1024} 1025 = x \Rightarrow$ **$5x = ?$**

A) $\frac{1}{10}$ B) $\frac{1}{8}$ C) $\frac{1}{4}$ D) $\frac{1}{2}$ E) 1

8. $f(x) = \log_2 x \cdot \log_{25} 4 \cdot \ln e^7$

$\Rightarrow 3^{\log_7 f(5)} = ?$

A) 1 B) 2 C) 3 D) 7 E) 9

9. $3^{\log_x 5} + 5^{\log_x 3} = 18 \Rightarrow$ **x = ?**

A) $\sqrt[3]{3}$ B) $\sqrt[3]{5}$ C) $\sqrt{3}$ D) $\sqrt{5}$ E) 5

10. $\log_3 5 = x$, $\log_2 3 = y$

$\Rightarrow$ **$\log_{20} 45$ = ?**

A) $\dfrac{y(x-2)}{xy-2}$ B) $\dfrac{y(x+2)}{xy+2}$ C) $\dfrac{x(y+2)}{xy+2}$ D) $\dfrac{x(y+2)}{xy-2}$ E) $\dfrac{y(x+2)}{xy-2}$

11. $\log_3 18 = a \Rightarrow$ **$\log_{24} 6$ = ?**

A) $\dfrac{a-1}{3a-5}$ B) $\dfrac{a-3}{5a-1}$ C) $\dfrac{5a-3}{a-1}$ D) $\dfrac{5a-1}{a-3}$ E) $\dfrac{3a-1}{5a-3}$

12. $9^{x+2} = 10^{2y} \Rightarrow$ **log3 = ?**

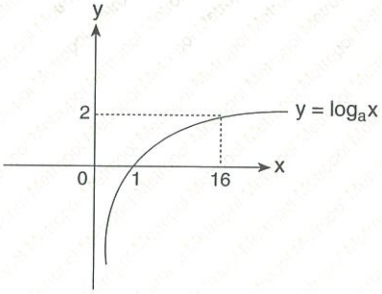
A) $\dfrac{2y}{x-2}$ B) $\dfrac{2y}{x+2}$ C) $\dfrac{y}{x-2}$ D) $\dfrac{y}{x+2}$ E) $\dfrac{y+2}{x}$

13. $\log(3a + 2b) = \log a + \log b + \log 5 - \log 2 \Rightarrow$ **b = ?**

A) $\dfrac{3a}{5a-4}$ B) $\dfrac{6a}{5a-4}$ C) $\dfrac{6a}{5a+4}$ D) $\dfrac{3a}{5a+4}$ E) $\dfrac{5a}{6a-4}$

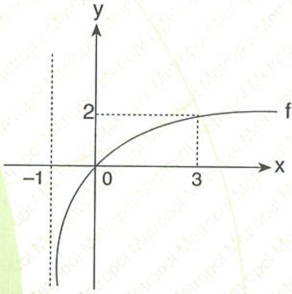
14. $y = \log_a x$

$\Rightarrow$ **a = ?**



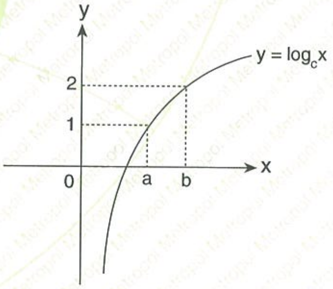
A) 1 B) 2 C) 3 D) 4 E) 5

15. **f(x) = y = ?**



A) $\log_3(x + 1)$ B) $\log_3(x + 1)^2$ C) $\log_3(x - 1)$ D) $\log_2(x - 1)$ E) $\log_2(x + 1)$

16. **b = ?**



A) $\dfrac{a}{2}$ B) a C) 2a D) a^2 E) $2a^2$

YANITLAR / ANSWERS

1	2	3	4	5	6	7	8
B	A	C	A	B	B	D	C
9	10	11	12	13	14	15	16
D	B	A	D	B	D	E	D

1. $\log_3(1+3\ln x)=2 \Rightarrow$ **$\log_x e^4 = ?$**

A) $\frac{3}{8}$ B) $\frac{3}{4}$ C) $\frac{3}{2}$ D) 3 E) 9

2. $x \in Z,\ \log_{x^2}2 - \log_4\sqrt{x} = -\frac{7}{12} \Rightarrow$ **$x = ?$**

A) $\sqrt{2}$ B) 1 C) 2 D) 4 E) 8

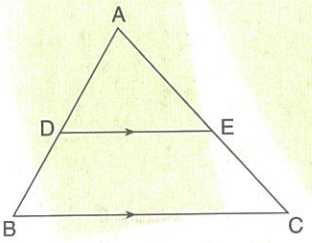
3. $[DE] \,//\, [BC]$,

$|AD| = \log_2 27$,

$|EC| = \log_3 64$

$|DB| = 3$ br (u)

$\Rightarrow$ **$|AE| = ?$ br (u)**



A) 2 B) 3 C) 5 D) 6 E) 8

4. $f(x)=1+\log_{\frac{1}{3}}(x-1) \Rightarrow$ **$f^{-1}(3) = ?$**

A) $\frac{4}{5}$ B) $\frac{9}{10}$ C) $\frac{10}{9}$ D) $\frac{5}{4}$ E) $\frac{3}{2}$

5. $\log_x e = \ln x \Rightarrow$ **$\sum x = ?$**

A) $\frac{1}{e}$ B) e C) 2e D) $\frac{e^2+1}{e}$ E) $\frac{e}{e^2+1}$

6. $x\cdot\log_2 3 - (\sqrt{x}+1)\log_4 3 = 0 \Rightarrow$ **ÇK(SS) = ?**

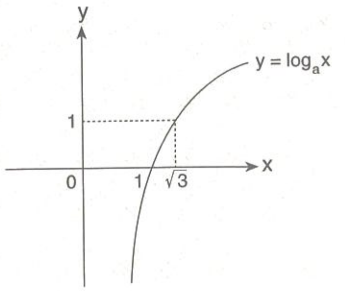
A) {1} B) {2} C) {3} D) {1, 2} E) {2, 3}

7. $\log_x 2 = m \Rightarrow$ **$\log_x\left(\frac{x}{2y}\right)+\log_x\left(\frac{y}{2x}\right) = ?$**

A) –4m B) –2m C) –m D) 2m E) 4m

8. $\log_a x = 4$

$\Rightarrow$ **$x = ?$**



A) 3 B) $2\sqrt{3}$ C) 9 D) $3\sqrt{3}$ E) 27

9. $(x^2\log_x 27)\cdot\log_9 x = x + 4 \Rightarrow$ **x = ?**

A) 1 B) $\frac{4}{3}$ C) $\frac{3}{2}$ D) 2 E) $\frac{5}{2}$

10. $2\cdot\log_9 x + 9\cdot\log_x 3 = 10$

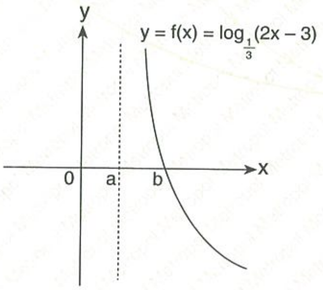
$\Rightarrow \prod x = ?$

A) 3^7 B) 3^8 C) 3^9 D) 3^{10} E) 3^{11}

11. $\frac{1}{x}\cdot\ln(10^x + 3x - 6) = \ln 2 + \ln 5 \Rightarrow$ **x = ?**

A) 1 B) 2 C) 3 D) 4 E) 5

12. **a + b = ?**



A) $\frac{3}{2}$ B) 2 C) $\frac{5}{2}$ D) 3 E) $\frac{7}{2}$

13. $\log_{\frac{1}{2}}\left[\log_3(x-7)\right]\geq 0$

$x \in Z \Rightarrow \sum x = ?$

A) 16 B) 17 C) 18 D) 19 E) 27

14. $\ln x^2 + 2\ln\sqrt{x} - 3 = 0 \Rightarrow$ **ÇK(SS) = ?**

A) $\{e\}$ B) $\left\{\frac{1}{e}\right\}$ C) $\{e^2\}$ D) $\{e^3\}$ E) $\{1\}$

15. $\log_{\frac{1}{2}}(4-x)\geq -2 \Rightarrow$ **ÇK(SS) = ?**

A) $[0,2)$ B) $[0,2]$ C) $[0,4)$ D) $[0,4]$ E) $[-4,4]$

16. $x^2 - 4x + \log_3 a = 0$

ÇK(SS) = $\{x_1, x_2\}$, $x_1 = x_2 \Rightarrow$ **a = ?**

A) 27 B) 36 C) 54 D) 72 E) 81

YANITLAR / ANSWERS

1	2	3	4	5	6	7	8
C	E	D	C	D	A	B	C
9	10	11	12	13	14	15	16
D	D	B	E	E	A	C	E

1. $\log_4(x+1) - \log_4(x-1) = 2 \Rightarrow$ **x = ?**

A) $\frac{16}{15}$ B) $\frac{17}{15}$ C) $\frac{6}{5}$ D) $\frac{19}{15}$ E) $\frac{4}{3}$

2. $\log_3(x-2) \leq 1 \Rightarrow$ **ÇK(SS) = ?**

A) (5,7] B) (4,6] C) (3,5]

D) (2,5] E) (4,7]

3. $\log_3(2x-25) - \log_3(x+24) = 0 \Rightarrow$ **$\log_7 x$ = ?**

A) 0 B) 2 C) 3 D) 4 E) 5

4. $\log_{\frac{1}{2}}(x+3) + \log_{\frac{1}{2}}(x+11) = -7 \Rightarrow$ **x = ?**

A) – 19 B) – 6 C) – 2 D) 4 E) 5

5. $\log_3(x^2-9) \leq 3 \Rightarrow$ **ÇK(SS) = ?**

A) [–6, –3) B) [–6, 6) C) (–3, 3)

D) $(-\infty, -6) \cup [6, +\infty)$ E) $[-6, -3) \cup (3, 6]$

6. $\log_5(2x^2 + 3x) < 1 \Rightarrow$ **ÇK(SS) = ?**

A) $\left(\frac{15}{2}, 1\right)$ B) $\left(-\frac{3}{2}, 0\right)$ C) $\left[-\frac{5}{2}, 1\right]$

D) $\left(-\infty, -\frac{5}{2}\right) \cup (1, \infty)$ E) $\left(-\frac{5}{2}, -\frac{3}{2}\right) \cup (0, 1)$

7. $\log 2 = a \Rightarrow$ **$\log\left(49^{1-\log_7 2} + 5^{-\log_5 4}\right) = ?$**

A) 1 + 3a B) 2 – 3a C) 2 – a

D) 3 + a E) $\frac{2-3a}{2}$

8. $\log_2(x+14) + \log_2(x+2) = 6 \Rightarrow$ **ÇK(SS) = ?**

A) {– 18} B) {2, 4} C) {2}

D) {– 18, 2} E) {4, 6}

9. $\log(3x+1) + \log(2x-1) = 2 - \log 2 \Rightarrow$ **ÇK(SS) = ?**

A) $\left\{-\frac{17}{6}, 3\right\}$ B) {4} C) {2, 4}

D) {3} E) $\left\{4, \frac{-5}{2}\right\}$

10. $\log_2(2x+3) - \log_2(x-2) \geq 1 \Rightarrow$ **ÇK(SS) = ?**

A) $[2, \infty)$ B) $[-\frac{3}{2}, 2)$ C) $(2, \infty)$

D) $(-\infty, -5]$ E) $[-5, -\frac{3}{2}]$

11. $3^{2x} - 8 \cdot 3^x - 9 = 0 \Rightarrow$ **ÇK(SS) = ?**

A) {–1, 9} B) {–1, 2} C) {2}

D) {9} E) {2, 9}

12. $2^{\ln x} + 2^{1-\ln x} = 3 \Rightarrow$ **ÇK(SS) = ?**

A) {0} B) {1} C) {e}

D) {0, e} E) {1, e}

13. $\ln x = 1 + \frac{2}{\ln x} \Rightarrow$ **ÇK(SS) = ?**

A) $\left\{\frac{1}{e}, e\right\}$ B) $\left\{\frac{1}{e^2}, e\right\}$ C) $\left\{\frac{1}{e}, e^2\right\}$

D) $\left\{\frac{1}{e^2}, e^2\right\}$ E) $\left\{\frac{1}{e}, e^3\right\}$

14. $\left|\frac{\log_2 2x}{3}\right| < 1 \Rightarrow$ **ÇK(SS) = ?**

A) $\left(\frac{1}{4}, 1\right)$ B) $\left(\frac{1}{4}, 2\right)$ C) $\left(\frac{1}{8}, 2\right)$

D) $\left(\frac{1}{8}, 4\right)$ E) $\left(\frac{1}{16}, 4\right)$

15. $x^{\log_3 x} = 81 \Rightarrow$ **ÇK(SS) = ?**

A) $\{3\}$ B) $\left\{9, \frac{1}{9}\right\}$ C) $\left\{\frac{1}{9}\right\}$

D) $\left\{3, \frac{1}{3}\right\}$ E) 4

16. $\log_x 9 = \log_9 x \Rightarrow$ **ÇK(SS) = ?**

A) $\left\{\frac{1}{9}, 9\right\}$ B) $\left\{-\frac{1}{9}, 9\right\}$ C) $(0, \infty) - \{1\}$

D) ∅ E) R

17. $f(x) = x^{1+\log 2x} - 20x$

$f(x_1) = f(x_2) = 0 \Rightarrow$ $\mathbf{x_1 \cdot x_2 = ?}$

A) 100 B) 50 C) 10 D) 1 E) $\frac{1}{2}$

18. $f(x) = \ln\left(x + \sqrt{x^2+4}\right) \Rightarrow$ $\mathbf{f^{-1}(x) = ?}$

A) $4e^{2x}$ B) $4e^x$ C) $\frac{e^{2x}-4}{e^x}$

D) $\frac{e^{2x}-4}{2e^x}$ E) $\frac{e^{2x}-2}{4e^x}$

YANITLAR / ANSWERS

1	2	3	4	5	6	7	8	9
B	D	B	E	E	E	B	C	D
10	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**
C	C	E	C	E	B	A	E	D

BÖLÜM CHAPTER 24

TARİHSEL NOT / HISTORICAL NOTE

BLAİSE PASCAL [1623 - 1662]

Fransız matematikçi, fizikçi ve düşünürdür. Pascal hayatı boyunca matematiği etkilemeye devam etmiştir. Pascal'ın 1653 yılındaki Traité du triangle arithmétique diye geçen Aritmetik üçgen üzerinde incelemesi, binom çarpanlarını uygun bir tablo halinde tanıtmıştır (Pascal üçgeni).

French mathematician, physicist and thinker. Pascal continued to influence mathematics throughout his life. Pascal's study of the Arithmetic triangle, called "Traité du triangle arithmétique" in 1653, introduced binomial factors as an appropriate table (Pascal's triangle).

TÜMEVARIM, DİZİLER ve SERİLER
INDUCTION, SEQUENCES and SERIES

Bu bölüm, 115 test sorusu içermektedir.
This chapter includes 115 test questions.

BÖLÜM / CHAPTER 24

TÜMEVARIM, DİZİLER ve SERİLER / INDUCTION, SEQUENCES and SERIES

- Tümevarım, Diziler ve Seriler / Induction, Sequences and Series . . . . 449 – 470

BÖLÜM

24

CHAPTER

TÜMEVARIM, DİZİLER ve SERİLER
INDUCTION, SEQUENCES and SERIES

TÜMEVARIM / INDUCTION

TOPLAM VE ÇARPIM SEMBOLLERİ / SUMMATION and MULTIPLICATION SYMBOLS

TOPLAM SEMBOLÜ / SUMMATION SYMBOL ($\sum$)

▼ $p, n \in Z$, $p \leq n$ ve $a_k \in R$ olmak üzere;

▼ If $p, n \in Z$, $p \leq n$ and $a_k \in R$, then;

$$\sum_{k=p}^{n} a_k = a_p + a_{p+1} + a_{p+2} + \ldots + a_n$$

Önemli / Important

- $\sum\limits_{k=1}^{n} a_k$ ifadesinde k harfi yerine başka harfler kullanılabilir ve alt sınır 1 olmayabilir.
- In the expression $\sum\limits_{k=1}^{n} a_k$, the letter k can be replaced by any other symbols and upper limit may not equal to 1.
- $\sum\limits_{i=5}^{20} i, \quad \sum\limits_{i=-3}^{30} i, \quad \sum\limits_{p=-7}^{13} 3^p$

Özellik / Property

★ $\sum\limits_{k=1}^{n} a_k = \sum\limits_{p=1}^{n} a_p = \sum\limits_{i=1}^{n} a_i = \ldots$

★ $\sum\limits_{k=1}^{n} c = n \cdot c$

★ $\sum\limits_{k=1}^{n} (c \cdot a_k) = c \cdot \left(\sum\limits_{k=1}^{n} a_k \right)$

★ $\sum\limits_{k=1}^{n} (a_k \mp b_k) = \left(\sum\limits_{k=1}^{n} a_k \right) \mp \left(\sum\limits_{k=1}^{n} b_k \right)$

★ $1 < m < n$ için/for $\sum\limits_{k=1}^{n} a_k = \sum\limits_{k=1}^{m} a_k + \sum\limits_{k=m+1}^{n} a_k$

★ $\sum\limits_{k=r}^{n} a_k = \sum\limits_{k=r \mp p}^{n \mp p} a_{(k \pm p)}$

BAZI ÖNEMLİ TOPLAM FORMÜLLERİ / SOME of the IMPORTANT SUMMATION FORMULAS

- $\sum_{k=1}^{n} k = 1+2+3+\ldots+n = \frac{n\cdot(n+1)}{2}$
- $\sum_{k=1}^{n} (2k-1) = 1+3+5+\ldots+(2n-1) = n^2$
- $\sum_{k=1}^{n} 2k = 2+4+6+8+\ldots+2n = n\cdot(n+1)$
- $\sum_{k=1}^{n} k^2 = 1^2+2^2+3^2+4^2+\ldots+n^2 = \frac{n\cdot(n+1)\cdot(2n+1)}{6}$
- $\sum_{k=1}^{n} k^3 = 1^3+2^3+3^3+4^3+\ldots+n^3 = \left[\frac{n(n+1)}{2}\right]^2$
- $\sum_{k=0}^{n-1} r^k = 1+r+r^2+\ldots+r^{n-1} = \frac{1-r^n}{1-r}, \quad [r \neq 1,\ r \neq 0]$
- $\sum_{k=1}^{n} \frac{1}{k\cdot(k+1)} = \frac{1}{1\cdot 2}+\frac{1}{2\cdot 3}+\ldots+\frac{1}{n\cdot(n+1)} = \frac{n}{n+1}$
- $\sum_{k=1}^{n} k\cdot k! = (n+1)!-1$
- $\sum_{k=1}^{n} k\cdot(k+1) = \frac{n\cdot(n+1)\cdot(n+2)}{3}$

  $\left[\sum_{k=1}^{n} k\cdot(k+1) = \left(\sum_{k=1}^{n} k^2\right)+\left(\sum_{k=1}^{n} k\right)\right]$
- $\sum_{k=1}^{n} k\cdot(k+1)\cdot(k+2) = \frac{n\cdot(n+1)\cdot(n+2)\cdot(n+3)}{4}$

  $\left[\sum_{k=1}^{n} k\cdot(k+1)\cdot(k+2) = \sum_{k=1}^{n} k\cdot(k^2+3k+2) = \sum_{k=1}^{n}(k^3+3k^2+2k)\right]$

  $= \left(\sum_{k=1}^{n} k^3\right)+\left(\sum_{k=1}^{n} 3k^2\right)+\left(\sum_{k=1}^{n} 2k\right)$

ÇARPIM SEMBOLÜ / MULTIPLICATION SYMBOL ($\prod$)

- $p, n \in Z$, $p \leq n$ ve $a_k \in R$ olmak üzere,
- If $p, n \in Z$, $p \leq n$ and $a_k \in R$, then;

$$\prod_{k=p}^{n} a_k = a_p \cdot a_{p+1} \cdot a_{p+2} \cdot \ldots\ldots \cdot a_n$$

Özellik / Property

★ $\prod_{k=1}^{n} a_k = \prod_{i=1}^{n} a_i = \prod_{p=1}^{n} a_p = \ldots$

★ $\prod_{k=1}^{n} c = c^n, \quad (c \in R)$

★ $\prod_{k=1}^{n} (c \cdot a_k) = c^n \cdot \left(\prod_{k=1}^{n} a_k\right) \quad (c \in R)$

★ $\prod_{k=1}^{n} (a_k \cdot b_k) = \left(\prod_{k=1}^{n} a_k\right) \cdot \left(\prod_{k=1}^{n} b_k\right)$

★ $1 < m < n \Rightarrow \prod_{k=1}^{n} a_k = \left(\prod_{k=1}^{m} a_k\right) \cdot \left(\prod_{k=m+1}^{n} a_k\right)$

★ $\prod_{k=1}^{n} \frac{a_k}{b_k} = \left(\frac{\prod_{k=1}^{n} a_k}{\prod_{k=1}^{n} b_k}\right) \quad (\forall b_k \neq 0)$

★ $\prod_{k=p}^{n} a_k = \prod_{k=p\mp r}^{n\mp r} a_{(k\pm r)}$

BAZI ÖNEMLİ ÇARPIM FORMÜLLERİ / SOME of the IMPORTANT MULTIPLICATION FORMULAS

▼ $\prod_{k=1}^{n} k = 1 \cdot 2 \cdot 3 \cdot 4 \cdot \ldots \cdot (n-1) \cdot n = \boldsymbol{n!}$

▼ $\prod_{k=p}^{n} \log_k (k+1) = \log_p (p+1) \cdot \log_{(p+1)} (p+2) \cdot \log_{(p+2)} (p+3) \cdot \ldots \cdot \log_n (n+1) = \boldsymbol{\log_p (n+1)}$

▼ $\prod_{k=1}^{n} r^k = r^1 \cdot r^2 \cdot r^3 \cdot r^4 \cdot \ldots \cdot r^n = r^{1+2+3+\ldots+n} = \boldsymbol{r^{\frac{n \cdot (n+1)}{2}}}$

▼ $\prod_{k=2}^{n} \left(1 - \frac{1}{k^2}\right) = \boldsymbol{\frac{n+1}{2n}}$

DİZİLER / SEQUENCES

Tanım: f: $N^+ \rightarrow R$, $f(n) = a_n$ biçiminde tanımlı her fonksiyona ***dizi*** denir.

$n = 1, 2, 3, 4, \ldots$ sayılarına karşılık gelen $f(1) = a_1$, $f(2) = a_2$, $f(3) = a_3$, …, $f(n) = a_n$ reel sayılarına ***dizinin terimleri*** denir. $f(n) = a_n$ reel sayısına da ***dizinin genel terimi*** denir.

$f(1) = a_1$, $f(2) = a_2$, …, $f(n) = a_n$ ise, f dizisi genel olarak $(a_1, a_2, a_3, \ldots, a_n, \ldots)$ veya kısaca (a_n) ile gösterilir.

Definition: A function which is defined in the form f: $N^+ \rightarrow R$, $f(n) = a_n$ is called a **sequence**. The real numbers $f(1) = a_1 \ldots f(n) = a_n$ which correspond to the numbers. $n = 1, 2, 3, 4 \ldots$ are called the ***terms of the sequence***. $f(n) = a_n$ is called the ***general term of the sequence***.

Let $f(1) = a_1$, $f(2) = a_2$, …… $f(n) = a_n$.

The sequence is denoted by $(a_1, a_2, a_3, \ldots a_n, \ldots)$ in general or it is denoted by (a_n), briefly.

ARİTMETİK DİZİ / ARITHMETIC SEQUENCE

Tanım: Ardışık terimleri arasındaki farkın aynı sabit sayıya eşit olduğu dizilere ***aritmetik dizi*** denir.

Definition: sequences in which the difference of any two consecutive terms of it is a constant number are called ***arithmetic sequence***.

▼ $\forall n \in N^+$ için/for:

$a_{n+1} - a_n = d, \quad (d \in R)$

[d: ortak fark / common difference]

▼ $a_2 - a_1 = a_3 - a_2 = \ldots = a_{n+1} - a_n = d$

▼ $a_n = a_1 + (n - 1) \cdot d$

▼ $p \in N^+, \quad a_n = a_p + (n - p) \cdot d$

GEOMETRİK DİZİ / GEOMETRIC SEQUENCE

Tanım: Ardışık her iki teriminin oranları eşit olan dizilere ***geometrik dizi*** denir.

Definition: sequences in which the ratio of any two consecutive terms of it is a constant number are called ***geometric sequence***.

▼ $\forall n \in N^+$ için/for:

$$\frac{a_{n+1}}{a_n} = r, \quad (r \in R)$$

[r: ortak oranı/ common ratio]

▼ $\frac{a_2}{a_1} = \frac{a_3}{a_2} = \frac{a_4}{a_3} = \ldots = \frac{a_{n+1}}{a_n} = r$

▼ $a_n = a_1 \cdot r^{n-1}$

▼ $p \in N^+, \; a_n = a_p \cdot r^{n-p}$

SABİT DİZİ / CONSTANT SEQUENCE

Tanım: Her $n \in N^+$ için $k \in R$ olmak üzere, $a_n = k$ ise, (a_n) dizisine ***sabit dizi*** denir.

Definition: A sequence (a_n) is said to be constant if $a_n = k$, $k \in R$, for every $n \in N^+$.

DİZİLERİN EŞİTLİĞİ / EQUALITY OF SEQUENCES

▼ $(a_n) = (b_n) \Leftrightarrow \forall n \in N^+$ için/for $a_n = b_n$

DİZİLERDE İŞLEMLER / OPERATION on SEQUENCES

(a_n) ve (b_n) herhangi iki dizi,

Let (a_n) and (b_n) be any two sequences,

▼ $(a_n) \mp (b_n) = (a_n \mp b_n)$

▼ $(a_n) \cdot (b_n) = (a_n \cdot b_n)$

▼ $\frac{(a_n)}{(b_n)} = \left(\frac{a_n}{b_n}\right)$, $[\forall n \in N^+$ için/for $b_n \neq 0]$

SERİLER / SERIES

▼ **Geometrik seri / Geometric Series:**

a_1 : 1.terim / 1st term

r : ortak çarpan / common multiplier

$$|r| < 1 \Rightarrow \sum_{n=1}^{\infty} a_1 \cdot r^{n-1} = \frac{a_1}{1-r}$$

1. $\sum_{k=1}^{23} k = ?$

A) 274 B) 276 C) 286 D) 298 E) 312

2. $\sum_{k=1}^{20} k = ?$

A) 120 B) 180 C) 210 D) 340 E) 420

3. $\sum_{k=1}^{18} k^2 = ?$

A) 2100 B) 2109 C) 2112 D) 2125 E) 2132

4. $\sum_{n=-3}^{18} n = ?$

A) 165 B) 170 C) 185 D) 196 E) 204

5. $\sum_{m=10}^{21} 4m = ?$

A) 690 B) 695 C) 700 D) 720 E) 744

6. $\sum_{k=1}^{20} (2k-5) = ?$

A) 320 B) 334 C) 342 D) 360 E) 372

7. $\sum_{k=1}^{16} (3k-4) = ?$

A) 472 B) 466 C) 408 D) 344 E) 342

8. $\sum_{k=3}^{12} (a+3) = 80 \Rightarrow a = ?$

A) 2 B) 3 C) 4 D) 5 E) 6

9. $\sum_{k=5}^{19}(2k+1)=?$

A) 375 B) 350 C) 325 D) 275 E) 250

10. $\sum_{k=-2}^{9}(3k+4)=?$

A) 124 B) 128 C) 148 D) 164 E) 174

11. $\sum_{n=5}^{9} a=45 \Rightarrow a=?$

A) 5 B) 6 C) 7 D) 8 E) 9

12. $\sum_{k=15}^{20}(2k-5)=?$

A) 120 B) 150 C) 160 D) 180 E) 220

13. $\sum_{k=1}^{10}4+\sum_{k=1}^{12}(2k-1)=?$

A) 161 B) 184 C) 209 D) 265 E) 300

14. $\sum_{k=2}^{10}(k^2-2k)=?$

A) 264 B) 270 C) 276 D) 282 E) 294

15. $f(x) = x^2 - 3x + 1$

$\Rightarrow \sum_{x=1}^{10} f(x)=?$

A) 220 B) 230 C) 625 D) 670 E) 720

16. $$\left.\begin{array}{l}\sum_{k=1}^{27} a_k=b\\ \sum_{k=12}^{27} a_k=c\end{array}\right\} \Rightarrow \sum_{k=1}^{11} 2\cdot a_k=?$$

A) b − c B) b + c C) 2b − 2c

D) 2b + 2c E) 2b − c

17. $\sum_{n=1}^{70}(3+n\cdot a)=1275 \Rightarrow a=?$

A) $\frac{2}{3}$ B) $\frac{3}{5}$ C) $\frac{5}{6}$ D) $\frac{5}{7}$ E) $\frac{3}{7}$

YANITLAR / ANSWERS

1	2	3	4	5	6	7	8	9
B	C	B	A	E	A	D	D	A
10	11	12	13	14	15	16	17	
E	E	D	B	C	B	C	E	

1. $\sum\limits_{k=-4}^{17}(3k-1)=?$

A) 407 B) 510 C) 521 D) 630 E) 1067

2. $\sum\limits_{k=-7}^{21}(k+8)=?$

A) 435 B) 445 C) 450 D) 470 E) 500

3. $\sum\limits_{k=3}^{12}k^2=?$

A) 1030 B) 940 C) 860 D) 720 E) 645

4. $\sum\limits_{a=5}^{20}(a-b)=120 \Rightarrow b=?$

A) 1 B) 2 C) 3 D) 4 E) 5

5. $\sum\limits_{n=0}^{6}(m+2n)=77 \Rightarrow m=?$

A) 3 B) 5 C) 6 D) 7 E) 11

6. $\sum\limits_{n=1}^{10}(a\cdot n-3a)=150 \Rightarrow a=?$

A) 4 B) 5 C) 6 D) 7 E) 8

7. $\sum\limits_{a=1}^{12}\left(\sum\limits_{b=2}^{4}b\right)=?$

A) 144 B) 128 C) 116 D) 112 E) 108

8. $\sum\limits_{m=1}^{15}(2m+a-2)=300 \Rightarrow a=?$

A) 7 B) 6 C) 5 D) 4 E) 2

9. $\sum\limits_{i=1}^{4}\left(\sum\limits_{j=-1}^{3}(3j+2i)\right)=?$

A) 240 B) 220 C) 180 D) 170 E) 160

10. $\sum\limits_{p=3}^{6}\left(\sum\limits_{n=0}^{3}(p\cdot n+2)\right)=?$

A) 148 B) 144 C) 142 D) 140 E) 136

11. $\sum\limits_{n=1}^{3}\left[\sum\limits_{m=3}^{4}(m^3\cdot n+2n)\right]=?$

A) 570 B) 610 C) 760 D) 840 E) 970

12. $\sum\limits_{a=2}^{4}\left(\sum\limits_{b=1}^{3}(3^b-2^a)\right)=?$

A) 27 B) 30 C) 33 D) 37 E) 40

13. $\sum\limits_{k=1}^{20}\frac{1}{k}-\sum\limits_{k=1}^{20}\frac{1}{k+1}=?$

A) $\frac{1}{18}$ B) $\frac{1}{20}$ C) $\frac{19}{20}$ D) $\frac{20}{21}$ E) $\frac{9}{10}$

14. $\sum\limits_{k=1}^{12}2^{k-1}=?$

A) $2^{11}-1$ B) $2^{12}-1$ C) $2^{11}+1$

D) $2^{12}+1$ E) 2^{13}

15. $\sum\limits_{k=7}^{23}2^{k}=?$

A) $2^{24}-1$ B) $2^{24}-2^{7}$ C) 2^{23}

D) $2^{23}-2^{6}$ E) 2^{55}

16. $f(x)=x^2-6x+p$

$f(a)=f(b)=0\Rightarrow\sum\limits_{k=3}^{12}(a+b)=?$

A) 54 B) 57 C) 60 D) 63 E) 66

17. $f(n)=\sum\limits_{k=1}^{n}(k^2+m-k),\ f(1)=4\ \Rightarrow\ f(2)=?$

A) m + 4 B) m + 8 C) 14

D) 12 E) 10

YANITLAR / ANSWERS

1	2	3	4	5	6	7	8	9
A	A	E	E	B	C	E	B	E
10	11	12	13	14	15	16	17	
D	A	C	D	B	B	C	E	

1. $\sum\limits_{n=1}^{12}\left(\prod\limits_{k=1}^{3} k\right)=?$

A) 54 B) 64 C) 72 D) 78 E) 84

2. $\prod\limits_{k=1}^{12} 2^k=?$

A) $2^{12}-1$ B) $2^{11}-1$ C) $2^{13}-1$ D) $2^{13}-2$ E) 2^{78}

3. $\prod\limits_{n=-9}^{12}\left(\frac{1}{3}\right)^n=?$

A) 3^{-33} B) 3^{-30} C) 3^{-27} D) 3^{-23} E) 3^{-18}

4. $\prod\limits_{k=4}^{37}\frac{k-1}{k-2}=?$

A) 12 B) 14 C) 16 D) 18 E) 20

5. $\prod\limits_{n=2}^{23}\left(1-\frac{1}{n+1}\right)=?$

A) $\frac{1}{14}$ B) $\frac{1}{13}$ C) $\frac{1}{12}$ D) $\frac{1}{11}$ E) $\frac{2}{11}$

6. $\prod\limits_{m=14}^{40}\frac{2m+1}{2m-1}=?$

A) 3 B) $\frac{5}{3}$ C) $\frac{27}{4}$ D) $\frac{40}{3}$ E) 81

7. $\prod\limits_{k=1}^{23}(k-14)=?$

A) 3482 B) 3462 C) 3452 D) 3442 E) 0

8. $\prod\limits_{m=4}^{6}\left(\sum\limits_{k=0}^{3}\left(\frac{2k-5}{m}\right)\right)=?$

A) -9 B) 10 C) $-\frac{64}{15}$ D) $\frac{100}{3}$ E) 290

9. $\prod\limits_{m=1}^{n} 3^{m} = 3^{300} \Rightarrow n = ?$

A) 22 B) 23 C) 24 D) 25 E) 26

10. $\sum\limits_{k=1}^{5} (k-1)\cdot(k-2)\cdot(k-3)\cdot(k-4) = ?$

A) 32 B) 24 C) 16 D) 12 E) 1

11. $\prod\limits_{k=2}^{63} \log_k(k+1) = ?$

A) 1 B) 2 C) 3 D) 4 E) 6

12. $\prod\limits_{k=5}^{44} 2^{\log_3\left(\frac{k+1}{k}\right)} = ?$

A) 2^{44} B) $2\log_3 11$ C) $2\log_3 5$ D) 4 E) 2^{9}

13. $\prod\limits_{k=3}^{87} \tan k^{\circ} = ?$

A) 1 B) $\frac{3}{2}$ C) $\frac{\sqrt{3}}{2}$ D) $\frac{\sqrt{2}}{2}$ E) 2

14. $\prod\limits_{k=3}^{16} \left(\log_2 \sqrt[k]{2^{k-1}}\right) = ?$

A) 8 B) 12 C) 16 D) $\frac{1}{16}$ E) $\frac{1}{8}$

15. $\prod\limits_{k=23^{\circ}}^{143^{\circ}} \cos k = ?$

A) $-\frac{\sqrt{3}}{2}$ B) $-\frac{1}{2}$ C 0 D) $\frac{1}{2}$ E) $\frac{\sqrt{3}}{2}$

16. $\prod\limits_{k=1}^{96} 2^{\log_7\left(\frac{k+1}{k+2}\right)} = ?$

A) $\frac{1}{16}$ B) $\frac{1}{8}$ C) $\frac{1}{4}$ D) 4 E) 8

17. $\prod\limits_{k=2}^{13} \log_{(k+2)}(k^2+6k+9) = ?$

A) 2 B) 2^{2} C) 2^{12} D) 2^{13} E) 2^{14}

YANITLAR / ANSWERS

1	2	3	4	5	6	7	8	9
C	E	A	D	C	A	E	C	C
10	11	12	13	14	15	16	17	
B	E	D	A	E	C	C	D	

1. $\sum_{k=-2}^{10} k(k+1) = ?$

A) 408 B) 412 C) 428 D) 442 E) 448

2. $\sum_{k=10}^{99} \frac{1}{k\cdot(k+1)} = ?$

A) $9{\cdot}10^{-3}$ B) $9{\cdot}10^{-2}$ C) $9{\cdot}10^{-1}$ D) 9 E) 90

3. $\sum_{n=0}^{16} (-1)^{n+1}\cdot(2n+3) = ?$

A) –21 B) –19 C) 0 D) 19 E) 21

4. $\sum_{k=-2}^{n-1} (2k+6) = an^2+bn+c$

$\Rightarrow$ **a + b + c = ?**

A) 12 B) 14 C) 16 D) 18 E) 20

5. $\sum_{k=4}^{n} (k-3)(k-1) = 133$

$\Rightarrow$ **n = ?**

A) 7 B) 8 C) 9 D) 10 E) 11

6. $\sum_{x=1}^{90} (\cos^2 x - \sin^2 x) = ?$

A) $-\frac{\sqrt{2}}{2}$ B –1 C) 0 D) 1 E) $\frac{\sqrt{2}}{2}$

7. $\sum_{k=2}^{100} \log\left(1-\frac{1}{k}\right) = ?$

A) –2 B) –1 C) 0 D) 1 E) 2

8. $\sum_{m=1}^{7} \sum_{n=5}^{9} (m+n-7) = ?$

A) 100 B) 112 C) 120 D) 132 E) 140

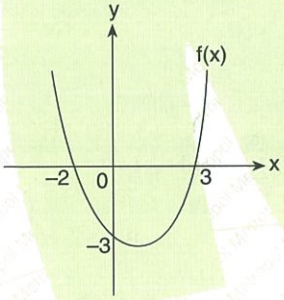
9. $\prod_{k=1}^{n}\left(1+\frac{1}{k+1}\right)=7 \Rightarrow$ **n = ?**

A) 3
B) 6
C) 9
D) 12
E) 18

10. $\prod_{x=-n}^{n+1} 2^x=16 \Rightarrow$ **n = ?**

A) 3
B) 4
C) 5
D) 6
E) 7

11. $\sum_{x=-2}^{1} f(x)=?$



A) –10
B) –8
C) –6
D) –4
E) –2

12. $\sum_{k=0}^{6}(m^2k-7)=\sum_{n=4}^{m}(2n-1)$

$\Rightarrow$ **m^2+1 = ?**

A) 3
B) 5
C) 6
D) 7
E) 9

13. $\sum_{k=1}^{4}\prod_{m=2}^{k}\log_m(m+1)=?$

A) $\log_2 20$
B) $\log_2 24$
C) $\log_2 36$
D) $\log_2 60$
E) $\log_2 120$

14. $\log_2 6=x \Rightarrow \sum_{n=2}^{3}\log_n(n+1)=?$

A) $\frac{x^2+2x+3}{x-1}$
B) $\frac{x^2-2x+3}{x-1}$
C) $\frac{x^2-2x+5}{x-1}$
D) $\frac{x^2+2x+5}{x-1}$
E) $\frac{x^2+2x+3}{x-2}$

15. $9^x=2 \Rightarrow \prod_{k=2}^{8}\log_{k^2}(k^2+2k+1)=?$

A) $\frac{1}{x}$
B) $\frac{2}{x}$
C) $\frac{3}{x}$
D) x
E) 2x

16. $\sum_{m=1}^{31}\prod_{n=1}^{30} n=A\cdot 3^x \Rightarrow$ **max(x) = ?**

A) 11
B) 12
C) 13
D) 14
E) 15

YANITLAR / ANSWERS

1	2	3	4	5	6	7	8
D	B	B	A	C	B	A	E
9	**10**	**11**	**12**	**13**	**14**	**15**	**16**
D	A	B	A	E	B	A	D

1. $\prod_{k=1}^{7}\left(\frac{1}{2}+\frac{1}{2k}\right)=?$

A) $\frac{1}{2}$ B) $\frac{1}{4}$ C) $\frac{1}{8}$ D) $\frac{1}{16}$ E) $\frac{1}{32}$

2. $\prod_{n=0}^{4}\left[\sum_{m=1}^{3}\left(\frac{x}{3}+1\right)\right]=32 \Rightarrow x=?$

A) –2 B) –1 C) 0 D) 1 E) 2

3. $x^2-6x+m=0$, $ÇK(SS)=\{x_1, x_2\}$

$\sum_{k=1}^{2} x_k(x_k-1)=0 \Rightarrow m=?$

A) 9 B) 12 C) 15 D) 18 E) 21

4. $\prod_{k=1}^{6} 5^k=\frac{1}{5^{x^2-10x}} \Rightarrow \sum x=?$

A) –10 B) –8 C) –5 D) 5 E) 10

5. $f(x)=\begin{cases} x^2+1, & x\geq 0 \\ -4x+1, & x<0 \end{cases}$

$g(x)=\sum_{k=-3}^{x} f(k) \Rightarrow g(3)=?$

A) 40 B) 45 C) 48 D) 52 E) 54

6. $\sum_{k=1}^{4}\frac{k!}{|k-7|}=?$

A) $\frac{151}{15}$ B) $\frac{314}{15}$ C) $\frac{316}{15}$ D) $\frac{65}{3}$ E) $\frac{326}{15}$

7. $\prod_{m=1}^{4}\left[\prod_{n=1}^{6}(2n-m)\right]=?$

A) 164 B) 128 C) 96 D) 48 E) 0

8. $\sum_{n=1}^{4}\prod_{m=0}^{2}(n+m-n\cdot m)=?$

A) –20 B) –15 C) –10 D) –5 E) 0

9. $f(x) = x^2 - (2a-1)x + 2$

$$f\left[\sum_{n=1}^{2}(n+1)\right] = \sum_{n=-1}^{2}(n^2+n+1) \Rightarrow \mathbf{a = ?}$$

A) –2 B) –1 C) 0 D) 1 E) 2

10. $\sum_{n=-1}^{10} 5^{n+1} = \frac{1}{4}(x^y - 1) \Rightarrow \mathbf{y - x = ?}$

A) 7 B) 8 C) 9 D) 10 E) 11

11. $\sum_{k=1}^{12}(a_k - x) = 25,$

$\sum_{k=1}^{12}(3a_k - 5) = x^2 + 13 \Rightarrow \prod \mathbf{x = ?}$

A) –36 B) –2 C) 2 D) 36 E) 72

12. $\sum_{k=-2}^{5}(xy - 3) = 40,$

$\sum_{k=3}^{11}(x+2) = -18 \Rightarrow \mathbf{y = ?}$

A) –5 B) –3 C) –2 D) 3 E) 5

13. $$\sum_{k=1}^{5}\left[\prod_{m=1}^{3}\left(\frac{m \cdot k}{3}\right)\right] = ?$$

A) 35 B) 40 C) 45 D) 50 E) 55

14. $\sum_{k=3}^{10} \frac{x}{k^2 - 3k + 2} = 24 \Rightarrow \mathbf{x = ?}$

A) 18 B) 24 C) 27 D) 36 E) 48

15. $g(x) = x + 3$, $gof(x) = x + 2$,

$\sum_{k=1}^{n} f(k) = 10,\ n \in Z^+ \Rightarrow \mathbf{n = ?}$

A) 5 B) 6 C) 7 D) 8 E) 9

16. $\sum_{k=-4}^{6}(2k - 1) = x,$

$\sum_{k=-6}^{4} k = y \Rightarrow \mathbf{x + y = ?}$

A) –12 B) –6 C) 0 D) 6 E) 12

YANITLAR / ANSWERS

1	2	3	4	5	6	7	8
D	B	C	E	B	A	E	C
9	**10**	**11**	**12**	**13**	**14**	**15**	**16**
E	A	B	C	D	C	A	C

1. $\prod_{k=1}^{n} 2^{k}=2^{210} \Rightarrow$ **n = ?**

A) 36 B) 32 C) 25 D) 24 E) 20

2. **4 – 10 + 8 – 20 + 12 – 30 + ... + 32 – 80 = ?**

A) –360 B) –344 C) –300 D) –216 E) 144

3. $f:N^{+} \longrightarrow N^{+},\ f(x)=\sum_{k=1}^{x} 2(k-1)$

$g:N^{+} \longrightarrow R,\ g(x)=\sum_{p=1}^{x}\left(\frac{p}{3}+6\right)$

$\Rightarrow$ **gof(3) = ?**

A) 39 B) 43 C) 57 D) 68 E) 98

4. $\sum_{k=2}^{16} \log_a\left(1-\frac{1}{k}\right)=-2 \Rightarrow$ **a = ?**

A) –2 B) 2 C) 3 D) 4 E) 8

5. $\sum_{k=3}^{9} \frac{1}{k^2-1}=?$

A) $\frac{21}{80}$ B) $\frac{19}{80}$ C) $\frac{17}{80}$ D) $\frac{1}{5}$ E) $\frac{14}{45}$

6. $\left(\sum_{n=1}^{10} a+\sum_{n=1}^{10}(a+1)+\sum_{n=1}^{10}(a+2)+\ldots+\sum_{n=1}^{10}(a+10)\right)=880$

A) 1 B) 2 C) 3 D) 4 E) 5

7. $\left.\begin{array}{l}\sum_{n=1}^{m}(a_n+3)=3m+6 \\ \sum_{n=1}^{m}(b_n-2)\cdot a_n=0\end{array}\right\} \Rightarrow \sum_{n=1}^{m} b_n=?$

A) 3 B) 4 C) 6 D) 10 E) 12

8. $A=\sum_{k=1}^{12} k^2 \Rightarrow \sum_{k=1}^{12}(k+1)^2=?$

A) 168 + A B) 169 + A C) 143 + A D) 144 + A E) 170 + A

9. $\frac{1}{1\cdot 2}+\frac{1}{2\cdot 3}+\frac{1}{3\cdot 4}+\ldots+\frac{1}{18\cdot 19}=?$

A) $\frac{17}{19}$ B) $\frac{18}{19}$ C) 1 D) $\frac{20}{19}$ E) $\frac{21}{19}$

10. $\sum_{n=1}^{12}\frac{n^2+n+13}{n^2+n}=?$

A) $\frac{171}{13}$ B) $\frac{291}{13}$ C) $\frac{341}{13}$ D) 80 E) 13

11. $\sum_{n=1}^{182}(\sqrt{4n+1}-\sqrt{4n-3})=?$

A) 21 B) 24 C) 26 D) 27 E) 28

12. $\sum_{k=1}^{89}\sin^2(k^\circ)=?$

A) 44 B) 45 C) $44+\frac{\sqrt{2}}{2}$ D) $\frac{89}{2}$ E) 1

13. $\sum_{k=2}^{18}k\cdot(k!)=?$

A) 19! – 1 B) 18! – 1 C) 18! D) 19! E) 19! – 2

14. $p, n \in N^+$,

$$\sum_{k=1}^{n}\frac{1}{k\cdot(k+1)}=\frac{n}{n+1}$$

$$\Rightarrow \frac{1}{p(p+1)}+\frac{1}{(p+1)(p+2)}+\ldots+\frac{1}{2p(2p+1)}=?$$

A) $\frac{p}{(p+1)(2p-1)}$ B) $\frac{p+1}{2p+1}$ C) $\frac{p+1}{p(2p+3)}$ D) $\frac{p-1}{p(2p+1)}$ E) $\frac{p+1}{p(2p+1)}$

15. $\sum_{k=1}^{n}\frac{1}{k}=x,\quad \sum_{k=1}^{n}\frac{1}{k+1}=y \;\Rightarrow\; n=?$

A) $\frac{x-y}{1-x+y}$ B) $\frac{x+y}{x-y}$ C) $\frac{x\cdot y}{x+y}$ D) $\frac{1+x+y}{x-y}$ E) $\frac{2xy}{1+x+y}$

YANITLAR / ANSWERS

1	2	3	4	5	6	7	8
E	D	B	D	E	C	E	A
9	**10**	**11**	**12**	**13**	**14**	**15**	
B	E	C	D	E	E	A	

1. $(a_n) = 3n - 1 \Rightarrow a_{12} + a_{15} = ?$

A) 77 B) 78 C) 79 D) 80 E) 81

2. $(a_n) = \sum_{k=1}^{n} k \Rightarrow a_{10} = ?$

A) 55 B) 64 C) 72 D) 96 E) 110

3. $\left.\begin{array}{l}(a_n) = 4n - 1 \\ (b_n) = \dfrac{n+4}{4}\end{array}\right\} \Rightarrow a_5 + b_{12} = ?$

A) 19 B) 20 C) 21 D) 22 E) 23

4. $(a_{2n+1}) = \left(\dfrac{3n-1}{4n+1}\right) \Rightarrow a_{11} = ?$

A) $\frac{1}{7}$ B) $\frac{2}{7}$ C) $\frac{3}{7}$ D) $\frac{4}{7}$ E) $\frac{2}{3}$

5. $(a_n) = \left(\sum_{k=1}^{n} (k^2 + 3)\right) \Rightarrow a_6 = ?$

A) 121 B) 109 C) 107 D) 95 E) 89

6. $(a_n) = \dfrac{3^{n-1}}{2^n} \Rightarrow \dfrac{a_{n+1}}{a_n} = ?$

A) 1 B) $\frac{3}{2}$ C) 2 D) $\frac{5}{2}$ E) 3

7. $(a_n) = (5n - 3) \Rightarrow \sum_{n=1}^{17} a_n = ?$

A) 676 B) 694 C) 704 D) 714 E) 728

8. $a_n = \dfrac{5^n \cdot n!}{n+2} \Rightarrow \dfrac{a_{n-2}}{a_{n-1}} = ?$

A) $\dfrac{n+1}{5n(n-1)}$ B) $\dfrac{n+1}{5n}$ C) $\dfrac{n+1}{n(n-1)}$

D) $\dfrac{1}{5n}$ E) $\dfrac{1}{n(n-1)}$

9. $\left.\begin{array}{l} a_1 = 4 \\ a_{n+1} - a_n = 2 \end{array}\right\} \Rightarrow \mathbf{a_{10} = ?}$

A) 18 B) 20 C) 21 D) 22 E) 24

10. $(a_{n+1}) = \left(\frac{n}{6} a_n\right),\ a_1 = 3 \Rightarrow \mathbf{a_7 = ?}$

A) $\frac{1}{36}$ B) $\frac{5}{72}$ C) $\frac{5}{108}$ D) $\frac{5}{81}$ E) $\frac{2}{243}$

11. $a_1 = 8,\ n > 1,\ a_n - a_{n-1} = 6 \Rightarrow \mathbf{\sum_{k=1}^{5} a_k = ?}$

A) 96 B) 100 C) 104 D) 112 E) 128

12. $(a_{2n-2}) = \frac{n+4}{n-1} \Rightarrow \mathbf{(a_n) = ?}$

A) $\frac{n+10}{n}$ B) $\frac{2n+20}{n}$ C) $\frac{n+10}{n+2}$ D) $\frac{n+10}{2n+4}$ E) $\frac{n+10}{2n+1}$

13. $a_n = (n+1)! \cdot 3^n,$

$\frac{a_n}{a_{n+1}} = \frac{1}{54} \Rightarrow \mathbf{n = ?}$

A) 18 B) 16 C) 14 D) 12 E) 10

14. $a_1 = 4,\ n \geq 1,$

$(n+1) \cdot a_{n+1} - a_n = 0 \Rightarrow \mathbf{a_5 = ?}$

A) $\frac{1}{12}$ B) $\frac{1}{24}$ C) $\frac{1}{30}$ D) $\frac{1}{42}$ E) $\frac{1}{45}$

15. $\prod_{k=1}^{n} a_n = 3^n \Rightarrow \mathbf{a_{23} = ?}$

A) 1 B) 3 C) 9 D) 27 E) 81

16. $2a_n = a_{n-1} + a_{n+1}$

$a_{11} = 21,$

$a_{19} = 39 \Rightarrow \mathbf{a_{15} = ?}$

A) 24 B) 25 C) 26 D) 28 E) 30

17. $(a_n) = \frac{4n-12}{n+2} \leq 0 \Rightarrow \mathbf{\sum n = ?}$

A) 0 B) 2 C) 3 D) 5 E) 6

YANITLAR / ANSWERS

1	2	3	4	5	6	7	8	9
C	A	E	E	B	B	D	A	D
10	**11**	**12**	**13**	**14**	**15**	**16**	**17**	
C	B	A	B	C	B	E	E	

BÖLÜM
CHAPTER
25

TARİHSEL NOT / HISTORICAL NOTE

BÎRÛNÎ [973 - 1051]

Matematik, gökbilim, doğa bilimleri, coğrafya ve tarih alanındaki çalışmalarıyla tanınır. Bîrûnî'nin matematikçi yönü en çok bilinen yönüdür. Yaşadığı yüzyılın en büyük matematikçisi olan Bîrûnî, trigonometrik fonksiyonlarda yarıçapın bir birim olarak kabul edilmesini öneren ilk kişi olup sinüs ve kosinüs gibi fonksiyonlara sekant, kosekant ve kotanjant fonksiyonlarını ilave etmiştir. Bîrûnî'nin bu yönü Batı Dünyası tarafından ancak iki asır sonra keşfedilip kullanılabilmiştir.

He is known for his work in mathematics, astronomy, natural sciences, geography and history. The mathematician aspect of Bîrûnî is the most known aspect. Bîrûnî, the greatest mathematician of the century in which he lived, was the first to propose that the radius be accepted as a unit in trigonometric functions and he added secant, cosecant and cotangent functions to functions such as sine and cosine. This work of Bîrûnî was not discovered and used by the Western World until after two centuries.

ÖZEL TANIMLI FONKSİYONLAR
THE SPECIAL DOMAIN FUNCTIONS

Bu bölüm, 54 test sorusu içermektedir.
This chapter includes 54 test questions.

BÖLÜM / CHAPTER 25

ÖZEL TANIMLI FONKSİYONLAR / THE SPECIAL DOMAIN FUNCTIONS

- Özel Tanımlı Fonksiyonlar / The Special Domain Functions .... 471 – 486

BÖLÜM

25

CHAPTER

ÖZEL TANIMLI FONKSİYONLAR
THE SPECIAL DOMAIN FUNCTIONS

FONKSİYONLARIN EN GENİŞ TANIM KÜMESİ
The LARGEST DOMAIN of FUNCTIONS

▼ $n \in N$, $f(x) = a_n \cdot x^n + a_{n-1} \cdot x^{n-1} + \ldots + a_1 \cdot x + a_0$

Polinom fonksiyonların tanım kümesi ***reel sayılardır***.

▼ Domain of polynomial functions such as $f(x) = a_n \cdot x^n + a_{n-1} \cdot x^{n-1} + \ldots + a_1 \cdot x + a_0$ is real numers.

▼ $f(x) = \dfrac{g(x)}{h(x)}$ rasyonel fonksiyonların tanım kümesi paydayı sıfır yapan değerler dışındaki reel sayılardır.

▼ Domain of the rational functions such as $f(x) = \dfrac{g(x)}{h(x)}$ is all real numbers except the ones that make denominator equal to 0.

$T \cdot K(D) = R - \{h(x) = 0\}$

▼ $f(x) = \sqrt[2n]{g(x)}$ şeklindeki fonksiyonların tanım kümesi, $T \cdot K = 0 \leq g(x)$ tir.

▼ Domain of the functions in the form of $f(x) = \sqrt[2n]{g(x)}$ is $D = 0 \leq g(x)$.

▼ $f(x) = \log_a g(x)$ logaritma fonksiyonların tanım kümesi, $T.K = \begin{cases} 0 < g(x) \\ 0 < a, \ a \neq 1 \end{cases}$.

▼ Domain of the logarithm functions such as $f(x) \ \log_a^{g(x)}$ is $D = \begin{cases} 0 < g(x) \\ 0 < a, \ a \neq 1 \end{cases}$.

PARÇALI FONKSİYON / PIECEWISE FUNCTION

Tanım: Tanım kümesinin belirli aralıklarında farklı bir kural uygulanan fonksiyonlara ***parçalı fonksiyon*** denir.

Definition: A function is called a piecewise function is when each range of its domain has different rules.

MUTLAK DEĞER FONKSİYONU / ABSOLUTE VALUE FUNCTION

▼ $x \in R$ olmak üzere, f(x)'in mutlak değeri **|f(x)|** biçiminde gösterilir.

▼ If $x \in R$, the absolute value of f(x), denoted by |f(x)| is as follows:

$$|f(x)| = \begin{cases} f(x), & f(x) \geq 0 \\ -f(x), & f(x) < 0 \end{cases}$$

Özellik / Property

★ $\forall x \in R$ için/for ***$y = |f(x)| \geq 0$***

★ $|f(x)| = a \Rightarrow$ ***$f(x) = \mp a$***

★ $|f(x)| \leq a \Rightarrow$ ***$-a \leq f(x) \leq a$***

★ $|f(x)| \geq a \Rightarrow$ ***$f(x) \geq a$ veya/or $f(x) \leq -a$***

- ★ $|f(x)\cdot g(x)| = |f(x)|\cdot|g(x)|$
- ★ $\left|\dfrac{f(x)}{g(x)}\right| = \dfrac{|f(x)|}{|g(x)|}, \quad (g(x) \neq 0)$
- ★ $a < b$ ve/and $a, b \in R^+$

  $a < |f(x)| < b \Rightarrow \begin{cases} -b < f(x) < -a \\ a < f(x) < b \end{cases}$

TAM DEĞER FONKSİYONU / GREATEST INTEGER FUNCTION

Tanım: $\forall x \in R$ için x tamsayı ise kendisine, tamsayı değilse kendisinden küçük en büyük tamsayıya ***x in tam değeri*** denir ve ***[|x|]*** biçiminde gösterilir.

Definition: For $\forall x \in R$, if x is an integer , the number itself; and if it is not an integer, the greatest integer smaller than that number is called ***greatest integer value*** and denoted by [|x|] .

$$[|x|] = a, \ (a \in Z) \Rightarrow a \leq x < a + 1$$

Özellik / Property

- ★ $\forall x \in R$ için/for $[|x|] \leq x < [|x|] + 1$
- ★ $x \in R, a \in Z \Rightarrow [|x + a|] = [|x|] + a$
- ★ $\forall x,y \in R$ için/for $[|x + y|] \geq [|x|] + [|y|]$
- ★ $\forall x,y \in R$ için/for $[|x \cdot y|] > [|x|] \cdot [|y|]$
- ★ $\forall x,y \in R$ için/for $[|x + [|y|]|] = [|x|] + [|y|]$
- ★ $\forall x \in R$ ve/and $a \in Z$ için/for $[|f(x)|] > a \Rightarrow f(x) \geq a + 1$
- ★ $\forall x \in R$ ve/and $a \in Z$ için/for $[|f(x)|] < a \Rightarrow f(x) < a$

İŞARET FONKSİYONU / SIGNUM FUNCTION

Tanım : $A \subset R$ olmak üzere, $f: A \rightarrow R$, $f: x \rightarrow y = f(x)$ fonksiyonu için,

Definition: If $A \subset R$ and $f: A \rightarrow R$, for the function $f: x \rightarrow y = f(x)$.

$$g(x) = sgn(f(x)) = \begin{cases} -1, & f(x) < 0 \\ 0, & f(x) = 0 \\ 1, & f(x) > 0 \end{cases}$$

şeklinde tanımlanır. Burada, $g: A \rightarrow R$, fonksiyonuna ***f nin işaret fonksiyonu*** denir.

is called signum function. Here, $y : A \rightarrow R$ is called **the signum function of f**.

1. $f(x)=\begin{cases} 2x-4, & x>2 \\ 3x-1, & x\leq 2 \end{cases}$

$\Rightarrow$ **f(2) + f(1) + f(3) = ?**

A) 3 B) 5 C) 6 D) 7 E) 9

2. $g(x)=\begin{cases} 2x-4, & x<3 \\ x^2-1, & x\geq 3 \end{cases}$

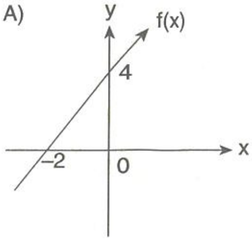
$f(x)=\begin{cases} 5x+1, & x\leq 2 \\ 2x-3, & x>2 \end{cases}$

$\Rightarrow$ **(fog)(1) = ?**

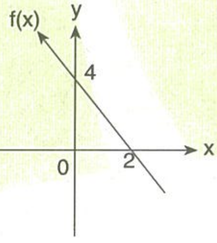
A) –5 B) –7 C) –9 D) –10 E) –11

3. f(x) = 2x + 4 ⇒ **f(x) = ?**

A)



B)



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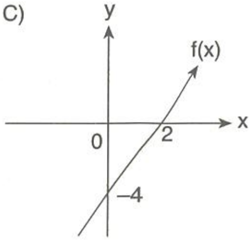

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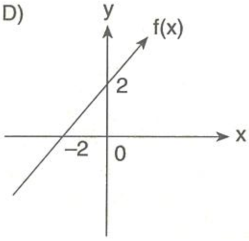

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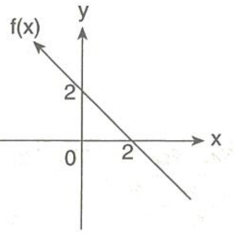
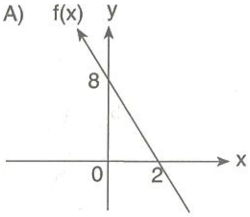
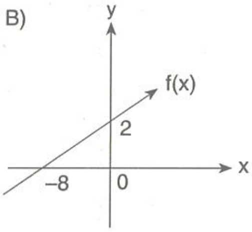


4. f(x) = 4x – 8 ⇒ **f(x) = ?**

A)



B)



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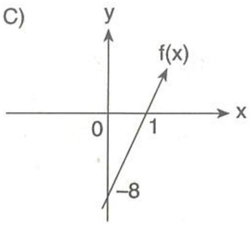

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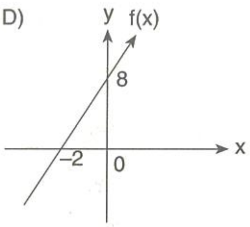
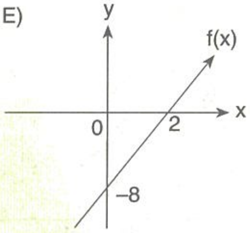
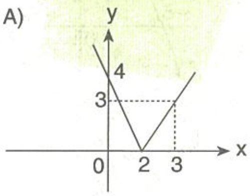


E)

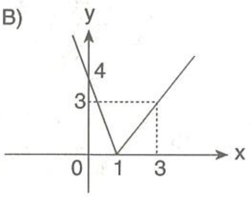


5. $f(x)=\begin{cases} 3x-6, & x>2 \\ 4-2x, & x\leq 2 \end{cases} \Rightarrow$ **f(x) = ?**

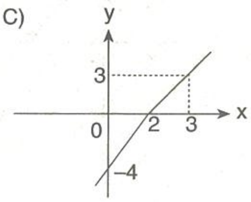
A)



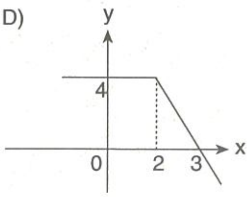
B)



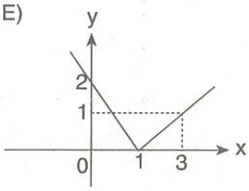
C)



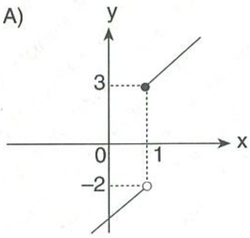
D)

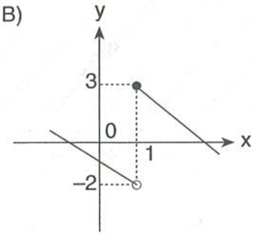


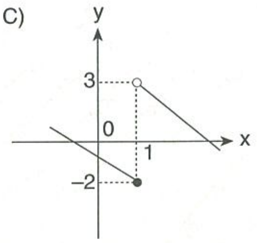
E)

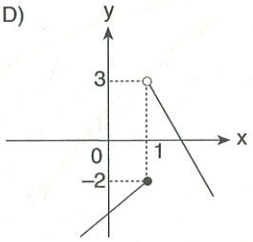


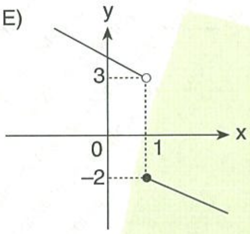
6. $f(x)=\begin{cases}-x+4, & x>1\\ 2x-4, & x\leqslant 1\end{cases} \Rightarrow \mathbf{f(x)=?}$

A)


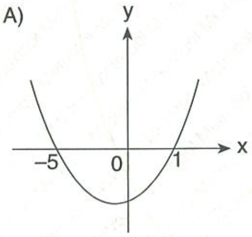
B)


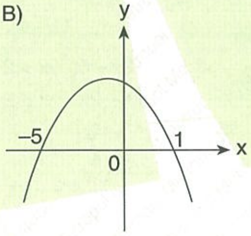
C)


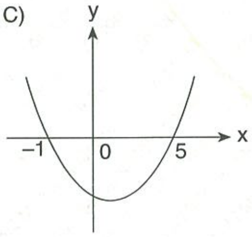
D)


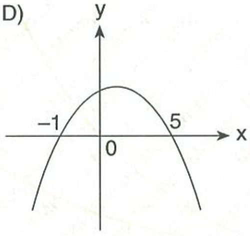
E)


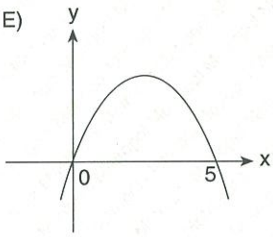
7. $f(x) = x^2 - 4x - 5 \Rightarrow \mathbf{f(x) = ?}$

A)


B)


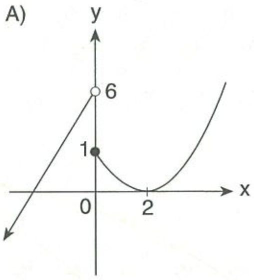
C)


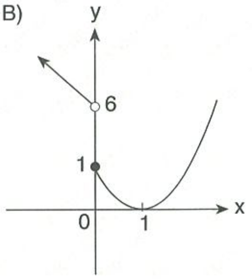
D)


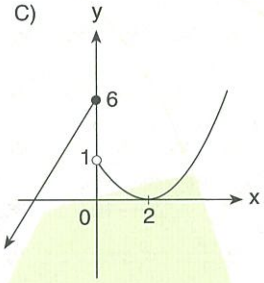
E)


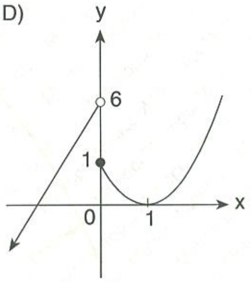
8. $f(x)=\begin{cases}x^2-2x+1, & x\geq 0\\ 2x+6, & x<0\end{cases}$

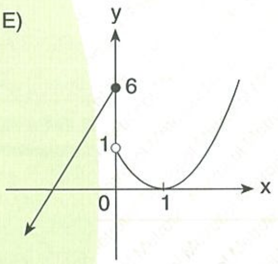
$\Rightarrow \mathbf{f(x) = ?}$

A)


B)


C)


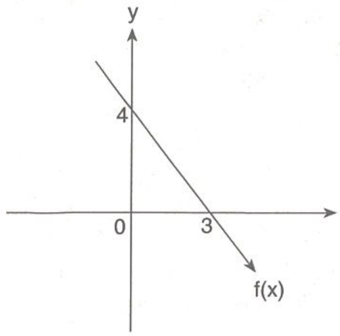
D)


E)


YANITLAR / ANSWERS

1	2	3	4	5	6	7	8
E	C	A	E	A	D	C	D

1.



$\Rightarrow$ **f(x) = ?**

A) $3x + 4$

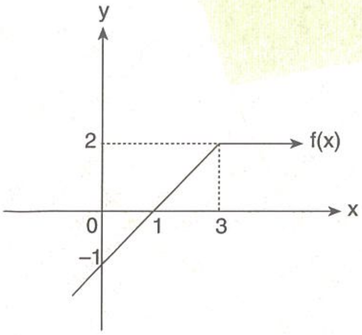
B) $4x + 3$

C) $\frac{12-3x}{4}$

D) $\frac{4x-12}{3}$

E) $\frac{12-4x}{3}$

2.



$\Rightarrow$ **f(x) = ?**

A) $f(x) = \begin{cases} 2, & x \geq 3 \\ x+1, & x < 3 \end{cases}$

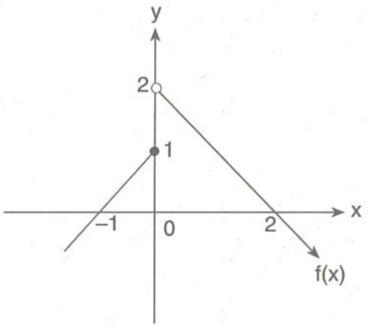
B) $f(x) = \begin{cases} 2, & x \geq 2 \\ x-1, & x < 2 \end{cases}$

C) $f(x) = \begin{cases} 2, & x \geqslant 3 \\ x-1, & x < 3 \end{cases}$

D) $f(x) = \begin{cases} 2, & x \geq 3 \\ 1-x, & x < 3 \end{cases}$

E) $f(x) = \begin{cases} 2, & x \geq 3 \\ 2x-1, & x < 3 \end{cases}$

3.



$\Rightarrow$ **f(x) = ?**

A) $f(x) = \begin{cases} 1+x, & x > 0 \\ 2-x, & x \leq 0 \end{cases}$

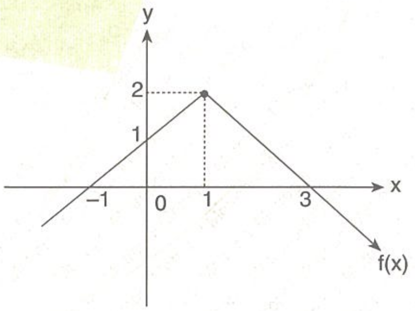
B) $f(x) = \begin{cases} 1+x, & x \leq 0 \\ 2-x, & x > 0 \end{cases}$

C) $f(x) = \begin{cases} 1+x, & x \geq 0 \\ 2-x, & x < 0 \end{cases}$

D) $f(x) = \begin{cases} 1+x, & x < 0 \\ 1-x, & x > 0 \end{cases}$

E) $f(x) = \begin{cases} 1-x, & x \leq 0 \\ 2-x, & x > 0 \end{cases}$

4.



$\Rightarrow$ **f(x) = ?**

A) $f(x) = \begin{cases} 3-x, & x \geq 1 \\ x+1, & x < 1 \end{cases}$

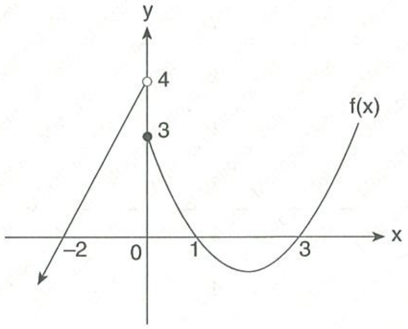
B) $f(x) = \begin{cases} 3-x, & x \leqslant 1 \\ x+1, & x > 1 \end{cases}$

C) $f(x) = \begin{cases} x-3, & x \geq 1 \\ x+1, & x < 1 \end{cases}$

D) $f(x) = \begin{cases} 3-x, & x \geq 1 \\ 1-x, & x < 1 \end{cases}$

E) $f(x) = \begin{cases} x+3, & x \geq 1 \\ x+1, & x < 1 \end{cases}$

5.



$\Rightarrow$ **f(x) = ?**

A) $f(x)=\begin{cases} x^2+4x-3, & x\geq 0 \\ 2x+4, & x<0 \end{cases}$

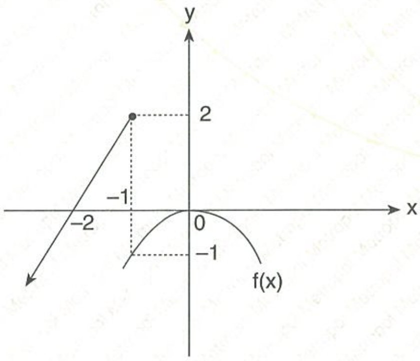
B) $f(x)=\begin{cases} x^2-4, & x\geq 0 \\ 2x+4, & x<0 \end{cases}$

C) $f(x)=\begin{cases} x^2-4x+3, & x>0 \\ 4+2x, & x\leq 0 \end{cases}$

D) $f(x)=\begin{cases} x^2-4x+3, & x\geq 0 \\ 2x-4, & x<0 \end{cases}$

E) $f(x)=\begin{cases} x^2-4x+3, & x\geq 0 \\ 4+2x, & x<0 \end{cases}$

6.



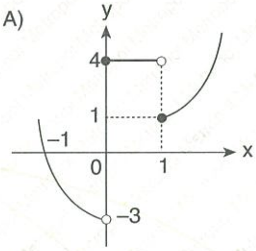
$\Rightarrow$ **f(3) + f(–5) = ?**

A) 15 B) 3 C) –3 D) –6 E) –15

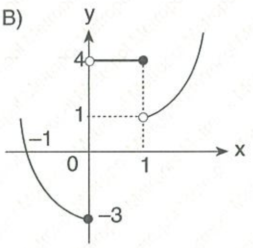
7. $f(x)=\begin{cases} x^2-2x-3, & x<0 \\ 4, & 1>x\geq 0 \\ x^2, & x\geq 1 \end{cases}$

$\Rightarrow$ **f(x) = ?**

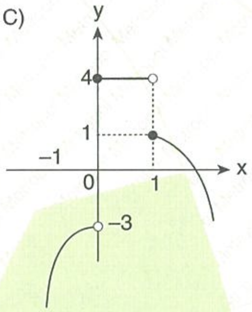
A)



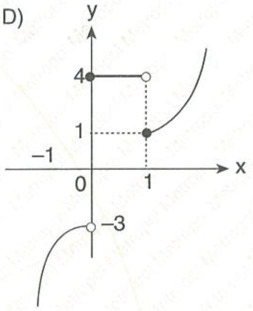
B)



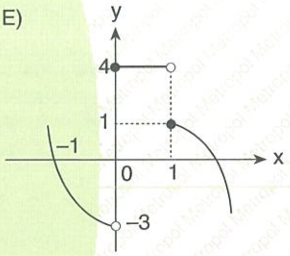
C)



D)



E)



YANITLAR / ANSWERS

1	2	3	4	5	6	7
E	C	B	A	E	E	A

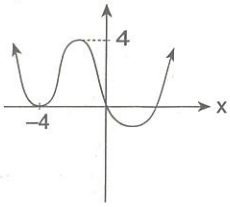
1. $f(x)=\sqrt[4]{4-|x+1|}$ ⇒ **tanım kümesi (domain) = ?**

A) [–5, 3] B) [–5, 2] C) (–5, 4)

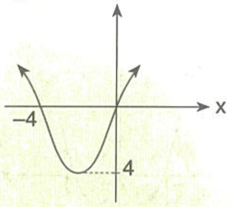
D) [–4, 2] E) [–4, 3]

2. $f(x)=|x^2+4x|$ ⇒ **grafik (graph) = ?**

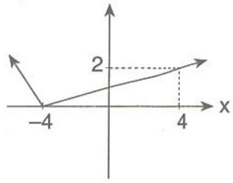
A)



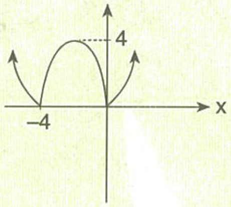
B)



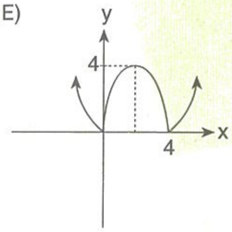
C)



D)



E)



3. $f(x)=\begin{cases} x^2-2x-6, & x\geq 2 \\ x+3, & x<2 \end{cases}$

⇒ **fofof(1) = ?**

A) –8 B) –6 C) –4 D) 4 E) 6

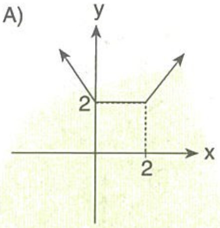
4. $f(x)=\sqrt[6]{\dfrac{1}{7-|x-2|}}+\sqrt[5]{x}+x-2$

$x\in Z$ ⇒ **$\sum x = ?$**

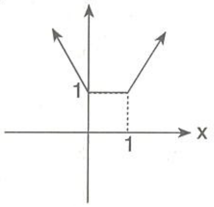
A) 33 B) 28 C) 26 D) 24 E) 19

5. $f(x)=|x-2|+|x|$ ⇒ **grafik (graph) = ?**

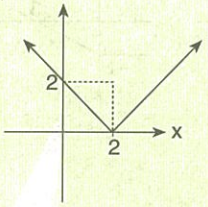
A)



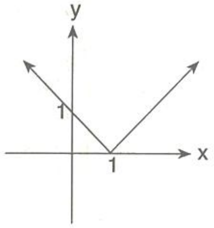
B)



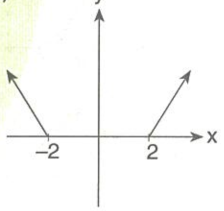
C)



D)



E)



6. $x<0$,

$f(x)=\sqrt[4]{x^4}+\sqrt{x^2-4x+4}+\sqrt[3]{x^3-6x^2+12x-8}$

⇒ **f(x) = ?**

A) –x B) –x –2 C) –x – 4

D) x – 2 E) x – 4

7. $f(x)=\sqrt{x^2-2mx+6m-8}$,

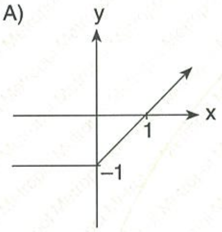
Tanım kümesi (domain) = R $\Rightarrow$ **? < m < ?**

A) $-4 \le m \le -2$ B) $-2 \le m < 0$ C) $0 \le m < 2$
D) $0 \le m \le 4$ E) $2 \le m \le 4$

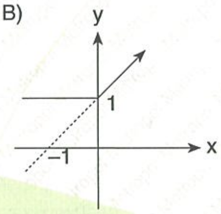
8. $f(x) = |x - 1| - x$,

$g(x) = 2x + 1 \Rightarrow$ **gof(x) = ?**

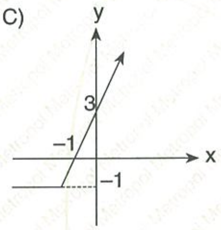
A)



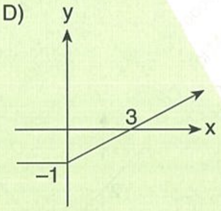
B)



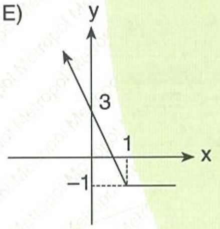
C)



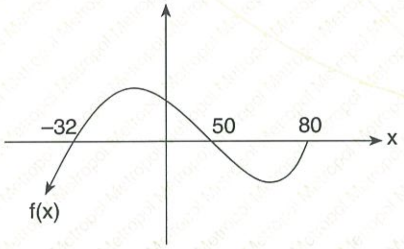
D)



E)



9.


y
–32
50
80
x
f(x)

$f(x) > 0$,

$x \in Z \Rightarrow \sum x = ?$

A) 635 B) 654 C) 696 D) 729 E) 732

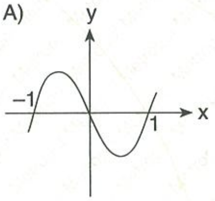
10. $x \in R$,

$A = |2x - 5| + |4x + 10| \Rightarrow$ **min(A) = ?**

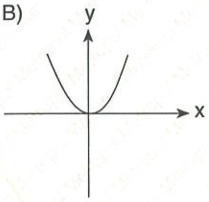
A) 6 B) 8 C) 10 D) 12 E) 14

11. $f(x) = (x - 1)\cdot|x| \Rightarrow$ **grafik (graph) = ?**

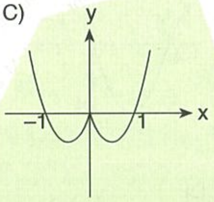
A)



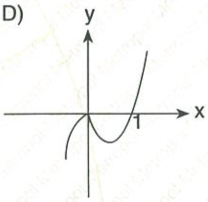
B)



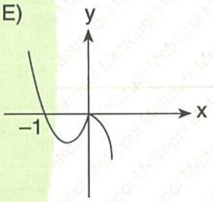
C)



D)



E)



12. $a \in Z$,

$$\frac{|x^2+4|}{2x^2-3x+a-5} > 0,$$

ÇK(SS) = R $\Rightarrow$ **min(a) = ?**

A) 3 B) 5 C) 7 D) 11 E) 12

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1	2	3	4	5	6
A	D	B	C	A	A
7	**8**	**9**	**10**	**11**	**12**
E	E	D	C	D	C

1. $-3 < x < 4$

$\Rightarrow$ **$|x - 4| + |x + 3| = ?$**

A) 1 B) 2 C) 3 D) 5 E) 7

2. $|2x - 3| = 7 \Rightarrow$ **ÇK(SS) = ?**

A) {–5, 2} B) {–3, 3} C) {–2, 5} D) {2, 5} E) {5}

3. $1 < x < 5$

$|x - 1| + |x - 5| + 2x = 8$

$\Rightarrow$ **$x = ?$**

A) 1 B) 2 C) 3 D) 4 E) 5

4. $||x - 4| + 3| = 7 \Rightarrow$ **ÇK(SS) = ?**

A) {–14, –6, 0, 8} B) {0, 8} C) {–14, –6} D) {–6, 8} E) {1, 7}

5. $f(x)=\begin{cases}2x-4, & x\geq 2\\ 4-2x, & x<2\end{cases}$

$f(x) = 6 \Rightarrow$ **ÇK(SS) = ?**

A) {1, 5} B) {–5, 1} C) {–1, 5} D) {–5, –1} E) {0, 5}

6. $f(x) = x^2 - 4 \cdot |x| - 5$

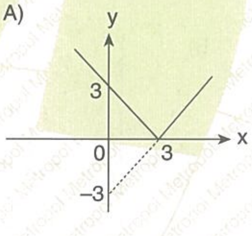
$f(x_1) = f(x_2) = 0$

$\Rightarrow$ **$x_1 \cdot x_2 = ?$**

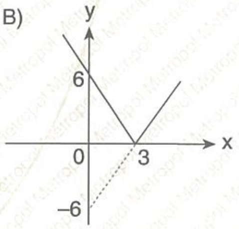
A) –25 B) –16 C) –1 D) 1 E) 25

7. $f(x) = |2x - 6| \Rightarrow$ **f(x) = ?**

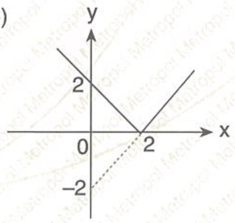
A)



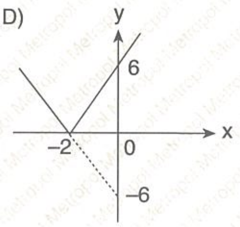
B)



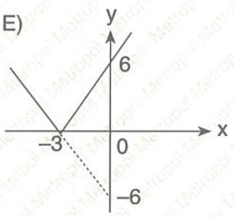
C)



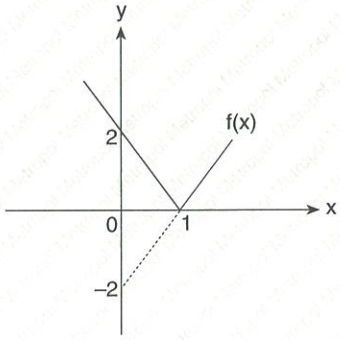
D)



E)



8.



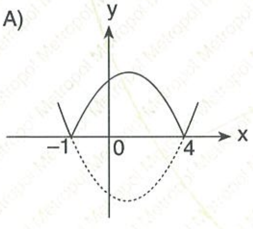
$\Rightarrow$ **f(x) = ?**

A) $|x-1|$ B) $|x+1|+x$ C) $|2x-2|$
D) $|2x+2|$ E) $|x|+2$

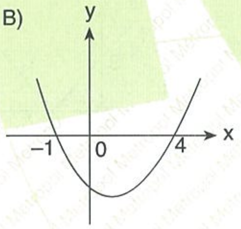
9. $f(x) = |x^2 - 3x - 4|$

$\Rightarrow$ **f(x) = ?**

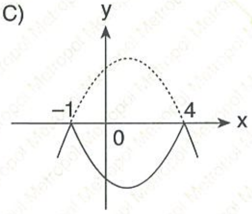
A)



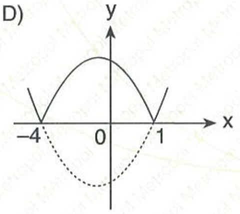
B)



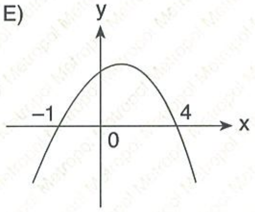
C)



D)



E)



10. $\left.\begin{array}{l} f(x) = x - |x+1| \\ g(x) = 3x+5 \end{array}\right\} \Rightarrow$ **(fog)(x) = ?**

A) $(fog)(x) = \begin{cases} -1, & x \geq -2 \\ 6x+11, & x < -2 \end{cases}$

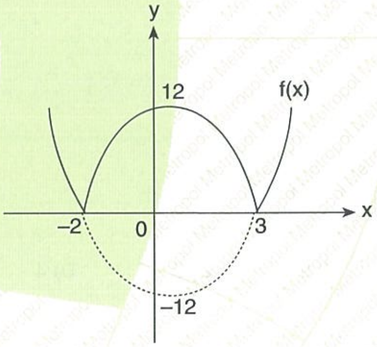
B) $(fog)(x) = \begin{cases} 6x+11, & x \geq -2 \\ -1, & x < -2 \end{cases}$

C) $(fog)(x) = \begin{cases} 3x-1, & x \geq -2 \\ -1, & x < -2 \end{cases}$

D) $(fog)(x) = \begin{cases} -1, & x \geq -2 \\ 3x-1, & x < -2 \end{cases}$

E) $(fog)(x) = \begin{cases} -1, & x \geq -2 \\ 6x-1, & x < -2 \end{cases}$

11.


y
12
f(x)
–2
0
3
x
–12

$\Rightarrow$ **f(x) = ?**

A) $|x^2 - x - 1|$ B) $|x^2 + x - 6|$
C) $|2x^2 - 2x - 12|$ D) $|x^2 + 2x - 12|$
E) $|x^2 - 2x - 12| + x$

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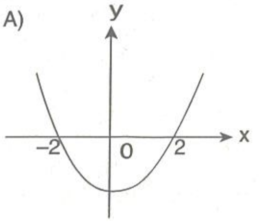
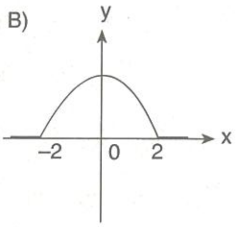
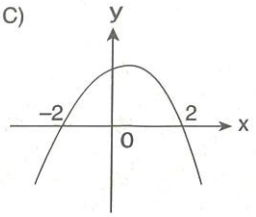
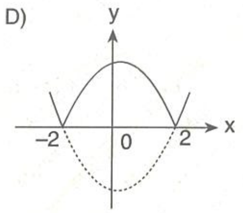
1	2	3	4	5	6
E	C	B	B	C	A
7	**8**	**9**	**10**	**11**	**12**
B	C	A	A	C	

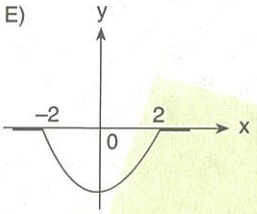
1. $f(x) = |x^2 - 4| \Rightarrow$ **f(x) = ?**

A)


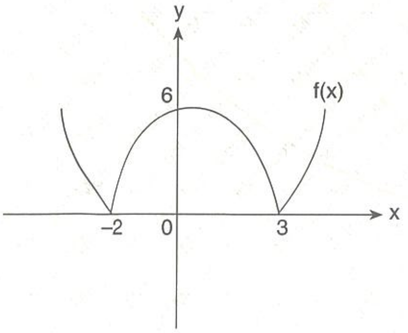
B)


C)


D)


E)


2.



$\Rightarrow$ **f(x) = ?**

A) $|x^2 + x + 6|$

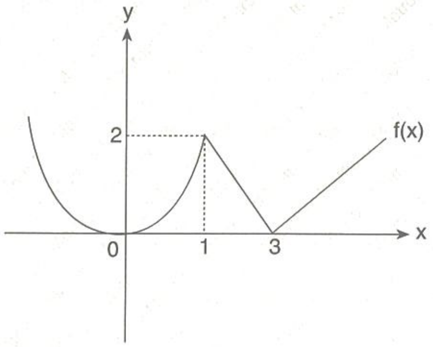
B) $|x^2 - x + 6|$

C) $|-x^2 + x + 6|$

D) $|2x^2 - 2x - 12|$

E) $|x^2 - 5x - 6|$

3.



$\Rightarrow$ **f(x) = ?**

A) $f(x) = \begin{cases} -x^2, & x < 1 \\ x-3, & x \geq 2 \end{cases}$

B) $f(x) = \begin{cases} x^2, & x < 1 \\ x-3, & x \geq 1 \end{cases}$

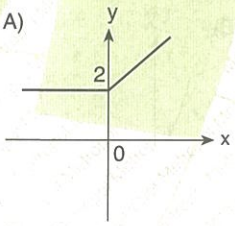
C) $f(x) = \begin{cases} |x^2|, & x < 1 \\ x-3, & x \geq 1 \end{cases}$

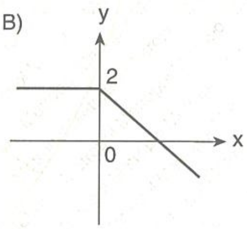
D) $f(x) = \begin{cases} 2x^2, & x < 1 \\ |x-3|, & x \geq 1 \end{cases}$

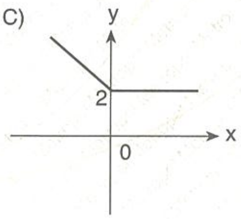
E) $f(x) = \begin{cases} x^2, & x < 1 \\ |x-3|, & x \geq 1 \end{cases}$

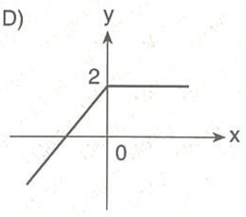
4. $f(x) = x + |x - 2|,$

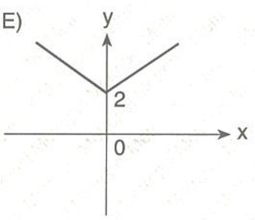
$g(x) = 2x + 2 \quad \Rightarrow$ **(fog)(x) = ?**

A)


B)


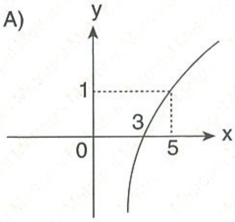
C)


D)


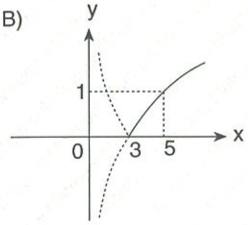
E)


5. $f(x) = |\log_3(x-2)| \Rightarrow$ **f(x) = ?**

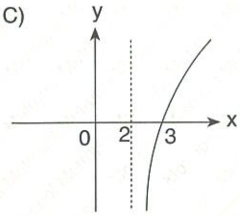
A)



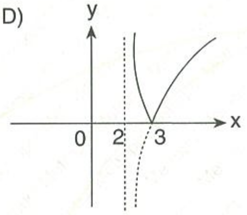
B)



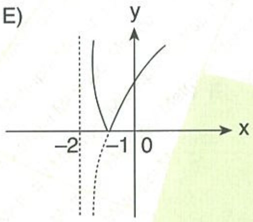
C)



D)

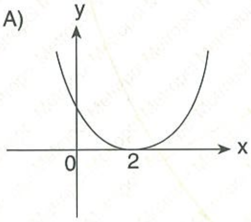


E)

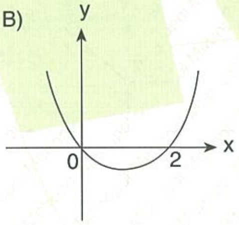


6. $f(x) = x \cdot |x-2| \Rightarrow$ **f(x) = ?**

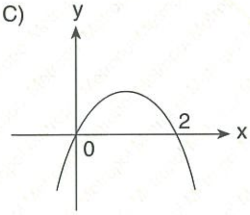
A)



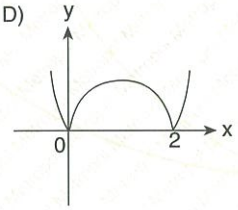
B)



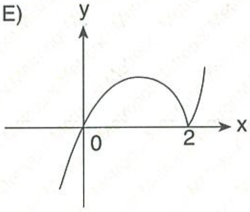
C)



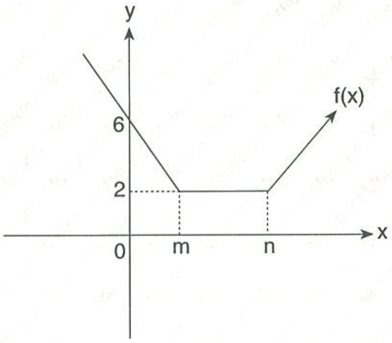
D)



E)



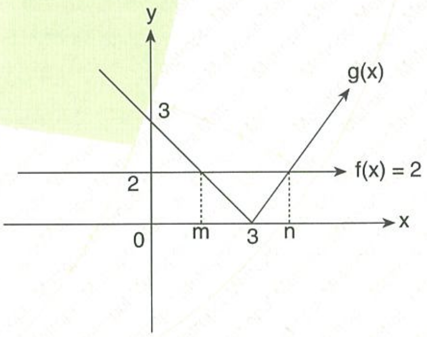
7.



$f(x) = |x-m| + |x-n| \Rightarrow$ **m·n = ?**

A) 6 B) 8 C) 10 D) 12 E) 16

8.



$\Rightarrow$ **n – m = ?**

A) 3 B) 4 C) 5 D) 6 E) 7

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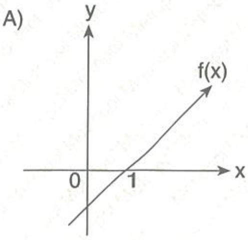
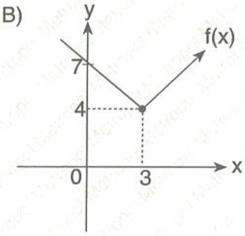
1	2	3	4	5	6	7	8
D	C	D	A	D	E	B	B

1. $f(x) = |x - 3| + 4 \Rightarrow$ **f(x) = ?**

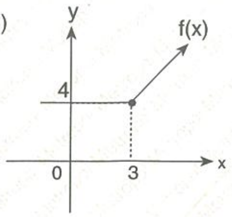
A)



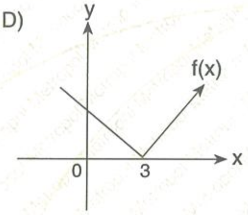
B)



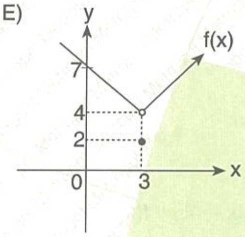
C)



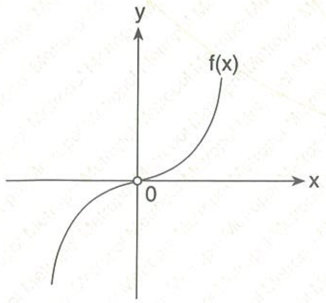
D)



E)



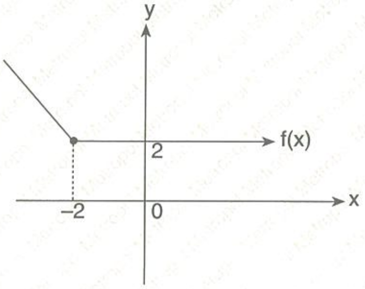
2.



$h(x) = 2x - \dfrac{2f(x)}{|f(x)|} \Rightarrow$ **h(−3) = ?**

A) −4 B) −5 C) −6 D) −7 E) −8

3.



⇒ **f(x) = ?**

A) $f(x) = |2 - x| + x$

B) $f(x) = |2 + x| + x$

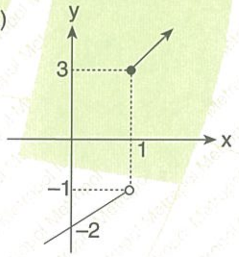
C) $f(x) = |x - 2| - x$

D) $f(x) = |x + 2| + 2x$

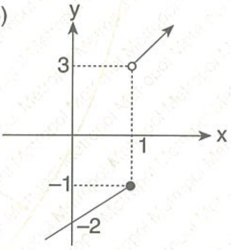
E) $f(x) = |x + 2| - x$

4. $f(x) = \begin{cases} 2x+1, & x \geq 1 \\ x-2, & x < 1 \end{cases} \Rightarrow$ **f(x) = ?**

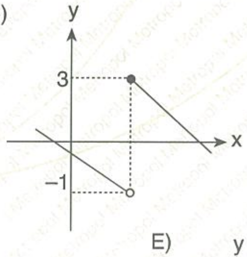
A)



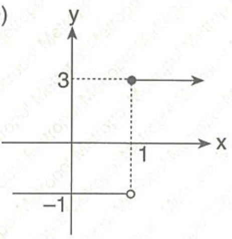
B)



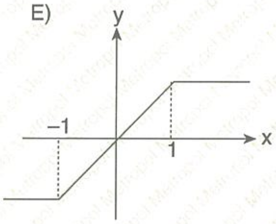
C)



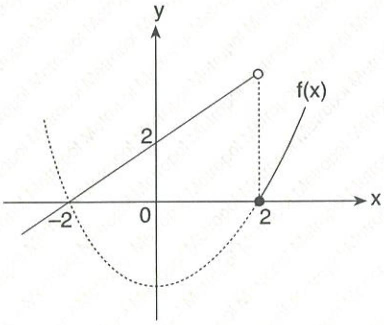
D)



E)



5.



$\Rightarrow$ **f(x) = ?**

A) $f(x)=\begin{cases} x^2-4, & x>2 \\ x-2, & x\le 2 \end{cases}$

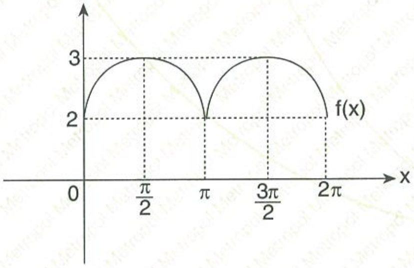
B) $f(x)=\begin{cases} x^2-4, & x\ge 2 \\ x+2, & x<2 \end{cases}$

C) $f(x)=\begin{cases} x^2-4, & x\le 2 \\ x+2, & x>2 \end{cases}$

D) $f(x) = x + |x^2 - 4|$

E) $f(x) = |x^2 - 4| - x^2$

6.



$\Rightarrow$ **f(x) = ?**

A) $f(x) = |\sin x|$

B) $f(x) = 1 + |\sin x|$

C) $f(x) = |2 + \sin x|$

D) $f(x) = 2 + |\sin x|$

E) $f(x) = 2 - |\sin x|$

7. $f(x) = |2x - 2\cdot|-x|| - 3 \Rightarrow$ **f(x) = ?**

A) $f(x)=\begin{cases} 4x-3, & x\ge 0 \\ -4x+3, & x<0 \end{cases}$

B) $f(x)=\begin{cases} -3, & x\ge 0 \\ -4x-3, & x<0 \end{cases}$

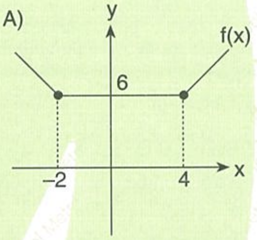
C) $f(x)=\begin{cases} 4x-3, & x\ge 0 \\ 3, & x<0 \end{cases}$

D) $f(x)=\begin{cases} 4x-3, & x\ge 0 \\ -3, & x<0 \end{cases}$

E) $f(x)=\begin{cases} -4x-3, & x\ge 0 \\ -3, & x<0 \end{cases}$

8. $f(x) = |x + 2| + |x - 4| \Rightarrow$ **f(x) = ?**

A)



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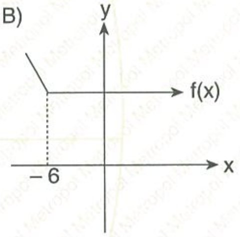
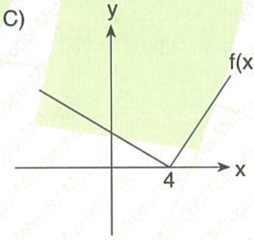
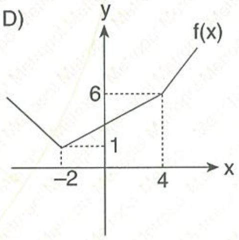


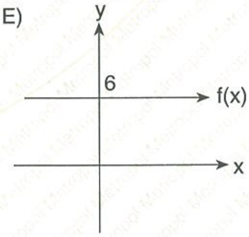
C)



D)



E)



YANITLAR / ANSWERS

1	2	3	4	5	6	7	8
B	A	E	A	B	D	B	A

BÖLÜM CHAPTER 26

TARİHSEL NOT / HISTORICAL NOTE

LEONHARD EULER [1707 - 1783]

İsviçreli matematikçi ve fizikçi. 18. yüzyılın en önemli ve tüm zamanların önde gelen matematikçilerinden biri kabul edilmektedir. En üretken matematikçilerden biri olarak çalışmalarının bütünü 70 cildi aşmaktadır. Euler pek çok yeni kavram geliştirmiş, basit aritmetikten sayılar teorisi ve topolojiye kadar farklı alanlarda uzun süre kabul gören birçok teorem ispatlamıştır. Bu çalışmaları esnasında, günümüzde kullanılan modern matematik terminolojisinin yaratıcısı olmuş, fonksiyon kavramı ve onun yazımını tanımlamıştır (yaptığı bu çalışma için verilebilecek örneklerden bazıları trigonometrik fonksiyonlar için yaptığı sin, cos ve tan tanımlamalarıdır).

Swiss mathematician and physicist. It is regarded as one of the most important and all-time leading mathematicians of the 18th century. As one of the most prolific mathematicians, his works exceed 70 volumes. Euler has developed a number of new concepts, and has proved many theorems that have long been accepted in different fields from simple arithmetic to number theory and topology. During these practices, he became the creator of modern mathematics terminology used today and he also defined the concept writing and of function (some of the examples that he made for this study are the definitions of sin, cos and tan for trigonometric functions).

LİMİT
LIMITS

Bu bölüm, 49 test sorusu içermektedir.
This chapter includes 49 test questions.

BÖLÜM / CHAPTER 26

LİMİT / LIMITS

- Limit / Limits .... 487 – 498

BÖLÜM

26

CHAPTER

LİMİT
LIMITS

LİMİT / LIMITS

▼ $A \subseteq R$, $a \in A$, $f: A \rightarrow R$ ya da $f: A - \{a\} \rightarrow R$

$y = f(x)$ şeklindeki fonksiyonlarda x değerleri a ya soldan artan değerlerle yaklaşırken, f fonksiyonunun değeri L_1 e yaklaşır. Buna fonksiyonun ***a noktasındaki soldan limiti*** denir.

▼ In functions in the form $y = f(x)$, if when x approaches to a from values less than a, the value of function approaches L_1. That is ***left-hand limit*** of the function in point a.

$$\lim_{x \to a^-} f(x) = L_1$$

▼ x değerleri a ya sağdan azalan değerlerle yaklaşırken f fonksiyonunun değeri L_2 ye yaklaşır. Buna fonksiyonun ***a noktasındaki sağdan limiti*** denir.

▼ If when x approaches to a from values greater than a, the value of function approaches L_2. That is ***right-hand limit*** of the function in point a.

$$\lim_{x \to a^+} f(x) = L_2$$

▼ f(x) fonksiyonunun x = a noktasında limiti varsa a noktasındaki sağdan ve soldan limitleri eşittir.

▼ If the function f(x) has a limit at point x = a, limits from the right and left are equal.

$$\lim_{x \to a} f(x) = L \iff \lim_{x \to a^+} f(x) = \lim_{x \to a^-} f(x) = L$$

▼ $\lim\limits_{x \to a^+} f(x) \neq \lim\limits_{x \to a^-} f(x) \Rightarrow \lim\limits_{x \to a} f(x)$ yoktur. / does not exist.

Özellik / Property

$\lim\limits_{x \to a} f(x) = L_1$,

$\lim\limits_{x \to a} g(x) = L_2$ olmak üzere / if

★ $\lim\limits_{x \to a} k = k \quad (k \in R)$

★ $\lim\limits_{x \to a} [(f(x) \mp g(x)] = [\lim\limits_{x \to a} f(x)] \mp [\lim\limits_{x \to a} g(x)] = L_1 + L_2$

★ $\lim\limits_{x \to a} [(f(x) \cdot g(x)] = [\lim\limits_{x \to a} f(x)] \cdot [\lim\limits_{x \to a} g(x)] = L_1 \cdot L_2$

★ $k \in R$, $\lim\limits_{x \to a} [k \cdot (f(x)] = k \cdot [\lim\limits_{x \to a} f(x)] = k \cdot L_1$

★ $\lim\limits_{x \to a} \dfrac{f(x)}{g(x)} = \dfrac{\lim\limits_{x \to a} f(x)}{\lim\limits_{x \to a} g(x)} = \dfrac{L_1}{L_2} \quad (\lim\limits_{x \to a} g(x) \neq 0)$

- ★ $\lim\limits_{x \to a} [f(x)^n] = [\lim\limits_{x \to a} f(x)]^n = L_1{}^n$
- ★ $n > 1, \lim\limits_{x \to a} \sqrt[n]{f(x)} = \begin{cases} \sqrt[n]{\lim\limits_{x \to a} f(x)}, & \text{n tek doğal sayı / odd natural number}, \\ \sqrt[n]{\lim\limits_{x \to a} f(x)}, & \text{n çift doğal sayı / even natural number}, \ f(x) \geq 0 \end{cases}$
- ★ $\lim\limits_{x \to a} [k^{f(x)}] = k^{\lim\limits_{x \to a} f(x)}, \ (k \in R^+)$
- ★ $\lim\limits_{x \to k} [\log_a f(x)] = \log_a [\lim\limits_{x \to k} f(x)]$

TRİGONOMETRİK FONKSİYONLARIN LİMİTİ / LIMIT of TRIGONOMETRIC FUNCTIONS

- ▼ $\lim\limits_{x \to a} \sin x = \sin a$
- ▼ $\lim\limits_{x \to a} \cos x = \cos a$
- ▼ $\lim\limits_{x \to a} \tan x = \tan a \ \left(a \neq (2k+1) \cdot \frac{\pi}{2}, \ k \in Z\right)$
- ▼ $\lim\limits_{x \to a} \cot x = \cot a \ (a \neq k\pi, \ k \in Z)$

Özellik / Property

Kritik Noktalar / The Critical Points:

- ★ $f(x) = \frac{g(x)}{h(x)}$ fonksiyonunda h(x) = 0 eşitliğini sağlayan x değerleri,
- ★ In functions $f(x) = \frac{g(x)}{h(x)}$, the values x satisfying h(x) = 0
- ★ f(x) = Ig(x)I fonksiyonunda g(x) = 0 eşitliğini sağlayan x değerleri,
- ★ In functions f(x) = Ig(x)I, the values x satisfying g(x) = 0
- ★ Parçalı fonksiyonun parçalandığı x değerleri
- ★ The values x of disjunction points of piecewise functions
- ★ f : R → R, $\lim\limits_{x \to a} f(x)$ bulunurken / when finding:
  - **i.** x = a noktası kritik nokta ise, sağdan ve soldan limit incelenir.

    Examine the left and the right limits if x = a is a critical point.
  - **ii.** x = a noktası kritik nokta değilse, $\lim\limits_{x \to a} f(x) = f(a)$ dır.

    $\lim\limits_{x \to a} f(x) = f(a)$ if x = a is not a critical point.

GENİŞLETİLMİŞ REEL SAYILAR KÜMESİ / EXTENDED REAL NUMBERS SET

Tanım: Her $x \in R$ için reel sayılar kümesi $-\infty < x < \infty$ veya $R = (-\infty, \infty)$ ile gösterilir. Burada $-\infty$ ve $+\infty$, reel sayılar kümesine dahil değildir. R gerçel sayılar kümesi ile $-\infty$ ve $+\infty$ un birleşmesiyle elde edilen kümeye ***genişletilmiş reel sayılar kümesi*** denir.

Definition: For every $x \in R$, the set of real numbers can be represented $-\infty < x < +\infty$ or $R = (-\infty, +\infty)$. Here, $-\infty$ and $+\infty$ are not included in the set of real numbers. The set $R \cup \{-\infty, +\infty\}$ is called **the extended real numbers set**.

SONUÇLAR / CONCLUSIONS

- $a \in R$
  $a \mp \infty = \mp\infty, \quad a \mp (-\infty) = \pm\infty$
- $a \in R^+ \Rightarrow a \cdot (+\infty) = +\infty, \quad a \cdot (-\infty) = -\infty$
  $a \in R^- \Rightarrow a \cdot (+\infty) = -\infty, \quad a \cdot (-\infty) = +\infty$
- $a \in R^+ \Rightarrow \frac{+\infty}{a} = +\infty, \quad \frac{-\infty}{a} = -\infty$
  $a \in R^- \Rightarrow \frac{+\infty}{a} = -\infty, \quad \frac{-\infty}{a} = +\infty$
- $a \in R \Rightarrow \frac{a}{\pm\infty} = 0, \ (a \neq 0)$
- $a \in R^+ \Rightarrow \frac{a}{0^+} = +\infty, \quad \frac{a}{0^-} = -\infty$
  $a \in R^- \Rightarrow \frac{a}{0^+} = -\infty, \quad \frac{a}{0^-} = +\infty$
- $\infty + \infty = \infty, (-\infty) + (-\infty) = -\infty$
- $(\mp\infty) \cdot (\mp\infty) = \infty, \quad (\mp\infty) \cdot (\pm\infty) = -\infty$
- $n \in Z$
  $\sqrt[2n-1]{\pm\infty} = \pm\infty$
  $\sqrt[2n]{+\infty} = \infty, \quad \sqrt[2n]{-\infty} =$ tanımsız / indefinite
- $n \in Z$
  $(-\infty)^{2n} = +\infty$
  $(-\infty)^{2n-1} = -\infty$
- $a > 1 \Rightarrow a^{+\infty} = \infty$
  $a^{-\infty} = 0$
  $0 < a < 1 \Rightarrow a^{+\infty} = 0$
  $a^{-\infty} = +\infty$

Önemli / Important

- $\lim\limits_{x \to 0^-} \frac{1}{x} = -\infty$
- $\lim\limits_{x \to 0^+} \frac{1}{x} = +\infty$
- $\lim\limits_{x \to -\infty} \frac{1}{x} = 0$
- $\lim\limits_{x \to +\infty} \frac{1}{x} = 0$

▼ x → +∞, – ∞ için polinom fonksiyonlarında limit bulunurken derecesi en büyük olan terimin limitine bakılması yeterlidir.

▼ To find the limit of the polynomial function for x → +∞, –∞, we can find the limit of the term with maximum degree.

$$\lim_{x \to \infty} 4x^3 + 5x^2 + 7x + 1 = \lim_{x \to \infty} 4x^3 = 4 \cdot (\infty) = \infty$$

MUTLAK DEĞER FONKSİYONUNUN LİMİTİ / LIMIT OF THE ABSOLUTE VALUE FUNCTION

▼ $\lim\limits_{x \to a} |f(x)| = \left| \lim\limits_{x \to a} f(x) \right| = |f(a)|$

TAM DEĞER FONKSİYONUNUN LİMİTİ / LIMIT of the GREATEST INTEGER FUNCTION

▼ f(x) = [|g(x)|]

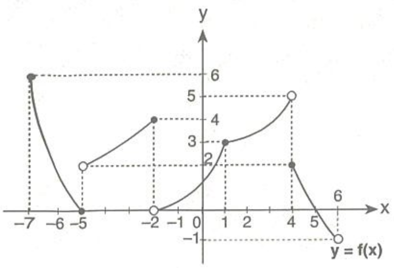
$$\lim_{x \to a} f(x) = \begin{cases} [|g(a)|], & g(a) \notin Z \\ \text{Limit yoktur./Limit does not exist}, & g(a) \in Z \end{cases}$$

İŞARET FONKSİYONUNUN LİMİTİ / LIMIT of the SIGNUM FUNCTION

▼ f(x) = sgn[g(x)]

$$\lim_{x \to a} f(x) = \begin{cases} \text{sgn}[g(a)], & g(a) \neq 0 \\ \text{Limit yoktur./Limit does not exist}, & g(a) = 0 \end{cases}$$

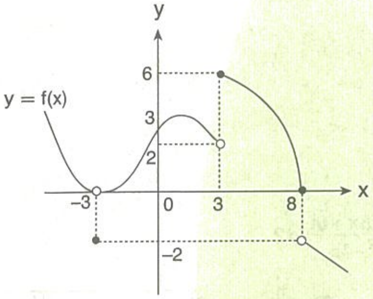
1.



$\Rightarrow \lim\limits_{x \to -5^+} f(x) + \lim\limits_{x \to 6^-} f(x) + \lim\limits_{x \to -2^+} f(x) = ?$

A) −2 B) −1 C) 0 D) 1 E) 2

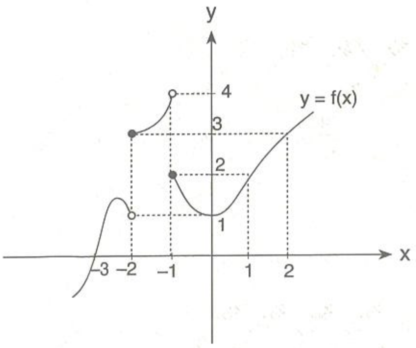
2.



$\Rightarrow \lim\limits_{x \to -3^+} f(x) + \lim\limits_{x \to 3^+} f(x) + \lim\limits_{x \to 8^-} f(x) = ?$

A) 4 B) 6 C) 8 D) 9 E) 10

3.



$\Rightarrow f(1) - \lim\limits_{x \to 0} f(x) + \lim\limits_{x \to 2} f(x) - \lim\limits_{x \to 1} f(x) = ?$

A) 1 B) 2 C) 3 D) 4 E) 5

4. $f(x) = \begin{cases} x^2 - 1, & x \geq 2 \\ 2x + 1, & x < 2 \end{cases}$

$\Rightarrow \lim\limits_{x \to 2^+} f(x) + \lim\limits_{x \to 1^+} f(x) = ?$

A) 3 B) 4 C) 5 D) 6 E) 7

5. $f(x) = x^2 - 2x + \cos(x - 3) \Rightarrow \lim\limits_{x \to 3^+} (f(x)) = ?$

A) 2 B) 3 C) 4 D) 5 E) 6

6. $\lim\limits_{x \to 2} \dfrac{x^2 - 4}{2x^2 + 3x - 13} = ?$

A) $\frac{4}{11}$ B) $\frac{1}{2}$ C) 0 D) $-\frac{1}{2}$ E) $-\frac{4}{11}$

7. $\lim\limits_{x \to 1^+} \dfrac{1}{3 - 3x} = ?$

A) $-\infty$ B) −2 C) 0 D) $\frac{2}{3}$ E) ∞

8. $\lim\limits_{x\to 3^+}\left(\dfrac{|x-3|}{x-3}\right)+\lim\limits_{x\to 5^-}\left(\dfrac{x-5}{|x-5|}\right)=?$

A) -2 B) -1 C) 0 D) 1 E) 2

9. $f(x)=\begin{cases}x^2-2x+m, & x\geq 3\\ 3x+1, & x<3\end{cases}$

$\lim\limits_{x\to 3}f(x)=10 \Rightarrow m=?$

A) 7 B) 8 C) 9 D) 10 E) 11

10. $\lim\limits_{x\to 3}(\log_2(5x+1))=?$

A) 16 B) 8 C) 6 D) 4 E) 2

11. $\lim\limits_{x\to e^+}\dfrac{5}{1-\ln x}=?$

A) $-\infty$ B) -2 C) 0 D) $\dfrac{2}{3}$ E) ∞

12. $\lim\limits_{x\to\infty}\dfrac{4}{x}=?$

A) $-\infty$ B) -4 C) 0 D) 4 E) ∞

13. $\lim\limits_{x\to-\infty}\dfrac{5^x+4}{3}=?$

A) 3 B) $\dfrac{4}{3}$ C) $\dfrac{5}{3}$ D) $\dfrac{8}{3}$ E) $\dfrac{1}{3}$

14. $\lim\limits_{x\to 3}\dfrac{\sqrt{x-3}}{x+3}=?$

A) 0 B) 1 C) $\sqrt{3}$ D) 3 E) $+\infty$

15. $\lim\limits_{x\to 2^-}\dfrac{|x^2-5x+6|}{|x^2-4|}=?$

A) $-\dfrac{1}{4}$ B) $-\dfrac{1}{2}$ C) 1 D) $\dfrac{1}{2}$ E) $\dfrac{1}{4}$

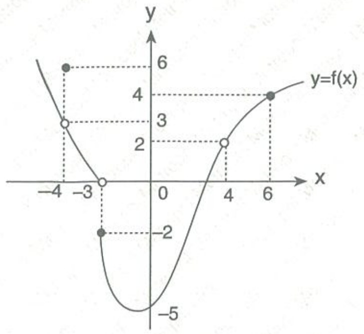
16. $\lim\limits_{x\to-1}\left[\dfrac{\ln(5x^2-4x-8)}{\sqrt[3]{x}+3}\right]=?$

A) 0 B) $-\dfrac{14}{3}$ C) $-\dfrac{3}{14}$ D) $\dfrac{3}{14}$ E) $\dfrac{14}{3}$

YANITLAR / ANSWERS

1	2	3	4	5	6	7	8
D	B	B	D	C	C	A	C
9	10	11	12	13	14	15	16
A	D	A	C	B	A	E	A

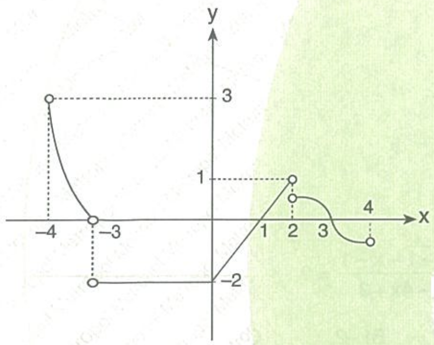
1.



$$\Rightarrow \lim_{x\to -3^+} f(x) + \lim_{x\to 6^-} f(x) + \lim_{x\to -4^+} f(x) = ?$$

A) 0 B) 1 C) 2 D) 3 E) 5

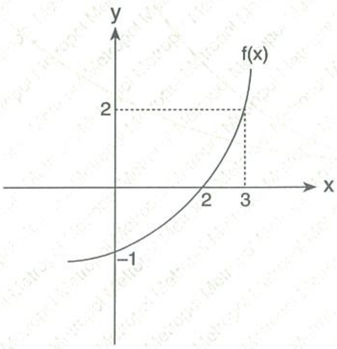
2.



$$\Rightarrow \lim_{x\to -4^+} f(x) + \lim_{x\to -3^-} f(x) + \lim_{x\to 2^-} f(x) - \lim_{x\to 0} f(x) = ?$$

A) 1 B) 2 C) 4 D) 5 E) 6

3. $\lim\limits_{x\to 3} \dfrac{(f\circ f)(x-1)}{x\cdot f(x)} = ?$



A) $-\frac{1}{6}$ B) $-\frac{1}{3}$ C) $\frac{1}{3}$ D) $\frac{1}{2}$ E) 1

4. $\lim\limits_{x\to 3} \dfrac{x^2+4x-21}{x^3+27} = ?$

A) $-\frac{1}{9}$ B) $-\frac{1}{3}$ C) $-\frac{7}{9}$ D) 0 E) $\frac{1}{3}$

5. $\lim\limits_{x\to \frac{1}{\sqrt{3}}} (\arctan x) = ?$

A) $-\frac{\pi}{6}$ B) $-\frac{\pi}{3}$ C) $\frac{\pi}{6}$ D) $\frac{\pi}{4}$ E) $\frac{\pi}{3}$

6. $\lim\limits_{x\to 0^-} \dfrac{|1-x|}{x-1}\cdot(2+x) = ?$

A) −2 B) −1 C) 0 D) 1 E) 2

7. $\lim\limits_{x\to \pi^+} \left(\dfrac{x}{2} - \dfrac{|\sin x|}{\sin x}\right) = ?$

A) $\frac{\pi-2}{2}$ B) $\frac{\pi-1}{2}$ C) $\frac{\pi}{6}$ D) $\frac{\pi+1}{2}$ E) $\frac{\pi+2}{2}$

8. $\lim\limits_{x\to 1^+} \dfrac{|1-x^2|}{x^2+x-2} = ?$

A) $\frac{1}{4}$ B) $\frac{2}{3}$ C) $\frac{4}{5}$ D) $\frac{3}{2}$ E) $\frac{5}{2}$

9. $\lim\limits_{x \to 3} \dfrac{x^2+2x-15}{3x^2-11x+6} = ?$

A) $\frac{2}{5}$ B) $\frac{7}{8}$ C) 1 D) $\frac{8}{7}$ E) $\frac{5}{2}$

10. $\lim\limits_{x \to 3} \dfrac{x^2-5x+6}{x^2-2x-3} = ?$

A) $\frac{1}{4}$ B) $\frac{1}{2}$ C) 1 D) $\frac{3}{2}$ E) $\frac{5}{2}$

11. $\lim\limits_{x \to 5} \left(\dfrac{3x-8}{x^2-3x-10} - \dfrac{1}{x-5} \right) = ?$

A) $\frac{1}{7}$ B) $\frac{2}{7}$ C) $\frac{3}{7}$ D) $\frac{4}{7}$ E) $\frac{5}{7}$

12. $\lim\limits_{x \to -3^+} \left[\dfrac{|x^2-9|}{x+3} - 3x \right] = ?$

A) – 6 B) –3 C) 3 D) 6 E) 15

13. $\lim\limits_{x \to -\infty} \left(5^x - 4^{\frac{1}{x}} + 3 \right) = ?$

A) 1 B) 2 C) 3 D) 4 E) 5

14. $f(x) = \begin{cases} \dfrac{2}{x} + \dfrac{x}{2} + a, & x > 2 \\ b+3, & x = 2 \\ 2x^2-3, & 2 < x \end{cases}$

$\lim\limits_{x \to 2} f(x) = f(2) \Rightarrow \mathbf{a - b = ?}$

A) 1 B) 2 C) 3 D) 4 E) 5

15. $\lim\limits_{y \to \frac{x}{2}} \dfrac{\tan(2y-x)+2}{\cos(4y-2x)} = ?$

A) x B) 2x C) 1 D) 2 E) 4

16. $\lim\limits_{x \to 5} \dfrac{\sqrt{7x+1} - x - 1}{x^2-4x+3} = ?$

A) –4 B) –2 C) 0 D) $\frac{1}{8}$ E) $\frac{1}{3}$

17. $\lim\limits_{x \to 3^+} \dfrac{(2x-1) \cdot \ln(x-3)^4}{\ln(x-3)^5 - \ln(x-3)^3} = ?$

A) 5 B) 8 C) 10 D) 12 E) 15

YANITLAR / ANSWERS

1	2	3	4	5	6	7	8	9
E	E	A	D	C	A	E	B	D
10	11	12	13	14	15	16	17	
A	B	E	B	A	D	D	C	

1. $\left.\begin{array}{l} f(x)=\sqrt{x+11} \\ g(x)=x^2-9 \end{array}\right\} \Rightarrow \lim\limits_{x \to 4}(gof)(x)=?$

A) $\sqrt{6}$ B) $2\sqrt{3}$ C) 6 D) 7 E) 8

2. $\lim\limits_{x \to \infty} 5^{1-x} = ?$

A) $-\infty$ B) -1 C) 0 D) 1 E) $+\infty$

3. $\lim\limits_{x \to -\infty} \left(11^{\frac{1}{x}} + 11^x + 11\right) = ?$

A) 11 B) 12 C) 13 D) 14 E) ∞

4. $\lim\limits_{x \to -\infty} \dfrac{2^{x-1}+3}{4+\dfrac{1}{x}} = ?$

A) $-\infty$ B) $\frac{3}{4}$ C) 1 D) $\frac{4}{3}$ E) $+\infty$

5. $\lim\limits_{x \to -\infty} \left(9^{\frac{1}{x}} + 4^x + 2\right) = ?$

A) 0 B) 1 C) 2 D) 3 E) 4

6. $\lim\limits_{x \to 4^-} \dfrac{|x-4|}{x^2-2x-8} = ?$

A) $-\frac{1}{8}$ B) $-\frac{1}{6}$ C) $-\frac{1}{4}$ D) $-\frac{1}{3}$ E) $-\frac{1}{2}$

7. $\lim\limits_{x \to 0^-} \left(e^{\frac{1}{x}} + \pi^x + 1\right) = ?$

A) 1 B) 2 C) 3
D) $e+1$ E) $\pi+1$

8. $\lim\limits_{x \to \infty} \sqrt[x]{5} + \dfrac{\sin x}{x} = ?$

A) 0 B) 1 C) 2 D) $\sqrt{5}$ E) $\sqrt{6}$

9. $\lim\limits_{x \to 3^-} \dfrac{\left|x^2-4x+3\right|}{x-3} = ?$

A) –3 B) –2 C) –1 D) 0 E) 1

10. $\lim\limits_{x \to -3} \dfrac{3x^3+81}{2x^2+4x-6} = ?$

A) $-\dfrac{81}{8}$ B) $-\dfrac{27}{4}$ C) $-\dfrac{9}{2}$ D) $-\dfrac{3}{2}$ E) – 1

11. $\lim\limits_{x \to \frac{\pi}{2}} \dfrac{3-3\sin x}{\sin 3x} = ?$

A) –3 B) –2 C) –1 D) 0 E) $\dfrac{1}{3}$

12. $\lim\limits_{x \to \pi} \dfrac{\tan x-\sin x}{2x} = ?$

A) –2 B) –1 C) 0 D) 1 E) 2

13. $\lim\limits_{x \to 2} \dfrac{\sin\left(\sqrt{2x}-2\right)}{\tan\left(\dfrac{\pi}{3x}\right)+\sqrt{3}} = ?$

A) 0 B) $\dfrac{3}{4\pi}$ C) $\dfrac{3}{2\pi}$ D) $\dfrac{2\pi}{3}$ E) $\dfrac{4\pi}{3}$

14. $f(x) = \begin{cases} x+m, & x \le 2 \\ \dfrac{ax-6}{x-2}, & x > 2 \end{cases}$

$\lim\limits_{x \to 2} f(x) = c,\ c \in \mathbb{Z} \Rightarrow$ **$m = ?$**

A) 1 B) 2 C) 3 D) 4 E) 5

15. $\lim\limits_{x \to 2^+} \left(\dfrac{|2-x|}{x-2}+3x\right) = ?$

A) 4 B) 5 C) 6 D) 7 E) 8

16. $f(x) = \begin{cases} ax-4, & x < 2 \\ x^2+2, & x = 2 \\ bx+a, & x > 2 \end{cases}$

$\lim\limits_{x \to 2} f(x) = f(2) \Rightarrow$ **$a+b = ?$**

A) 4 B) $\dfrac{9}{2}$ C) 5 D) $\dfrac{11}{2}$ E) 6

YANITLAR / ANSWERS

1	2	3	4	5	6	7	8
C	C	B	B	D	B	B	B
9	**10**	**11**	**12**	**13**	**14**	**15**	**16**
B	A	D	C	A	A	D	D

BÖLÜM
CHAPTER
27

TARİHSEL NOT / HISTORICAL NOTE

ÖMER HAYYAM [1048 – 1131]

İranlı şair, filozof, matematikçi ve astronom. Hayyam, üçüncü dereceden bilinmeyen denklemlerle ilgili yazdığı bir eserinde bilinmeyen rakamın yerine Arapça'da "şey" anlamına gelen kelimeyi kullanmıştır. Daha sonra bu eseri diğer dillere çevrilirken İspanyolcaya "Xay" olarak geçmiştir. Daha sonra bu kelime ilk harfine indirgenerek bilinmeyen rakamın simgesi "x" olarak kullanılmaya başlamıştır. Binom açılımını ilk kullanan bilim insanıdır.

Iranian poet, philosopher, mathematician and astronomer. Khayyam, in a book he wrote about the equations of the third degree unknowns, used the word "thing" in Arabic instead of the unknown figure. Later this work was translated into other languages and passed to Spanish as "Xay". Then this word is reduced to the first letter of the unknown number "x" as the symbol began to be used. He is the first scientist to use binomial expansion.

LİMİTTE BELİRSİZLİKLER ve FONKSİYONLARIN SÜREKLİLİĞİ
INDETERMINATE FORMS and CONTINUITY of FUNCTIONS

Bu bölüm, 231 test sorusu içermektedir.
This chapter includes 231 test questions.

BÖLÜM / CHAPTER 27

LİMİTTE BELİRSİZLİKLER ve FONKSİYONLARIN SÜREKLİLİĞİ / INDETERMINATE FORMS and CONTINUITY of FUNCTIONS

- Limitte Belirsizlikler ve Fonksiyonların Sürekliliği / Indeterminate Forms and Continuity of Functions .... 499 – 532

BÖLÜM

27

CHAPTER

LİMİTTE BELİRSİZLİKLER
INDETERMINATE FORMS

LİMİTTE BELİRSİZLİKLER / INDETERMINATE FORMS

▼ $\lim_{x \to a} f(x)$ limitinin sonucu $\frac{0}{0}$, $\frac{\infty}{\infty}$, $\infty - \infty$, $0 \cdot \infty$, 1^{∞}, 0^0, ∞^0 olabilir. Bu sonuçların her birine ***belirsizlik*** denir.

▼ Result of $\lim_{x \to a} f(x)$ can be $\frac{0}{0}$, $\frac{\infty}{\infty}$, $\infty - \infty$, $0 \cdot \infty$, 1^{∞}, 0^0, ∞^0. Each of these results is called an ***indeterminate form***.

$\frac{0}{0}$ BELİRSİZLİĞİ / INDETERMINATE FORM of $\frac{0}{0}$

▼ $\lim_{x \to a} \frac{f(x)}{g(x)}$ limitinin sonucu $\frac{0}{0}$ belirsizliği oluyorsa, f(x) ve g(x) fonksiyonlarında (x – a) çarpanı vardır.

▼ To find $\lim_{x \to a} \frac{f(x)}{g(x)}$, when you apply direct substitution and obtain $\frac{0}{0}$ you can conclude that (x – a) must be a common factor to both f(x) and g(x).

▼ $\frac{0}{0}$ belirsizlikleri çözülürken,

▼ While we solve indeterminate form $\frac{0}{0}$,

$f(x) = (x - a) \cdot f_1(x)$, $g(x) = (x - a) \cdot g_1(x)$

$\lim_{x \to a} \frac{f(x)}{g(x)} = \lim_{x \to a} \frac{(x-a) \cdot f_1(x)}{(x-a) \cdot g_1(x)} = \lim_{x \to a} \frac{f_1(x)}{g_1(x)} = \lim_{x \to a} \frac{f_1(a)}{g_1(a)}$ yolu izlenir (is written).

▼ Eğer belirsizlik kalkmıyorsa bu işleme devam edilir.

▼ If indeterminate condition continues, apply the similar process one more time.

Özellik / Property

★ $\lim\limits_{x \to 0} \dfrac{\sin ax}{bx} = \dfrac{a}{b}, \quad \lim\limits_{x \to 0} \dfrac{\tan ax}{bx} = \dfrac{a}{b}$

★ $\lim\limits_{x \to 0} \dfrac{\sin ax}{\sin bx} = \dfrac{a}{b}, \quad \lim\limits_{x \to 0} \dfrac{\tan ax}{\tan bx} = \dfrac{a}{b}$

★ $\lim\limits_{x \to 0} \dfrac{\sin ax}{\tan bx} = \dfrac{a}{b}, \quad \lim\limits_{x \to 0} \dfrac{\tan ax}{\sin bx} = \dfrac{a}{b}$

$\frac{\infty}{\infty}$ BELİRSIZLİĞİ / INDETERMINATE FORM of $\frac{\infty}{\infty}$

▼ $\lim\limits_{x \to \infty} \dfrac{f(x)}{g(x)} = \dfrac{\lim\limits_{x \to \infty} f(x)}{\lim\limits_{x \to \infty} g(x)} = \left(\dfrac{\infty}{\infty}\right)$ olsun.

Bu durumda, f(x) ve g(x) fonksiyonlarındaki en yüksek dereceli terim parantezine alınır ve sonra sadeleştirilir.

In this case, factor out the terms with highest degree in f(x) and g(x) and then simplify them.

▼ m, n ∈ N için,

▼ For m, n ∈ N ,

$$\lim_{x \to \pm\infty} \frac{a_n \cdot x^n + a_{n-1} \cdot x^{n-1} + \ldots + a_0}{b_m \cdot x^m + b_{m-1} \cdot x^{m-1} + \ldots + b_0} = \begin{cases} 0, & n < m \\ \dfrac{a_n}{b_m}, & n = m \\ \pm\infty, & n > m \end{cases}$$

$\infty - \infty$ BELİRSİZLİĞİ / INDETERMINATE FORM of $\infty - \infty$

▼ $\lim\limits_{x \to a}(f(x) - g(x))$ limitinin sonucu $\infty - \infty$ belirsizliği oluyorsa, $\frac{0}{0}$ ya da $\frac{\infty}{\infty}$ belirsizliklerine dönüştürülerek çözülür.

▼ If the result of the limit $\lim\limits_{x \to a}(f(x) - g(x))$ is in the indeterminate form of $\infty - \infty$, it can be solved by converting it to the indeterminate form of $\frac{0}{0}$ or $\frac{\infty}{\infty}$

Özellik / Property

★ $a > 0$,

$$\lim_{x\to\pm\infty}\sqrt{ax^2+bx+c}=\sqrt{a}\cdot\lim_{x\to\pm\infty}\left|x+\frac{b}{2a}\right|$$

0·∞ BELİRSİZLİĞİ / INDETERMINATE FORM of 0·∞

▼ $\lim_{x\to a} f(x)\cdot g(x) = 0\cdot\infty$ belirsizliği, $\frac{0}{0}$ ya da $\frac{\infty}{\infty}$ belirsizliklerine dönüştürülerek çözülür.

▼ The indeterminate form of $\lim_{x\to a} f(x)\cdot g(x) = 0\cdot\infty$ is written in the form of $\frac{0}{0}$ or $\frac{\infty}{\infty}$, and then calculated.

▼ $\lim_{x\to a} f(x)\cdot g(x) = 0\cdot\infty \Rightarrow$

$$\lim_{x\to a} f(x)\cdot g(x) = \lim_{x\to a}\frac{f(x)}{\frac{1}{g(x)}}=\left(\frac{0}{0}\right) \text{ veya/or } \lim_{x\to a} f(x)\cdot g(x) = \lim_{x\to a}\frac{g(x)}{\frac{1}{f(x)}}=\left(\frac{\infty}{\infty}\right)$$

1^{∞} BELİRSİZLİĞİ / INDETERMINATE FORM of 1^{∞}

▼

$$\left.\begin{array}{l}\lim_{x\to\infty} f(x) = 0\\ \lim_{x\to\infty} g(x) = \infty\\ \lim_{x\to\infty} (f(x)\cdot g(x)) = k\end{array}\right\} \Rightarrow \lim_{x\to\infty}\left[1+f(x)\right]^{g(x)} = e^{k}$$

0^0 ve ∞^0 BELİRSİZLİKLERİ / INDETERMINATE FORMS of 0^0 and ∞^0

▼ $y = (f(x))^{g(x)}$ olmak üzere,

$\lim\limits_{x \to a} (f(x))^{g(x)}$ limiti hesaplandığında 0^0 veya ∞^0 belirsizliklerinin oluşması durumunda her iki tarafında doğal logaritması alınarak limit hesaplanır.v

▼ $y = (f(x))^{g(x)}$,

If we calculate $\lim\limits_{x \to a} (f(x))^{g(x)}$ and we obtain 0^0 or ∞^0 , we can take logarithms of both sides and then calculate the limit.

▼ $\lim\limits_{x \to a} (f(x))^{g(x)} = (0^0) \Rightarrow y = (f(x))^{g(x)} \Rightarrow \ln y = \ln(f(x))^{g(x)}$

$$= g(x) \cdot \ln(f(x))$$

$$\Rightarrow \lim_{x \to a} [\ln y] = \lim_{x \to a} [g(x) \cdot \ln f(x)] = 0 \cdot \infty$$

▼ $\lim\limits_{x \to a} (f(x))^{g(x)} = (\infty^0) \Rightarrow y = (f(x))^{g(x)} \Rightarrow \ln y = \ln(f(x))^{g(x)}$

$$= g(x) \cdot \ln(f(x))$$

$$\Rightarrow \lim_{x \to a} [\ln y] = \lim_{x \to a} [g(x) \cdot \ln(f(x))] = 0 \cdot \infty$$

FONKSİYONLARIN SÜREKLİLİĞİ / CONTINUITY of FUNCTIONS

▼ $f: A \to R, \; a \in A$

$f(x)$, $x = a$ da süreklidir. / is continuous. $\Leftrightarrow \lim\limits_{x \to a^+} f(x) = \lim\limits_{x \to a^-} f(x) = f(a)$

1. $\lim\limits_{x \to 3} \dfrac{x^3-27}{x-3} = ?$

A) 0 B) 3 C) 6 D) 18 E) 27

2. $\lim\limits_{x \to 2} \dfrac{x^3-8}{2x^2-8} = ?$

A) $\frac{3}{2}$ B) 2 C) 3 D) $\frac{5}{2}$ E) 4

3. $\lim\limits_{x \to 3} \dfrac{x^2-x-6}{x^2-9} = ?$

A) $\frac{1}{2}$ B) $\frac{1}{3}$ C) $\frac{5}{6}$ D) $\frac{1}{9}$ E) $\frac{5}{9}$

4. $\lim\limits_{x \to 2} \sqrt{\dfrac{x^3-8}{x^2-4}} = ?$

A) $\frac{\sqrt{3}}{2}$ B) $\frac{\sqrt{6}}{2}$ C) $\sqrt{3}$ D) $2\sqrt{3}$ E) 3

5. $\lim\limits_{x \to \frac{\pi}{2}} \dfrac{1-\sin^2 x}{\cos x} = ?$

A) –2 B) –1 C) $-\frac{1}{2}$ D) 0 E) $\frac{1}{2}$

6. $\lim\limits_{x \to 0} \dfrac{x^2+4x}{\sin 3x} = ?$

A) $\frac{4}{3}$ B) $\frac{5}{3}$ C) 2 D) $\frac{5}{2}$ E) 3

7. $\lim\limits_{a \to 16} \dfrac{a-16}{\sqrt{a}-4} = ?$

A) 1 B) 2 C) 4 D) 6 E) 8

8. $\lim\limits_{x \to 0} \dfrac{\sqrt{x+16}-4}{\sqrt{x+9}-3} = ?$

A) $\frac{1}{2}$ B) 1 C) $\frac{3}{2}$ D) $\frac{4}{3}$ E) $\frac{3}{4}$

9. $\lim\limits_{x \to 0} \dfrac{\sqrt{x+4}-2}{\sin 3x} = ?$

A) $\frac{3}{4}$ B) $\frac{5}{4}$ C) $\frac{1}{2}$ D) $\frac{1}{4}$ E) $\frac{1}{12}$

10. $\lim\limits_{x\to 4} \dfrac{\sqrt{x}-2}{\sqrt[4]{x}-\sqrt{2}} = ?$

A) $\sqrt[4]{2}$ B) $\sqrt{2}$ C) $2\sqrt{2}$ D) 2 E) 4

11. $\lim\limits_{x\to 0} \dfrac{e^x-1+\sin x}{\ln(x+1)} = ?$

A) 1 B) 2 C) 3 D) 4 E) 5

12. $\lim\limits_{x\to -2} \dfrac{-x^3-4x^2-5x-2}{x^3-5x^2-9x+10} = ?$

A) $-\frac{1}{23}$ B) $-\frac{1}{12}$ C) $-\frac{1}{6}$ D) $\frac{1}{6}$ E) $\frac{1}{12}$

13. $\lim\limits_{x\to 8} \dfrac{\sqrt[3]{x}-2}{x-8} = ?$

A) $\frac{1}{2}$ B) $\frac{1}{4}$ C) $\frac{1}{8}$ D) $\frac{1}{12}$ E) $\frac{1}{16}$

14. $\lim\limits_{x\to 1} \dfrac{\sqrt{x}-1}{\sqrt[3]{x}-1} = ?$

A) $\frac{1}{2}$ B) $\frac{2}{3}$ C) $\frac{3}{4}$ D) $\frac{3}{2}$ E) $\frac{4}{3}$

15. $\lim\limits_{x\to 2} \dfrac{\sqrt[3]{x+6}-2}{x-2} = ?$

A) $-\frac{1}{18}$ B) $-\frac{1}{12}$ C) $-\frac{1}{4}$ D) $\frac{1}{12}$ E) $\frac{1}{18}$

16. $\lim\limits_{x\to 0} \dfrac{\sqrt[3]{x+27}-3}{\sqrt[4]{x+16}-2} = ?$

A) $-\frac{15}{17}$ B) $-\frac{27}{32}$ C) $-\frac{32}{27}$ D) $\frac{32}{27}$ E) $\frac{35}{32}$

17. $\lim\limits_{x\to \frac{\pi}{4}} \dfrac{2\cdot\cos x-\sqrt{2}}{\sin 2x-\tan x} = ?$

A) 0 B) $\frac{1}{2}$ C) $\sqrt{2}$ D) $\frac{\sqrt{2}}{2}$ E) $\frac{\sqrt{2}}{4}$

18. $\lim\limits_{x\to 0} \left(\dfrac{\sin 4x+\tan 6x}{5x}\right) = ?$

A) 1 B) 2 C) $\frac{9}{5}$ D) $\frac{12}{5}$ E) 3

YANITLAR / ANSWERS

1	2	3	4	5	6	7	8	9
E	A	C	C	D	A	E	E	E
10	11	12	13	14	15	16	17	18
C	B	A	D	D	D	D	D	B

1. $\lim\limits_{x \to 0}\left(\dfrac{\sin x}{2x} + \dfrac{3x}{\tan 4x}\right) = ?$

A) $\frac{1}{4}$ B) $\frac{1}{2}$ C) $\frac{3}{4}$ D) 1 E) $\frac{5}{4}$

2. $\lim\limits_{x \to 3} \dfrac{\tan(x-3)}{3x-9} = ?$

A) $\frac{1}{3}$ B) $\frac{2}{3}$ C) 1 D) $\frac{3}{2}$ E) $\frac{5}{2}$

3. $\lim\limits_{x \to 1} \dfrac{\sin^3(x-1)}{(2x-2)^3} = ?$

A) $\frac{1}{2}$ B) $\frac{1}{4}$ C) $\frac{1}{8}$ D) $\frac{1}{16}$ E) $\frac{5}{8}$

4. $\lim\limits_{x \to 0} \dfrac{\tan x^3}{2x^3} = ?$

A) 2 B) 1 C) $\frac{1}{2}$ D) $\frac{1}{4}$ E) $\frac{1}{8}$

5. $\lim\limits_{x \to 2} \dfrac{\sin^3(x-2)}{\tan(2x-4)^3} = ?$

A) 1 B) $\frac{1}{2}$ C) $\frac{1}{4}$ D) $\frac{1}{6}$ E) $\frac{1}{8}$

6. $\lim\limits_{x \to \frac{\pi}{3}} \dfrac{\sin\left(x-\frac{\pi}{3}\right)}{3x-\pi} = ?$

A) $-\frac{\sqrt{3}}{2}$ B) $-\frac{1}{2}$ C) $\frac{1}{2}$ D) $\frac{\sqrt{3}}{2}$ E) $\frac{1}{3}$

7. $\lim\limits_{x \to 2} \dfrac{2x^2-8}{m-x^2} = -2 \Rightarrow$ **m = ?**

A) 1 B) 2 C) 4 D) 9 E) 16

8. $\lim\limits_{x \to 2} \dfrac{x^2+mx-6}{x-2} = a,\ a \in R \Rightarrow$ **m = ?**

A) –2 B) –1 C) 0 D) 1 E) 2

9. $\lim\limits_{x \to -2} \dfrac{x^3-ax+2}{x+2} = 9 \Rightarrow$ **a = ?**

A) –4 B) –3 C) 1 D) 2 E) 3

10. $\lim\limits_{x \to 2} \dfrac{a^{x-2} - 3^{x-2}}{4^{x-2} - b^{x-2}} = 1 \Rightarrow \mathbf{a \cdot b = ?}$

A) 14 B) 12 C) 10 D) 8 E) 6

11. $\lim\limits_{x \to m} \dfrac{x^2 - (m+1)x + m}{x^3 - m^3} = ?$

A) $\dfrac{m+1}{m}$ B) $\dfrac{m-1}{m}$ C) $\dfrac{m+1}{3m-1}$ D) $\dfrac{m-1}{3m^2}$ E) $\dfrac{m+1}{m^2}$

12. $\lim\limits_{x \to \frac{\pi}{4}} \dfrac{\sin^2 x - \frac{1}{2}}{\sin 4x} = ?$

A) $-\dfrac{1}{8}$ B) $-\dfrac{1}{4}$ C) $-\dfrac{1}{2}$ D) $\dfrac{1}{2}$ E) $\dfrac{1}{4}$

13. $\lim\limits_{x \to 0} \dfrac{4 - 4\cos 2x}{x^2} = ?$

A) –8 B) –4 C) 0 D) 4 E) 8

14. $\lim\limits_{x \to 0} \dfrac{5x - 2\sin x}{x + 4\sin 2x} = ?$

A) $\dfrac{1}{3}$ B) $\dfrac{2}{3}$ C) 1 D) $\dfrac{3}{2}$ E) 3

15. $\lim\limits_{x \to 0} \left[\dfrac{\sin 4x - \tan 4x}{x^3}\right] = ?$

A) –64 B) –48 C) –32 D) 32 E) 48

16. $f(x) = \dfrac{(2m-3)x - 8}{\sqrt{x+5} - 1}$,

$\lim\limits_{x \to -4} f(x) = a,\ a \in R \Rightarrow \mathbf{m = ?}$

A) $-\dfrac{1}{2}$ B) $-\dfrac{1}{4}$ C) 1 D) $\dfrac{1}{2}$ E) $\dfrac{1}{4}$

17. $\lim\limits_{h \to 2} \dfrac{\cos\left(\frac{\pi}{h}\right)}{h - 2} = ?$

A) $-\dfrac{\pi}{4}$ B) $\dfrac{\pi}{4}$ C) $\dfrac{-1}{4}$ D) $\dfrac{1}{4}$ E) $-\dfrac{\pi}{2}$

18. $\lim\limits_{x \to 4} \dfrac{x^2 - 16}{4 - x} = ?$

A) –8 B) –4 C) 0 D) 4 E) 8

YANITLAR / ANSWERS

1	2	3	4	5	6	7	8	9
E	A	C	C	E	E	C	D	E
10	11	12	13	14	15	16	17	18
B	D	B	E	A	C	D	B	A

1. $\lim\limits_{x \to 1} \dfrac{\sqrt{x}-1}{x^2-1} = ?$

A) $-\frac{1}{4}$ B) $-\frac{14}{3}$ C) $-\frac{3}{14}$ D) $\frac{3}{14}$ E) $\frac{1}{4}$

2. $\lim\limits_{x \to 1} \left(\dfrac{\sqrt{x}-x}{x-1}\right) = ?$

A) $-\frac{1}{4}$ B) $-\frac{1}{2}$ C) 0 D) $\frac{1}{2}$ E) $\frac{1}{4}$

3. $\lim\limits_{x \to 0} \dfrac{\sin 5x}{x^2-3x} = ?$

A) $-\frac{8}{3}$ B) $-\frac{5}{3}$ C) $-\frac{1}{3}$ D) $\frac{1}{3}$ E) $\frac{5}{3}$

4. $\lim\limits_{x \to \infty} \dfrac{\sin\frac{1}{x} \cdot \cos\frac{1}{x}}{\frac{2}{x}} = ?$

A) $\frac{1}{4}$ B) $\frac{1}{2}$ C) 1 D) 2 E) 4

5. $m \in R$,

$\lim\limits_{x \to 2} \left(\dfrac{x^3+ax-16}{x^2+2x-8}\right) = m \Rightarrow m = ?$

A) $\frac{4}{3}$ B) $\frac{3}{2}$ C) $\frac{8}{3}$ D) 2 E) 3

6. $f(x) = \dfrac{x^2-4}{x^3-8} \Rightarrow \lim\limits_{h \to 0} f(2+h) = ?$

A) 1 B) $\frac{3}{4}$ C) $\frac{1}{2}$ D) $\frac{7}{16}$ E) $\frac{1}{3}$

7. $f(x) = \dfrac{\sqrt[3]{x}-\sqrt[3]{2}}{x-2} \Rightarrow \lim\limits_{x \to 2} f(x) = ?$

A) $2\sqrt[3]{2}$ B) $\frac{\sqrt[3]{2}}{6}$ C) $\frac{1}{3}$ D) $\frac{\sqrt[3]{4}}{3}$ E) $\frac{1}{6}$

8. $\lim\limits_{x \to 16} \dfrac{x^{\frac{3}{2}}-16x^{\frac{1}{2}}}{x-16} = ?$

A) 4 B) 3 C) 2 D) 0 E) -2

9. $\lim\limits_{x \to 2} \dfrac{e^{\ln x}-2}{x^2-4} = ?$

A) 2 B) 1 C) $\frac{1}{2}$ D) $\frac{1}{4}$ E) $\frac{1}{8}$

10. $\lim\limits_{x\to 2} \dfrac{5^{\log_5 x^3}-8}{\sqrt{2x}-2} = ?$

A) 4 B) 5 C) 6 D) 12 E) 24

11. $\lim\limits_{x\to 0}\left(\dfrac{\sin 4x}{x^2\cdot\cot 2x}\right) = ?$

A) 2 B) 4 C) 5 D) 6 E) 8

12. $\lim\limits_{x\to \frac{\pi}{6}} \dfrac{2\cos x-\cot x}{\sin 2x-\cos x} = ?$

A) $\frac{9}{2}$ B) 2 C) $\frac{3}{2}$ D) 1 E) $\frac{2}{3}$

13. $\lim\limits_{x\to 0} \dfrac{16x\cdot\sin 4x}{1-\cos 8x} = ?$

A) 0 B) 1 C) 2 D) 4 E) 8

14. $\lim\limits_{h\to 0} \dfrac{\cos(a+h)-\cos a}{h} = ?$

A) 0 B) 1 C) sina D) cosa E) – sina

15. $\lim\limits_{x\to 1} \dfrac{x\cdot\sin\frac{3\pi}{2}x+1}{x-1} = ?$

A) 1 B) $\frac{1}{2}$ C) 0 D) $-\frac{1}{2}$ E) –1

16. $\lim\limits_{x\to 0}\left[\dfrac{\sin 3x+4x}{\tan x+3x}\right] = ?$

A) $\frac{7}{4}$ B) 2 C) $\frac{5}{2}$ D) 3 E) $\frac{10}{3}$

17. $\lim\limits_{x\to 0} \dfrac{\sin 4x\cdot\tan 8x}{16x^2} = ?$

A) 2 B) 4 C) 8 D) 16 E) 32

18. $\lim\limits_{x\to 1} \dfrac{x^3-1}{\sqrt{x}-1} = ?$

A) 2 B) 3 C) 4 D) 5 E) 6

YANITLAR / ANSWERS

1	2	3	4	5	6	7	8	9
E	B	B	B	C	E	B	A	D
10	11	12	13	14	15	16	17	18
E	E	B	C	E	E	A	A	E

1. $\lim\limits_{x\to 1}\dfrac{x^2-3ax+5}{x-1}=-4 \Rightarrow$ **a = ?**

A) –2 B) –1 C) 0 D) 1 E) 2

2. $a\in\mathbb{R}$,

$\lim\limits_{x\to 2}\dfrac{4x^2-2x+m-2}{x^2-4}=a \Rightarrow$ **a = ?**

A) $\frac{5}{2}$ B) 3 C) $\frac{7}{2}$ D) 4 E) $\frac{9}{2}$

3. $\lim\limits_{x\to -1}\dfrac{x^3+x^2-2x-2}{x^3+x^2+2x+2}=$ **?**

A) –1 B) $-\frac{1}{2}$ C) $-\frac{1}{3}$ D) 0 E) $\frac{1}{3}$

4. $\lim\limits_{x\to 2}\dfrac{x^3-3x^2+a}{x^3-5x^2+8x-4}=3 \Rightarrow$ **a = ?**

A) 8 B) 6 C) 4 D) 3 E) 2

5. $\lim\limits_{x\to 6}\dfrac{\sqrt{5x+6}-6}{\sqrt{x+3}-3}=$ **?**

A) $\frac{5}{2}$ B) 3 C) $\frac{7}{2}$ D) 4 E) $\frac{9}{2}$

6. $\lim\limits_{x\to 0}\dfrac{\sqrt[3]{x+1}-1}{\sqrt{x+1}-1}=$ **?**

A) 2 B) $\frac{3}{2}$ C) 1 D) $\frac{2}{3}$ E) $\frac{3}{4}$

7. $\lim\limits_{x\to 1}\dfrac{x^5-x}{x^2+7x-8}=$ **?**

A) $-\frac{2}{3}$ B) $-\frac{5}{6}$ C) $\frac{4}{3}$ D) $\frac{2}{3}$ E) $\frac{4}{9}$

8. $\lim\limits_{x\to 8}\dfrac{x-8}{\sqrt[3]{x-16}+2}=$ **?**

A) $\frac{5}{6}$ B) $\frac{4}{3}$ C) 6 D) 12 E) 18

9. $\lim\limits_{x\to 125}\dfrac{\sqrt[3]{x}-5}{\sqrt{x}-5\sqrt{5}}=$ **?**

A) $\frac{1}{5}$ B) $\frac{\sqrt{5}}{5}$ C) $\frac{1}{2}$ D) $\frac{2\sqrt{5}}{15}$ E) $\frac{1}{4}$

10. $\lim\limits_{x\to 2}\left(\dfrac{4-2^x}{4^x-16}\right)=?$

A) $-\frac{1}{16}$ B) $-\frac{1}{12}$ C) $-\frac{1}{8}$ D) $-\frac{1}{6}$ E) $-\frac{1}{4}$

11. $\lim\limits_{x\to 0}\dfrac{e^{\frac{x}{2}}-1}{e^x-1}=?$

A) 2 B) 1 C) $\frac{1}{2}$ D) $\frac{1}{4}$ E) $\frac{1}{8}$

12. $\lim\limits_{x\to 2}\dfrac{27^{\frac{x}{2}}-27}{3^x-9}=?$

A) $\frac{9}{2}$ B) $\frac{7}{2}$ C) $\frac{3}{2}$ D) 3 E) 9

13. $\lim\limits_{x\to 1}\dfrac{1-\sqrt{3x-2}}{x^2-1}=?$

A) $-\frac{1}{6}$ B) $-\frac{1}{3}$ C) $-\frac{3}{4}$ D) $-\frac{4}{3}$ E) -3

14. $\lim\limits_{x\to 0^+}\dfrac{\sqrt{x}}{\sqrt{4+\sqrt{x}}-2}=?$

A) 0 B) 2 C) 3 D) 4 E) 5

15. $\lim\limits_{x\to 0^+}\dfrac{\sqrt{1-\cos x}}{\sqrt{2}\cdot\sin x}=?$

A) $\frac{\sqrt{2}}{4}$ B) $\frac{\sqrt{2}}{2}$ C) $\frac{1}{2}$ D) 1 E) $\sqrt{2}$

16. $\lim\limits_{x\to 0}\dfrac{1-\cos x}{x^2\cdot\cos x}=?$

A) $\frac{1}{2}$ B) $\frac{1}{3}$ C) 0 D) $-\frac{1}{3}$ E) $-\frac{1}{2}$

17. $\lim\limits_{x\to y}\dfrac{x^2-y^2}{\sin 2x-\sin 2y}=?$

A) siny B) cosy C) sin2y

D) y · sec2y E) y · cosec2y

18. $\lim\limits_{y\to x}\dfrac{e^{\sin(x-y)}-1}{x-y}=?$

A) $\frac{1}{x}$ B) x C) 0 D) 1 E) 2

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1	2	3	4	5	6	7	8	9
E	C	C	C	A	D	E	D	D
10	11	12	13	14	15	16	17	18
C	C	A	C	D	C	A	D	D

1. $\lim\limits_{x \to \frac{\pi}{4}} \dfrac{\cos 2x}{\tan x - 1} = ?$

A) 4 B) 3 C) –1 D) $-\frac{1}{2}$ E) $-\frac{3}{4}$

2. $\lim\limits_{y \to \frac{\pi}{2}} \dfrac{\cos y}{\frac{\pi}{2} - y} = ?$

A) $\frac{2}{\pi}$ B) $\frac{1}{2}$ C) $\frac{3}{4}$ D) 1 E) $\frac{\pi}{2}$

3. $\lim\limits_{x \to 0} \dfrac{x \cdot \sin x}{1 - \cos x} = ?$

A) $\frac{1}{2}$ B) $\frac{3}{4}$ C) $\frac{3}{2}$ D) 2 E) $\frac{5}{2}$

4. $\lim\limits_{\theta \to 0} \dfrac{1 - \cos\theta}{\theta^2} = ?$

A) 1 B) $\frac{1}{2}$ C) 0 D) $-\frac{1}{2}$ E) $-\frac{1}{4}$

5. $\lim\limits_{h \to 0} \dfrac{h^2 + 6h}{\sin(2h)} = ?$

A) 1 B) $\frac{3}{2}$ C) 2 D) $\frac{5}{2}$ E) 3

6. $\lim\limits_{x \to 0} \dfrac{x \cdot \sin 2x}{1 - \cos 2x} = ?$

A) 2 B) $\frac{3}{2}$ C) 1 D) $\frac{1}{2}$ E) $\frac{1}{4}$

7. $\lim\limits_{x \to 0} \dfrac{\sin 2x}{\sqrt{1 - \cos x}} = ?$

A) 4 B) 3 C) 2 D) $\sqrt{2}$ E) $2\sqrt{2}$

8. $\lim\limits_{x \to a} \dfrac{\sin x - \sin a}{x - a} = ?$

A) cosa B) sina C) seca D) 1 E) –1

9. $\lim\limits_{x\to 0}\dfrac{1-\cos x}{1-\cos\dfrac{x}{2}}=?$

A) $\frac{7}{2}$ B) 4 C) $\frac{9}{2}$ D) $\frac{11}{2}$ E) 6

10. $\lim\limits_{x\to \frac{\pi}{2}}\dfrac{\sin 3x+\sin x}{\cos 4x-\cos 2x}=?$

A) –2 B) –1 C) 0 D) 1 E) 2

11. $\lim\limits_{x\to 0}\dfrac{\sin^2 3x}{1-\cos 3x}=?$

A) –1 B) –2 C) 2 D) 3 E) $\frac{7}{2}$

12. $\lim\limits_{x\to 0}\dfrac{\sin 2x^2}{6x^2+3x}=?$

A) $\frac{1}{2}$ B) $\frac{1}{3}$ C) $\frac{1}{4}$ D) $\frac{1}{6}$ E) 0

13. $\lim\limits_{x\to \frac{\pi}{4}}\dfrac{1-\sqrt{2}\sin x}{\cos 2x}=?$

A) –1 B) $-\frac{1}{2}$ C) $\frac{1}{2}$ D) 1 E) $\frac{3}{2}$

14. $\lim\limits_{x\to \frac{\pi}{4}}\dfrac{\cos 2x}{\dfrac{\pi}{4}-x}=?$

A) $\frac{1}{3}$ B) $\frac{1}{2}$ C) 1 D) $\frac{3}{2}$ E) 2

15. $\lim\limits_{x\to \frac{\pi}{2}}\dfrac{1-\sin x}{\sin\left(x-\dfrac{\pi}{2}\right)}=?$

A) –1 B) 0 C) $\frac{3}{2}$ D) 2 E) $\frac{5}{2}$

16. $a, b \in \mathbb{R}$,

$\lim\limits_{x\to 3}\dfrac{\sqrt{x^2+a}-4}{x^2-5x+6}=b \Rightarrow \mathbf{b=?}$

A) $\frac{3}{4}$ B) $\frac{1}{2}$ C) $\frac{3}{2}$ D) 2 E) 4

17. $\lim\limits_{x\to 4}\dfrac{\tan(\sqrt{x}-2)}{x-4}=a \Rightarrow \lim\limits_{x\to a}\dfrac{\cos(15\pi\cdot x)}{\sin(13\pi\cdot x)}=?$

A) –2 B) –1 C) 1 D) $\sqrt{3}$ E) 2

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1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
C	D	D	B	E	C	E	A	B
10	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**
C	C	E	C	E	B	A	B	

1. $\lim\limits_{x \to \infty} \dfrac{5^x + 3^x}{5^{x+2} + 3^x} = ?$

A) 1 B) $\frac{1}{3}$ C) $\frac{1}{5}$ D) $\frac{1}{9}$ E) $\frac{1}{25}$

2. $\lim\limits_{x \to \infty} \dfrac{3^{x+1} + 5^{x-1}}{3^x + 5^{x+1}} = ?$

A) $\frac{1}{25}$ B) $\frac{1}{20}$ C) $\frac{1}{15}$ D) $\frac{1}{10}$ E) $\frac{1}{5}$

3. $\lim\limits_{x \to \infty} \dfrac{2\pi^x - 3^{x+1}}{2^{x-1} - \pi^x} = ?$

A) π B) $-\pi$ C) -3 D) -2 E) $\frac{\pi}{2}$

4. $\lim\limits_{x \to \infty} \dfrac{4e^x + e^{-x}}{2e^x - e^{-x}} = ?$

A) 2 B) 3 C) 4 D) 6 E) 8

5. $\lim\limits_{x \to \infty} \left(\dfrac{5^{x+1} + 6^{x+1}}{5 \cdot 6^x + 2^x} \right) = ?$

A) $-\frac{1}{2}$ B) $\frac{3}{4}$ C) 1 D) $\frac{6}{5}$ E) $\frac{11}{2}$

6. $\lim\limits_{x \to -\infty} \left(\dfrac{2 \cdot 5^x + 3 \cdot 3^x}{3 \cdot 5^x - 2 \cdot 3^x} \right) = ?$

A) $\frac{2}{3}$ B) $\frac{3}{2}$ C) $\frac{1}{2}$ D) $-\frac{3}{2}$ E) $-\frac{2}{3}$

7. $\lim\limits_{x \to \infty} \dfrac{5x^2 + 4x - 7}{8x^2 - 3x + 2} = ?$

A) $\frac{2}{3}$ B) $\frac{5}{8}$ C) $\frac{3}{4}$ D) $\frac{8}{5}$ E) 2

8. $\lim\limits_{x \to \infty} \dfrac{-6x^3 + 2x^2 - 1}{2x^2 + x - 3} = ?$

A) $-\infty$ B) -6 C) -3 D) -2 E) -1

9. $\lim\limits_{x \to -\infty} \dfrac{3x^2 + 2x - 5}{2x^2 + 3x - 5} = ?$

A) $\frac{5}{2}$ B) $\frac{3}{2}$ C) 1 D) $-\frac{3}{2}$ E) $-\frac{5}{2}$

10. $\lim\limits_{x \to -\infty} \dfrac{3x^3+4x^2+1}{6x^2+x+3} = ?$

A) $-\infty$ B) $-\frac{1}{3}$ C) $\frac{1}{3}$ D) $\frac{1}{2}$ E) ∞

11. $\lim\limits_{x \to \infty} \dfrac{2x^4+6x^3+3x^2+1}{6x^5+3x^2+2x-1} = ?$

A) 0 B) $\frac{1}{2}$ C) $\frac{1}{3}$ D) $\frac{1}{4}$ E) ∞

12. $\lim\limits_{x \to -\infty} \left(\dfrac{3\sqrt{x^2+x+1}-x}{5x-\sqrt{4x^2+3x-1}} \right) = ?$

A) $-\frac{5}{7}$ B) $-\frac{4}{7}$ C) $-\frac{3}{7}$ D) $-\frac{2}{7}$ E) $-\frac{1}{7}$

13. $\lim\limits_{x \to -\infty} \dfrac{\sqrt{25x^2+4x}-\sqrt{16x^2+x+1}}{\sqrt{9x^2-3x+2}+4x} = ?$

A) -2 B) -1 C) 1 D) 2 E) 5

14. $\lim\limits_{x \to \infty} \left[\dfrac{(x+a)^2-(x+2)^2}{(3x+1)} \right] = 4 \Rightarrow a = ?$

A) 2 B) 6 C) 8 D) 12 E) 16

15. $\lim\limits_{x \to \infty} \left(\dfrac{3x^2+4x-1}{5x^2-1} \right)^x = ?$

A) e^{-1} B) 0 C) $e^{\frac{3}{5}}$ D) e E) ∞

16. $\lim\limits_{x \to \infty} \dfrac{\sqrt{4x+1}+2}{\sqrt{9x-3}} = ?$

A) $\frac{4}{9}$ B) $\frac{3}{2}$ C) $\frac{2}{3}$ D) 1 E) $\frac{9}{4}$

17. $\lim\limits_{x \to \infty} \dfrac{\sqrt{x+1}}{\sqrt{9x-1}} = ?$

A) 3 B) 2 C) $\frac{1}{2}$ D) $\frac{1}{3}$ E) $\frac{1}{6}$

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1	2	3	4	5	6	7	8	9
E	A	D	A	D	D	B	A	B
10	11	12	13	14	15	16	17	18
A	A	B	B	C	B	C	D	

1. $\lim\limits_{x \to \infty} \left(\dfrac{2}{1 - 3^{\frac{x}{x-1}}} \right) = ?$

A) –3 B) –1 C) 0 D) 1 E) 3

2. $\lim\limits_{x \to \infty} \dfrac{4x^2 - 2x + 1}{x^2 + 1} = ?$

A) 0 B) 1 C) 3 D) 4 E) ∞

3. $\lim\limits_{x \to -\infty} \dfrac{3x^2 + 4x - 1}{x^2 - x^3} = ?$

A) – ∞ B) –3 C) 0 D) 3 E) ∞

4. $\lim\limits_{x \to \infty} \dfrac{3x^5 - 4x + 1}{x^2 + 3x} = ?$

A) – ∞ B) –3 C) 0 D) 3 E) ∞

5. $\lim\limits_{x \to -\infty} \dfrac{2x^3 + x + 1}{x^2 - 1} = ?$

A) – ∞ B) – 2 C) 0 D) 2 E) ∞

6. $\lim\limits_{x \to -\infty} \dfrac{3x^5 + 4x + 1}{x^3 - 2x - 1} = ?$

A) – ∞ B) –3 C) 0 D) 3 E) ∞

7. $\lim\limits_{x \to -\infty} \dfrac{\sqrt{4x^2 - x + 2} + \sqrt{x^2 - 3x + 7}}{3x - \sqrt{4x^2 - 16}} = ?$

A) $-\dfrac{1}{5}$ B) $-\dfrac{3}{5}$ C) $\dfrac{1}{5}$ D) $\dfrac{3}{5}$ E) $\dfrac{4}{5}$

8. $n \in N$, $m \in R$,

$\lim\limits_{x \to \infty} \left[\dfrac{x^{5n-2} + 3}{x^{4n+3} - 5} \right] = m \Rightarrow \sum n = ?$

A) 5 B) 7 C) 10 D) 12 E) 15

9. $\lim\limits_{x \to \infty} \dfrac{5^{x-1} - 3^{x+1} + 1}{5^{x+1} + 3^{x-1} + 2} = ?$

A) $-\dfrac{1}{9}$ B) $-\dfrac{1}{25}$ C) 1 D) $\dfrac{1}{9}$ E) $\dfrac{1}{25}$

10. $\lim\limits_{x \to -\infty} \dfrac{5^{x+1} - 2^{x+1}}{5^{x-1} - 2^{x-1}} = ?$

A) −4 B) $-\dfrac{1}{2}$ C) 1 D) 4 E) $\dfrac{1}{25}$

11. $\lim\limits_{x \to \infty} \dfrac{(a-3)\cdot x^2 + (b+1)\cdot x - 8}{3x+4} = 1 \Rightarrow \mathbf{a \cdot b = ?}$

A) 0 B) 2 C) 3 D) 6 E) 8

12. $\lim\limits_{x \to \infty} \dfrac{(m+3)x^3 - 2x + 4}{2x^3 + x + 1} = 2 \Rightarrow \mathbf{m = ?}$

A) 1 B) 2 C) 3 D) 4 E) 5

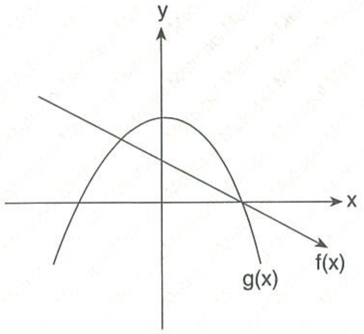
13. $\lim\limits_{x \to \infty} \left(\log_4 \left(\dfrac{2x - 20x^2}{3 - 5x^2} \right) \right) = ?$

A) −∞ B) −1 C) 0 D) 1 E) ∞

14. $\lim\limits_{x \to \infty} \dfrac{x}{x+2} + \dfrac{x+4}{x} = ?$

A) −∞ B) −1 C) 1 D) 2 E) ∞

15. $\lim\limits_{x \to \infty} \dfrac{g(x)}{f(x)} = ?$



A) −∞

B) −1

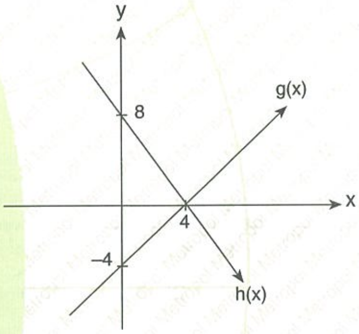
C) 0

D) 1

E) ∞

16. $f(x) = \dfrac{g(x)}{h(x)}$

$\Rightarrow \lim\limits_{x \to \infty} f(x) = ?$



A) $-\dfrac{1}{2}$

B) −1

C) 0

D) $\dfrac{1}{2}$

E) 2

17. $\lim\limits_{n \to \infty} \dfrac{[1 + 4 + 9 + \dots + n^2 + (n+1)^2]}{7n^2 + 4n - 3} = ?$

A) 0 B) ∞ C) 1 D) −1 E) $\dfrac{1}{21}$

YANITLAR / ANSWERS

1	2	3	4	5	6	7	8	9
B	D	C	E	A	E	B	E	E
10	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**
D	D	A	D	D	E	A	B	

1. $\lim\limits_{x\to 0} \dfrac{\tan(x^2)+\tan^2 x}{x^2} = ?$

A) 1 B) $\frac{3}{2}$ C) 2 D) $\frac{5}{2}$ E) 3

2. $\lim\limits_{x\to 0} \dfrac{2x+\sin 4x}{\sin 8x} = ?$

A) $\frac{1}{2}$ B) $\frac{3}{2}$ C) $\frac{3}{4}$ D) 1 E) $\frac{2}{3}$

3. $\lim\limits_{x\to \frac{\pi}{2}} \dfrac{\cos^2 x}{1-\sin^3 x} = ?$

A) $\frac{1}{2}$ B) $\frac{2}{3}$ C) 1 D) $\frac{3}{2}$ E) $\frac{5}{2}$

4. $\lim\limits_{x\to \frac{\pi}{4}} \dfrac{\sin x-\cos x}{1-\tan x} = ?$

A) $-\frac{\sqrt{2}}{2}$ B) 0 C) $\frac{\sqrt{2}}{2}$ D) 1 E) $\frac{3}{2}$

5. $\lim\limits_{x\to 0} \dfrac{2-2\cos x}{4x^2} = ?$

A) 1 B) $\frac{1}{4}$ C) $\frac{1}{6}$ D) $\frac{1}{8}$ E) $\frac{3}{4}$

6. $\lim\limits_{x\to 0} \dfrac{1}{x^2}\cdot \sin x\cdot \tan 2x = ?$

A) 1 B) 2 C) $\frac{3}{2}$ D) $\frac{4}{3}$ E) $\frac{1}{2}$

7. $\lim\limits_{x\to 3} \dfrac{\tan(27-x^3)}{x^4-81} = ?$

A) $-\frac{1}{4}$ B) $-\frac{1}{2}$ C) 0 D) $\frac{1}{4}$ E) $\frac{3}{8}$

8. $\lim\limits_{x\to 0} \dfrac{\sin(e^{6x}-1)}{\sin(e^{2x}-1)} = ?$

A) $\frac{1}{2}$ B) 1 C) $\frac{3}{2}$ D) 3 E) 6

9. $\lim\limits_{x\to 0} \dfrac{\sin 2x}{2x^2} = ?$

A) $\frac{5}{2}$ B) 2 C) $\frac{3}{2}$ D) $\frac{2}{3}$ E) 0

10. $\lim\limits_{x \to 0} \dfrac{x^2+4x}{\sin 3x} = ?$

A) $\frac{3}{2}$ B) $\frac{4}{3}$ C) $\frac{2}{3}$ D) $\frac{1}{2}$ E) 0

11. $\lim\limits_{x \to \frac{\pi}{2}} \dfrac{4x-2\pi}{\cos x} = ?$

A) 4 B) 2 C) 0 D) –2 E) –4

12. $\lim\limits_{x \to 0} \dfrac{\sin 2x \cdot \tan 2x}{1-\cos 4x} = ?$

A) $\frac{1}{8}$ B) $\frac{1}{4}$ C) $\frac{1}{2}$ D) 4 E) 2

13. $\lim\limits_{x \to \pi} \dfrac{\cos^2 x-1}{\cos x+1} = ?$

A) 2 B) $\frac{1}{2}$ C) 0 D) $-\frac{1}{4}$ E) –2

14. $\lim\limits_{x \to \frac{\pi}{2}} \dfrac{\cos 2x+1}{\sin x-1} = ?$

A) 4 B) 2 C) 1 D) –3 E) –4

15. $\lim\limits_{a \to x} \dfrac{x^4-a^4}{\sin(2x^2-2a^2)} = ?$

A) 2 B) a^2 C) x^2 D) $2a^2$ E) $\frac{x^2}{2}$

16. $\lim\limits_{x \to 2} \left[\dfrac{2x^3-16}{\sin(4x-8)}\right] = ?$

A) 2 B) 4 C) 6 D) 8 E) 12

17. $\lim\limits_{x \to 0} \dfrac{\tan x+2x}{\sin x \cdot \cos x} = ?$

A) 1 B) $\frac{3}{2}$ C) 2 D) $\frac{5}{2}$ E) 3

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1	2	3	4	5	6	7	8	9
C	C	B	A	B	B	A	D	B
10	11	12	13	14	15	16	17	18
B	E	C	E	E	C	C	E	

1. $\lim\limits_{x\to\infty}\left(8^{-x}+5^{\frac{x+3}{x+4}}+1\right)=?$

A) 3 B) 5 C) 6 D) 9 E) 14

2. $\lim\limits_{x\to\infty}\left(\dfrac{7^x+7^{-x}}{7^x-7^{-x}}\right)=?$

A) $-\infty$ B) -1 C) 0 D) 1 E) ∞

3. $\lim\limits_{x\to\infty}\left(\dfrac{2^x-3\cdot5^{x+1}}{6\cdot3^{x+2}+5^{x+2}}\right)=?$

A) $-\frac{3}{5}$ B) $-\frac{5}{3}$ C) -2 D) $\frac{3}{5}$ E) $\frac{7}{5}$

4. $\lim\limits_{x\to-\infty}\left(\dfrac{7^x+4}{7^x+2}\right)=?$

A) ∞ B) -1 C) 0 D) 2 E) 4

5. $\lim\limits_{x\to\infty}\dfrac{2^x+3^{x-1}}{3^{x+a}+2^{x+3}}=3 \Rightarrow a=?$

A) -2 B) -1 C) 0 D) 1 E) 2

6. $\lim\limits_{x\to-\infty}\dfrac{2\cdot5^x-3\cdot5^{-x}}{5^{x+1}+4\cdot5^{-x}}=?$

A) $\frac{2}{5}$ B) 2 C) $\frac{1}{2}$ D) $-\frac{3}{5}$ E) $-\frac{3}{4}$

7. $\lim\limits_{x\to\infty}\left(\sqrt[x]{3^{x+1}}-9\right)^2=?$

A) 9 B) 16 C) 25 D) 36 E) 49

8. $\lim\limits_{x\to\infty}\dfrac{(a+2)\cdot x^3+6x^2-7}{bx^2+3x-5}=\dfrac{2}{3} \Rightarrow a+b=?$

A) 3 B) 5 C) 7 D) 9 E) 12

9. $\lim\limits_{x\to\infty} \dfrac{(2x+3)^5\cdot(x+5)^3}{4x^8-7} = ?$

A) 2 B) 4 C) 6 D) 8 E) 12

10. $\lim\limits_{x\to-\infty} \left(\dfrac{7^{\frac{1}{x}}+5^x-2}{3^{\frac{1}{x}+1}+6} \right) = ?$

A) $-\frac{1}{12}$ B) $-\frac{1}{9}$ C) $-\frac{1}{6}$ D) $-\frac{1}{4}$ E) $-\frac{1}{3}$

11. $\lim\limits_{x\to\infty} \left(\log_3 \dfrac{x\sqrt{4x^2+3}+x^2}{x^2-x-3} \right) = ?$

A) 1 B) $\frac{1}{2}$ C) 3 D) 0 E) 2

12. $\lim\limits_{x\to\infty} \dfrac{ax-1+\sqrt{4x^2+3}}{2x+6} = 3 \Rightarrow a = ?$

A) 2 B) 3 C) 4 D) 6 E) 9

13. $\lim\limits_{x\to\infty} \dfrac{4x+1+\sqrt{ax^2+3x}}{\sqrt{4x^2+1}+3x} = 2 \Rightarrow a = ?$

A) 8 B) 16 C) 36 D) 48 E) 54

14. $\dfrac{9x^2+2x-1}{3x^2-5} \le f(x) \le \dfrac{3x^3-4x+1}{x^3-6x+7} \Rightarrow \lim\limits_{x\to\infty} f(x) = ?$

A) 3 B) 2 C) 1 D) 0 E) – 1

15. $\lim\limits_{x\to\infty} \dfrac{2x-3+\sqrt{9x^2+1}}{\sqrt[3]{8x^3+x}+x} = ?$

A) $\frac{2}{3}$ B) $\frac{3}{4}$ C) $\frac{5}{3}$ D) 2 E) $\frac{7}{4}$

16. $\lim\limits_{x\to\infty} \dfrac{x+\sin x}{2x+5} = ?$

A) 1 B) $\frac{1}{2}$ C) $-\frac{1}{2}$ D) 0 E) ∞

YANITLAR / ANSWERS

1	2	3	4	5	6	7	8
C	D	A	D	A	E	D	C
9	**10**	**11**	**12**	**13**	**14**	**15**	**16**
D	B	A	C	C	A	C	B

1. $\lim\limits_{x \to \infty} \frac{1}{x} \cdot (3x - 1) = ?$

A) $-\infty$ B) -3 C) 0 D) 3 E) ∞

2. $\lim\limits_{x \to 3} (x-3) \cdot \left(\tan\left(\frac{\pi}{6} \cdot x\right)\right) = ?$

A) $\frac{\pi}{3}$ B) $\frac{\pi}{4}$ C) $\frac{\pi}{6}$ D) $\frac{6}{\pi}$ E) $-\frac{6}{\pi}$

3. $\lim\limits_{x \to \infty} x \cdot \sin\left(\frac{3}{x}\right) = ?$

A) $-\infty$ B) -3 C) 0 D) 3 E) ∞

4. $\lim\limits_{x \to \infty} \frac{5x-2}{3} \cdot \ln\left(\frac{x+3}{x}\right) = ?$

A) $e^{\frac{5}{3}}$ B) e^{3} C) e^{5} D) $\frac{5}{3}$ E) 5

5. $\lim\limits_{x \to \left(\frac{\pi}{9}\right)^{+}} \left[\left(x - \frac{\pi}{9}\right) \cdot \tan\left(\frac{9x}{2}\right)\right] = ?$

A) $\frac{9}{2}$ B) $\frac{3}{2}$ C) $\frac{1}{3}$ D) $-\frac{1}{3}$ E) $-\frac{2}{9}$

6. $\lim\limits_{x \to 0} (\sin x \cdot \cot 2x) = ?$

A) $-\frac{1}{2}$ B) 1 C) $\frac{3}{2}$ D) 2 E) $\frac{1}{2}$

7. $\lim\limits_{x \to \infty} (x-2) \cdot \ln\left(\frac{x+2}{x}\right) = ?$

A) -2 B) 0 C) 2 D) 4 E) ∞

8. $\lim\limits_{x \to 1} (1-x) \cdot \left(\tan \frac{\pi}{2} x\right) = ?$

A) $\frac{\pi}{4}$ B) $\frac{\pi}{3}$ C) $\frac{\pi}{2}$ D) $\frac{2}{\pi}$ E) $\frac{3}{\pi}$

9. $\lim\limits_{x\to 0} \sin 5x \cdot \cot 3x = ?$

A) $\frac{4}{3}$ B) $\frac{3}{2}$ C) $\frac{5}{3}$ D) $\frac{7}{3}$ E) $\frac{5}{2}$

10. $\lim\limits_{x\to 0} x \cdot \cot 2x = ?$

A) $\frac{3}{2}$ B) 1 C) $\frac{2}{3}$ D) $\frac{1}{2}$ E) $\frac{1}{4}$

11. $\lim\limits_{x\to \infty} x \cdot \sin\left(\frac{1}{2x}\right) = ?$

A) $\frac{1}{4}$ B) $\frac{1}{3}$ C) $\frac{1}{2}$ D) 1 E) $\frac{3}{2}$

12. $\lim\limits_{x\to \frac{\pi}{4}} \left(2x - \frac{\pi}{2}\right) \cdot \tan 2x = ?$

A) -2 B) $-\sqrt{2}$ C) -1 D) 0 E) 2

13. $\lim\limits_{x\to \infty} 3x \cdot \ln\left(\frac{5x+1}{5x}\right) = ?$

A) $\frac{4}{3}$ B) $\frac{3}{2}$ C) $\frac{3}{5}$ D) $\frac{6}{5}$ E) $\frac{2}{5}$

14. $\lim\limits_{x\to \infty} \left(\sqrt{x^2 - 4x + 5} - x\right) = ?$

A) -2 B) -1 C) 0 D) 1 E) 2

15. $\lim\limits_{x\to -\infty} \sqrt{9x^2 - 4x + 4} + 3x + 1 = ?$

A) $-\infty$ B) $-\frac{5}{3}$ C) -1 D) $\frac{5}{3}$ E) ∞

YANITLAR / ANSWERS

1	2	3	4	5	6	7	8
D	E	D	E	E	E	C	D
9	**10**	**11**	**12**	**13**	**14**	**15**	**16**
C	D	C	C	C	A	D	

1. $\lim\limits_{x\to\infty}\left[4x\cdot\sin\frac{5}{x}\right]=?$

A) $\frac{5}{4}$ B) $\frac{4}{5}$ C) 10 D) 15 E) 20

2. $\lim\limits_{\theta\to 0}\tan 3\theta\cdot\operatorname{cosec}6\theta=?$

A) 0 B) $\frac{1}{2}$ C) 1 D) $\frac{3}{2}$ E) 3

3. $\lim\limits_{x\to-\infty}\left(\sqrt{x^2+mx+3}-\sqrt{x^2-4x}\right)=1 \Rightarrow m=?$

A) –6 B) –5 C) –2 D) 2 E) 5

4. $\lim\limits_{x\to\infty}\left(\sqrt{x^2+2x}-x\right)=?$

A) – 2 B) –1 C) 1 D) 2 E) $\frac{1}{16}$

5. $\lim\limits_{x\to\infty}\left(\frac{1+2^2+3^2+\ldots+n^2}{n^2+1}-\frac{n}{3}\right)=?$

A) $\frac{1}{6}$ B) $\frac{1}{4}$ C) $\frac{1}{3}$ D) $\frac{1}{2}$ E) 1

6. $\lim\limits_{x\to\infty}\left(3\log_4(x^2+1)-2\log_4(2x^3-1)\right)=?$

A) –5 B) –4 C) –3 D) –2 E) –1

7. $\lim\limits_{x\to-\infty}\left(\sqrt{9x^2+6x+7}+3x\right)=?$

A) –3 B) –1 C) 1 D) 3 E) 6

8. $\lim\limits_{x\to 5}\left(\frac{1}{x-5}-\frac{10}{x^2-25}\right)=?$

A) $\frac{1}{5}$ B) $\frac{1}{10}$ C) $\frac{2}{13}$ D) $-\frac{1}{10}$ E) $-\frac{1}{5}$

9. $\lim\limits_{x\to-\infty}(\sqrt{x^2-x+1}-mx-n)=0 \Rightarrow \mathbf{n=?}$

A) $-\frac{1}{2}$ B) $-\frac{3}{2}$ C) 0 D) $\frac{1}{2}$ E) 5

10. $\lim\limits_{x\to\infty}\left[\ln\sqrt{e^2x^2+1}-\ln\sqrt{e^3x^2}\right]=?$

A) –1 B) $-\frac{1}{2}$ C) –2 D) $\frac{1}{2}$ E) 1

11. $\lim\limits_{x\to\infty}\left[\sqrt{ax^2+bx+c}-(3x+2)\right]=4 \Rightarrow \mathbf{b=?}$

A) 4 B) 9 C) 16 D) 25 E) 36

12. $\lim\limits_{x\to-\infty}\left(\sqrt{x^2-4x+6}-\sqrt{x^2+ax+3}\right)=6 \Rightarrow \mathbf{a=?}$

A) –16 B) –12 C) –8 D) 8 E) 16

13. $\lim\limits_{x\to\infty}\left(x-\frac{x^4}{x^3+1}\right)=?$

A) $-\frac{4}{3}$ B) –1 C) 0 D) 1 E) $\frac{4}{3}$

14. $\lim\limits_{x\to\infty}\left(\sqrt{4x^2-5x+1}-\sqrt{4x^2-2x-2}\right)=?$

A) $-\frac{3}{4}$ B) $-\frac{1}{2}$ C) –1 D) $\frac{1}{2}$ E) 1

15. $\lim\limits_{x\to\infty}(x-\sqrt{x^2-4x+1})=?$

A) – 4 B) –2 C) 0 D) 2 E) 4

16. $\lim\limits_{x\to-\infty}\left(\frac{x^2}{x-2}-\frac{x}{3}\right)=?$

A) $-\infty$ B) $-\frac{2}{3}$ C) –1 D) $\frac{2}{3}$ E) ∞

YANITLAR / ANSWERS

1	2	3	4	5	6	7	8
E	B	A	C	D	E	B	B
9	10	11	12	13	14	15	16
D	B	E	D	C	A	D	A

1. $\lim\limits_{x \to \infty} \left(\sqrt{x^2+17x-19}-\sqrt{x^2+7x}\right) = ?$

A) 5 B) 8 C) 10 D) 12 E) 15

2. $\lim\limits_{x \to \infty} \left(\log_2 \sqrt{x^2+5}-\log_2 \sqrt{16x^2-3}\right) = ?$

A) –4 B) –2 C) –1 D) 2 E) 4

3. $\lim\limits_{x \to \infty} \left(\sqrt{x+1}-\sqrt{x}\right) = ?$

A) $-\frac{1}{2}$ B) 0 C) $\frac{1}{2}$ D) 2 E) ∞

4. $\lim\limits_{x \to \infty} \left(\sqrt{x^2+3}-\sqrt{x^2+3x}\right) = ?$

A) $-\frac{3}{2}$ B) $-\frac{1}{2}$ C) 0 D) $\frac{1}{2}$ E) $\frac{3}{2}$

5. $\lim\limits_{x \to \infty} \left(\sqrt{4x^2+2}-\sqrt{4x^2-3x+1}\right) = ?$

A) $-\frac{3}{4}$ B) $-\frac{1}{4}$ C) 0 D) $\frac{1}{4}$ E) $\frac{3}{4}$

6. $\lim\limits_{x \to -\infty} \left(\sqrt{x^2+3x+1}-\sqrt{x^2-x}\right) = ?$

A) $-\infty$ B) –2 C) $-\frac{1}{2}$ D) $\frac{1}{2}$ E) ∞

5. $\lim\limits_{x \to \infty} \left(\sqrt{x^2+x}-\sqrt{x^2+5}\right) = ?$

A) $\frac{1}{2}$ B) $\frac{2}{3}$ C) 1 D) $\frac{3}{2}$ E) 2

8. $\lim\limits_{x \to \infty} \left(\sqrt{4x^2+9x-1}-2x\right) = ?$

A) $\frac{1}{8}$ B) $\frac{1}{4}$ C) $\frac{2}{3}$ D) $\frac{9}{4}$ E) 3

9. $\lim\limits_{x \to \infty} \left(\sqrt{x^2+x+1}-x\right) = ?$

A) $\frac{1}{8}$ B) $\frac{1}{4}$ C) $\frac{1}{2}$ D) 1 E) 2

10. $\lim\limits_{x \to 3} \left(\frac{1}{x-3}-\frac{6}{x^2-9}\right) = ?$

A) $-\frac{1}{6}$ B) $-\frac{1}{3}$ C) 0 D) $\frac{1}{3}$ E) $\frac{1}{6}$

11. $\lim\limits_{x \to -4} \left(\frac{1}{x+4}-\frac{8}{16-x^2}\right) = ?$

A) $-\frac{1}{8}$ B) $-\frac{1}{4}$ C) 0 D) $\frac{1}{4}$ E) $\frac{1}{8}$

12. $\lim\limits_{x \to \infty} \left(\sqrt{2x^2+6x+5}-\sqrt{2x^2-2x+1}\right) = ?$

A) $\sqrt{2}$ B) 2 C) $2\sqrt{2}$
D) $3\sqrt{2}$ E) $4\sqrt{2}$

13. $\lim\limits_{x \to \infty} \left(\sqrt{x^2+7x}-\sqrt{x^2+2x}\right) = ?$

A) 5 B) 3 C) $\frac{5}{2}$ D) 2 E) $\frac{1}{2}$

14. $\lim\limits_{x \to -\infty} \left(\sqrt{4x^2+6x}-\sqrt{4x^2+9}\right) = ?$

A) $-\frac{3}{2}$ B) $-\frac{1}{2}$ C) 1 D) $\frac{3}{2}$ E) $\frac{5}{2}$

15. $\lim\limits_{x \to \infty} \left(\sqrt{x^2+3x}-\sqrt{x^2-3x}\right) = ?$

A) 3 B) 4 C) 5 D) 6 E) 8

16. $\lim\limits_{x \to 4} \left(\sqrt{x^2-3x}-\sqrt{2x^2-3x+5}\right) = ?$

A) –7 B) –3 C) 0 D) 7 E) ∞

YANITLAR / ANSWERS

1	2	3	4	5	6	7	8
A	B	B	A	E	B	A	D
9	10	11	12	13	14	15	16
C	E	A	C	C	A	A	B

1. $\lim\limits_{x\to\infty}\left(1+\frac{1}{3x-1}\right)^{6x+1}=?$

A) e^{-1} B) 1 C) e D) e^2 E) e^3

2. $\lim\limits_{x\to\infty}\left(1-\frac{2x}{x^2+1}\right)^{x-1}=?$

A) e^{-2} B) e^{-1} C) e D) e^2 E) e^3

3. $\lim\limits_{x\to\infty}\left(\frac{3x+5}{3x+1}\right)^{3x}=?$

A) e B) e^2 C) e^3 D) e^4 E) ∞

4. $\lim\limits_{x\to\infty}\left(\frac{4x-3}{4x+1}\right)^{x+1}=?$

A) $\frac{1}{e^4}$ B) $\frac{1}{e^2}$ C) $\frac{1}{e}$ D) e E) e^2

5. $\lim\limits_{x\to\infty}\left(1+\frac{x}{2x+1}\right)^{4x+1}=?$

A) e B) e^2 C) e^3 D) e^4 E) ∞

6. $\lim\limits_{x\to\infty}\left(\frac{x^2+3x+1}{x^2+1}\right)^{2x+1}=?$

A) e^{-3} B) e^{-1} C) 0 D) e^3 E) e^6

7. $\lim\limits_{x\to\infty}\left(\frac{x+3}{x+1}\right)^{2x+1}=?$

A) e^{-4} B) e^{-2} C) e^{-1} D) e^2 E) e^4

8. $\lim\limits_{x\to\infty}\left(\frac{3x+a}{3x-7}\right)^{6x+1}=e^8 \Rightarrow a=?$

A) –4 B) –3 C) 3 D) 4 E) 8

9. $\lim\limits_{x\to\infty}\left(\dfrac{4x-6}{4x-3}\right)^{8x-2} = ?$

A) e^{-6} B) e^{-4} C) e^{6} D) $e^{-\frac{3}{4}}$ E) $e^{-\frac{4}{3}}$

10. $\lim\limits_{x\to\infty}\left(\dfrac{x^2+x+3}{x^2+3}\right)^{2x} = ?$

A) 2 B) $\frac{7}{3}$ C) e D) e^2 E) e^3

11. $\lim\limits_{x\to\infty}\left(\dfrac{3x+4}{3x+2}\right)^{2x-3} = ?$

A) e^2 B) e^3 C) $e^{\frac{3}{4}}$ D) $e^{\frac{3}{2}}$ E) $e^{\frac{4}{3}}$

12. $\lim\limits_{x\to\infty}(1+4x)^{\frac{1}{2x+3}} = ?$

A) ∞ B) 1 C) e D) e^2 E) e^4

13. $\lim\limits_{t\to\infty}\left(\dfrac{t^3-2}{t^3+1}\right)^{t^3+3} = ?$

A) 1 B) 2 C) e D) e^{-2} E) e^{-3}

14. $\lim\limits_{x\to\infty}\left(1+\dfrac{2x+3}{x^2+5}\right)^{\frac{x}{2}} = ?$

A) e B) $\frac{e}{2}$ C) $\frac{1}{e}$ D) $\sqrt{e}$ E) $\sqrt[3]{e^2}$

15. $\lim\limits_{x\to\infty}\left(\dfrac{2x+1}{2x-1}\right)^{2x+1} = ?$

A) $\frac{1}{e^2}$ B) $\frac{1}{2}$ C) 1 D) e E) e^2

16. $\lim\limits_{x\to 0}(1+2x)^{\frac{3}{x}} = ?$

A) 0 B) 1 C) e D) e^6 E) ∞

YANITLAR / ANSWERS

1	2	3	4	5	6	7	8
D	A	D	C	E	E	E	B
9	10	11	12	13	14	15	16
A	D	E	B	E	A	E	D

1. $f: R \to R$,

$$f(x)=\begin{cases} ax-3, & x<3 \\ 6, & x=3 \\ 4x-b, & x>3 \end{cases}$$

$\forall x_0 \in R$ için / for $\lim\limits_{x \to x_0} f(x)=f(x_0)$

$\Rightarrow$ **a + b = ?**

A) 9 B) 8 C) 7 D) 6 E) 5

2. $$f(x)=\begin{cases} 2x+a, & x<0 \\ 3, & x=0 \\ 4x-b, & x>0 \end{cases}$$

$\lim\limits_{x \to 0} f(x)=f(0) \Rightarrow$ **$a \cdot b = ?$**

A) – 6 B) – 7 C) – 8 D) – 9 E) –10

3. $\forall x \in R$ için / for

$g(x)=\sqrt{(5-x)\cdot(6x+18)}$ sürekli / continuous

$\Rightarrow$ **ÇK(SS) = ?**

A) $-3 \le x$ B) $x \le 5$ C) $0 \le x \le 5$ D) $-3 \le x \le 0$ E) $-3 \le x \le 5$

4 $$f(x)=\begin{cases} ax^2-b, & x<-1 \\ b, & x=-1 \\ x+5, & x>-1 \end{cases}$$

$\lim\limits_{x \to -1} f(x)=f(-1) \Rightarrow \mathbf{\frac{a}{b}=?}$

A) $\frac{1}{2}$ B) 2 C) 1 D) 0 E) – 2

5. $$h(x)=\begin{cases} mx+n, & x=2 \\ \dfrac{x^2-4}{2-x}, & x \ne 2 \end{cases}$$

$\forall x_0 \in R$ için / for $\lim\limits_{x \to x_0} h(x)=h(x_0)$

$\Rightarrow$ **2m+n=?**

A) – 2 B) – 4 C) – 6 D) –8 E) – 10

6. $$f(x)=\begin{cases} \log_3^{(x+6)}, & x>-3 \\ \dfrac{-9}{x(x+6)}, & x \le -3 \end{cases}$$

fonksiyonu kaç tam sayı değeri için süreksizdir?

For how many integer values, the function is discontinuous?

A) 6 B) 4 C) 3 D) 2 E) 1

7. $f(x)=|x^2-4|+\dfrac{3-x}{|x-5|}+\dfrac{10}{|x^3-1|}$

fonksiyonu kaç tam sayı değeri için süreksizdir?

For how many integer values, the function is discontinuous?

A) 1 B) 2 C) 3 D) 4 E) 5

8. $f(x)=\begin{cases} \dfrac{1}{x^2-25}, & x<0 \\ 6+3x, & 0\le x\le 1 \\ 10-x^2, & x>1 \end{cases}$

fonksiyonunun süreksiz olduğu noktaların apsisleri çarpımı kaçtır?

What is the product of abscissa values of points in which the function is discontinuous?

A) – 25 B) – 5 C) 0 D) 5 E) 25

9. $f : \mathbb{R} \to \mathbb{R}$,

$f(x)=\begin{cases} 2x^2+x, & x<1 \\ x^3-1, & x=1 \\ x^4+2, & x>1 \end{cases}$

fonksiyonu kaç noktada süreksizdir?

In how many points is the function discontinuous?

A) 1 B) 2 C) 3 D) 4 E) 5

10. $f(x)=\begin{cases} -\sin x, & x<\dfrac{\pi}{2} \\ m+n, & x=\dfrac{\pi}{2} \\ \dfrac{|\cos x|}{\cos x}-\sin 2x, & x>\dfrac{\pi}{2} \end{cases}$

$\lim\limits_{x\to\frac{\pi}{2}} f(x)=f\left(\dfrac{\pi}{2}\right) \Rightarrow m+n=?$

A) – 5 B) – 4 C) – 3 D) – 2 E) – 1

11. $f(x)=\begin{cases} 1+\sin 3x, & x<0 \\ \ln(xe), & 0\le x<\pi \\ \sin x+\cos x, & x\ge\pi \end{cases}$

fonksiyonu kaç noktada süreksizdir?

In how many points is the function discontinuous?

A) 1 B) 2 C) 3 D) 4 E) 5

12. $f(x)=\begin{cases} x^2+3x+1, & x<2 \\ 11, & x=2 \\ ax+b, & x>2 \end{cases}$

$\forall x_0\in\mathbb{R}$ için (for) $\lim\limits_{x\to x_0} f(x)=f(x_0)$

$\Rightarrow a = ?$

A) $\dfrac{1}{2}-b$ B) $\dfrac{13-b}{2}$ C) $\dfrac{11-b}{2}$

D) $\dfrac{13-3b}{2}$ E) b

YANITLAR / ANSWERS

1	2	3	4	5	6
A	D	E	B	B	E
7	**8**	**9**	**10**	**11**	**12**
B	C	A	E	B	C

BÖLÜM CHAPTER 28

TARİHSEL NOT / HISTORICAL NOTE

CARL FRİEDRİCH GAUSS (GAUß) [1777 -1855]

Alman matematikçi ve bilim insanı. Katkıda bulunduğu alanlardan bazıları; sayılar kuramı, analiz, diferansiyel geometri, jeodezi, elektrik, manyetizma, astronomi ve optiktir. "Matematikçilerin prensi" ve "antik çağlardan beri yaşamış en büyük matematikçi" olarak da anılan Gauss, matematiğin ve bilimin pek çok alanına etkisini bırakmıştır ve tarihin en nüfuzlu matematikçilerinden biri olarak kabul edilir.

German mathematician and scientist. Some of the areas where he participated are; numbers theory, analysis, differential geometry, geodesy, electricity, magnetism, astronomy and optics. Gauss, also known as "the prince of mathematicians" and "the greatest mathematician since antiquity period", has influenced many areas of mathematics and science and is considered one of the most influential mathematicians of history.

TÜREVİN TANIMI ve FONKSİYONLARIN TÜREVİ
DEFINITION of DERIVATIVE and DERIVATIVE of FUNCTIONS

Bu bölüm, 247 test sorusu içermektedir.
This chapter includes 247 test questions.

BÖLÜM / CHAPTER 28

TÜREVİN TANIMI ve FONKSİYONLARIN TÜREVİ / DEFINITION of DERIVATIVE and DERIVATIVE of FUNCTIONS

- Türevin Tanımı ve Fonksiyonların Türevi / Definition of Derivative and Derivative of Functions .... 533 – 572

BÖLÜM

28

CHAPTER

TÜREVİN TANIMI ve FONKSİYONLARIN TÜREVİ

DEFINITION of DERIVATIVE and DERIVATIVE of FUNCTIONS

TÜREVİN TANIMI ve FONKSİYONLARIN TÜREVİ / DEFINITION of DERIVATIVE and DERIVATIVE of FUNCTIONS

Tanım: $f : A \to R$, $y = f(x)$ ve $x_0 \in A$ da sürekli olmak üzere,

Definition: Let $f: A \to R$, $y = f(x)$. And the function be continuous in $x_0 \in A$.

$\lim\limits_{x \to x_0} \dfrac{f(x)-f(x_0)}{x-x_0} \in R$ limit değerine ***f(x) in x_0 noktasındaki türevi*** denir.

if $\lim\limits_{x \to x_0} \dfrac{f(x)-f(x_0)}{x-x_0}$ exists, this limit is called **the derivative of f(x) at the point x_0**.

$f'(x_0) = \dfrac{df(x_0)}{dx} = \dfrac{dy}{dx}\Big|_{x_0} = \lim\limits_{x \to x_0} \dfrac{f(x)-f(x_0)}{x-x_0}$ simgelerinden biri ile gösterilir.

is shown by one of the symbols $f'(x_0) = \dfrac{df(x_0)}{dx} = \dfrac{dy}{dx}\Big|_{x_0} = \lim\limits_{x \to x_0} \dfrac{f(x)-f(x_0)}{x-x_0}$.

▼ $x - x_0 = h \Rightarrow x \to x_0 \Leftrightarrow h \to 0$

$$\lim_{x \to x_0} \frac{f(x)-f(x_0)}{x-x_0} = \lim_{h \to 0} \frac{f(x_0+h)-f(x_0)}{h} = f'(x_0)$$

▼ $\lim\limits_{h \to 0} \dfrac{f(x+h)-f(x)}{h} = f'(x)$

TÜREV ALMA KURALLARI / RULES of TAKING DERIVATIVE

▼ $c \in R$, $f(x) = c \Rightarrow f'(x) = 0$

▼ $n \in R$,

$f(x) = x^n \Rightarrow \mathbf{f'(x) = n \cdot x^{n-1}}$

▼ $a, n \in R$,

$f(x) = a \cdot x^n \Rightarrow \mathbf{f'(x) = a \cdot n \cdot x^{n-1}}$

▼ $(f(x) \pm g(x))' = \mathbf{f'(x) \pm g'(x)}$

▼ $(f(x) \cdot g(x))' = \mathbf{f'(x) \cdot g(x) + g'(x) \cdot f(x)}$

▼ $\left(\dfrac{f(x)}{g(x)}\right)' = \dfrac{\mathbf{f'(x) \cdot g(x) - g'(x) \cdot f(x)}}{\mathbf{(g(x))^2}}$ $[g(x) \neq 0]$

▼ $((f(x))^m)' = \mathbf{m \cdot (f(x))^{m-1} \cdot f'(x)}$

▼ $f(x) = \sqrt{g(x)} = g(x)^{\frac{1}{2}} \Rightarrow \mathbf{f'(x) = \dfrac{g'(x)}{2 \cdot \sqrt{g(x)}}}$

$f(x) = \sqrt[m]{g^n(x)} = g(x)^{\frac{n}{m}} \Rightarrow \mathbf{f'(x) = \dfrac{n \cdot g'(x)}{m \cdot \sqrt[m]{g(x)^{m-n}}}}$

▼ $m, n \in R$,

$f(x) = g(mx + n)$

$\Rightarrow \mathbf{f'(x) = (mx + n)' \cdot g'(mx + n) = m \cdot g'(mx + n)}$

▼ $f(x) = |g(x)|$

$$\Rightarrow f'(x) = \begin{cases} g'(x), & g(x) \geq 0 \\ -g'(x), & g(x) < 0 \end{cases}$$

▼ $f: A \to R$, $y = f(x)$ ve/and $f(x)$, $x_0 \in A$ da sürekli, / is continuous,

$\lim\limits_{x \to x_0} \dfrac{f(x)-f(x_0)}{x-x_0} = f'(x_0)$ veya/or $\lim\limits_{h \to 0} \dfrac{f(x_0+h)-f(x_0)}{h} = f'(x_0)$

Önemli / Important

- $f(x) = g(x) \cdot h(x) \cdot t(x)$
  $\Rightarrow f'(x) = g'(x) \cdot h(x) \cdot t(x) + g(x) \cdot h'(x) \cdot t(x) + g(x) \cdot h(x) \cdot t'(x)$

Önemli / Important

- $f(x) = \sqrt{x} \Rightarrow f'(x) = \dfrac{1}{2\sqrt{x}}$

BİLEŞKE FONKSİYONUN TÜREVİ / DERIVATIVE of COMPOSITE FUNCTION

▼ $\left.\begin{array}{l} y = f(t) \\ t = g(x) \end{array}\right\} \Rightarrow y = f(g(x)) = (fog)(x)$

$\dfrac{dy}{dx} = \dfrac{dy}{dt} \cdot \dfrac{dt}{dx} = f'(t) \cdot g'(x) \Rightarrow (fog)'(x) = f'(g(x)) \cdot g'(x)$

TRİGONOMETRİK FONKSİYONLARIN TÜREVİ / DERIVATIVE of TRIGONOMETRIC FUNCTIONS

SİNÜS FONKSİYONUNUN TÜREVİ / THE DERIVATIVE of the SINE FUNCTION

▼ $f(x) = \sin x \Rightarrow f'(x) = \cos x$

▼ $f(x) = \sin(u(x)) \Rightarrow f'(x) = u'(x) \cdot \cos(u(x))$

▼ $f(x) = \sin^n x \Rightarrow f'(x) = n \cdot \sin^{n-1} x \cdot \cos x$

▼ $f(x) = \sin^n(u(x)) \Rightarrow f'(x) = n \cdot \sin^{n-1}(u(x)) \cdot \cos(u(x)) \cdot u'(x)$

KOSİNÜS FONKSİYONUNUN TÜREVİ / THE DERIVATIVE of The COSINE FUNCTION

▼ $f(x) = \cos x \Rightarrow f'(x) = -\sin x$

▼ $f(x) = \cos(u(x)) \Rightarrow f'(x) = -u'(x) \cdot \sin(u(x))$

▼ $f(x) = \cos^n x \Rightarrow f'(x) = -n \cdot \cos^{n-1} x \cdot \sin x$

▼ $f(x) = \cos^n(u(x)) \Rightarrow f'(x) = -n \cdot \cos^{n-1}(u(x)) \cdot \sin(u(x)) \cdot u'(x)$

TANJANT FONKSİYONUNUN TÜREVİ / THE DERIVATIVE of the TANGENT FUNCTION

▼ $f(x) = \tan x \Rightarrow f'(x) = 1 + \tan^2 x = \dfrac{1}{\cos^2 x} = \sec^2 x$

▼ $f(x) = \tan(u(x)) \Rightarrow f'(x) = u'(x) \cdot (1 + \tan^2(u(x)))$

▼ $f(x) = \tan^n x \Rightarrow f'(x) = n \cdot \tan^{n-1} x \cdot (1 + \tan^2 x)$

▼ $f(x) = \tan^n(u(x)) \Rightarrow f'(x) = n \cdot \tan^{n-1}(u(x)) \cdot (1 + \tan^2(u(x))) \cdot u'(x)$

KOTANJANT FONKSİYONUNUN TÜREVİ / THE DERIVATIVE of the COTANGENT FUNCTION

▼ $f(x)=\cot x \Rightarrow f'(x)=-(1+\cot^2 x)=-\frac{1}{\sin^2 x}=-\operatorname{cosec}^2 x$

▼ $f(x)=\cot(u(x)) \Rightarrow f'(x)=-u'(x)\cdot(1+\cot^2(u(x)))$

▼ $f(x)=\cot^n x \Rightarrow f'(x)=-n\cdot\cot^{n-1}x\cdot(1+\cot^2 x)$

▼ $f(x)=\cot^n(u(x)) \Rightarrow f'(x)=-n\cdot\cot^{n-1}(u(x))\cdot(1+\cot^2(u(x)))\cdot u'(x)$

TERS TRİGONOMETRİK FONKSİYONLARIN TÜREVİ / DERIVATIVE of INVERSE TRIGONOMETRIC FUNCTIONS

▼ $f(x)=\arcsin x \Rightarrow f'(x)=\frac{1}{\sqrt{1-x^2}}$

$f(x)=\arcsin(u(x)) \Rightarrow f'(x)=\frac{u'(x)}{\sqrt{1-(u(x))^2}}$

▼ $f(x)=\arccos x \Rightarrow f'(x)=\frac{-1}{\sqrt{1-x^2}}$

$f(x)=\arccos(u(x)) \Rightarrow f'(x)=\frac{-u'(x)}{\sqrt{1-(u(x))^2}}$

▼ $f(x)=\arctan x \Rightarrow f'(x)=\frac{1}{1+x^2}$

$f(x)=\arctan(u(x)) \Rightarrow f'(x)=\frac{-u'(x)}{1+(u(x))^2}$

▼ $f(x)=\operatorname{arccot} x \Rightarrow f'(x)=\frac{-1}{1+x^2}$

$f(x)=\operatorname{arccot}(u(x)) \Rightarrow f'(x)=\frac{-u'(x)}{1+(u(x))^2}$

LOGARİTMİK FONKSİYONLARIN TÜREVİ / DERIVATIVE of LOGARITHMIC FUNCTIONS

▼ $f(x)=\log_a x \Rightarrow f'(x)=\frac{1}{x\cdot\ln a}=\frac{1}{x}\cdot\log_a e$

$f(x)=\log_a(u(x)) \Rightarrow f'(x)=\frac{u'(x)}{u(x)\cdot\ln a}=\frac{u'(x)}{u(x)}\cdot\log_a e$

▼ $f(x)=\ln x \Rightarrow f'(x)=\frac{1}{x}$

ÜSTEL FONKSİYONLARIN TÜREVİ / DERIVATIVE of EXPONENTIAL FUNCTIONS

▼ $f(x)=a^x \Rightarrow f'(x)=a^x\cdot\ln a$

$f(x)=a^{u(x)} \Rightarrow f'(x)=u(x)\cdot a^{u(x)}\cdot\ln a$

▼ $f(x)=e^x \Rightarrow f'(x)=e^x$

$f(x)=e^{u(x)} \Rightarrow f'(x)=u'(x)\cdot e^{u(x)}$

TERS FONKSİYONUN TÜREVİ / DERIVATIVE of INVERSE FUNCTION

- ▼ A, B ⊂ R ve f : A → B fonksiyonu birebir ve örtendir.
- ▼ Let f : A → B and A, B ⊂ R a bijective function.

$$(f^{-1})'(y_0)=\frac{1}{f'(x_0)}$$

YÜKSEK BASAMAKTAN TÜREVLER / HIGHER ORDER DERIVATIVES

▼ $y'=f'(x)$: 1. basamaktan türev (1st order derivative)

$y''=f''(x)$: 2. basamaktan türev (2nd order derivative)

$y'''=f'''(x)$: 3. basamaktan türev (3rd order derivative)

.
.
.

$y^{(n)}=f^{(n)}(x)$: n. basamaktan türev (nth order derivative)

PARAMETRİK FONKSİYONLARIN TÜREVİ / DERIVATIVE of PARAMETRIC FUNCTIONS

▼ $y=f(x)$

$$\left.\begin{matrix} y=v(t) \\ x=u(t) \end{matrix}\right\} \Rightarrow \frac{dy}{dx}=\frac{\frac{dy}{dt}}{\frac{dx}{dt}}=\frac{dy}{dt}\cdot\frac{dt}{dx}=\frac{v'(t)}{u'(t)}$$

Önemli / Important

- $x=f(t),\ y=g(t) \Rightarrow \dfrac{d^2y}{dx^2}=\dfrac{\frac{d\left(\frac{dy}{dx}\right)}{dt}}{\frac{dx}{dt}}$

ZİNCİR KURALI / THE CHAIN RULE

▼ $$\left.\begin{matrix} y=f(u) \\ u=g(t) \\ t=h(x) \end{matrix}\right\} \Rightarrow \frac{dy}{dx}=\frac{dy}{du}\cdot\frac{du}{dt}\cdot\frac{dt}{dx}$$

KAPALI FONKSİYONLARIN TÜREVİ / DERIVATIVE of IMPLICIT FUNCTIONS

- ▼ F(x,y) = 0 biçiminde tan'mlanan fonksiyonlara ***kapalı fonksiyon*** denir.
- ▼ ***Implicit function*** is a function that is formed by F(x,y) = 0.

$$F(x,y)=0 \Rightarrow F'(x,y)=\frac{dy}{dx}=-\frac{F_x(x,y)}{F_y(x,y)}$$

F_x : x e göre türev, $y \in R$ / derivative of F with respect to x

F_y : y ye göre türev, $x \in R$ / derivative of F with respect to y

1. $f(x)=x^2-x+3 \Rightarrow \lim\limits_{x\to 3}\dfrac{f(x)-f(3)}{x-3}=?$

A) 1 B) 2 C) 3 D) 4 E) 5

2. $f(x)=2x^2-3x+4 \Rightarrow \lim\limits_{h\to 0}\dfrac{f(x+h)-f(x-3h)}{2h}=?$

A) 8x – 6 B) 4x – 3 C) 0 D) 4 E) 3 – 4x

3. $f(x)=3x^2+mx-4$

$\lim\limits_{x\to 2}\dfrac{f(x)-f(2)}{x-2}=10 \Rightarrow m=?$

A) –2 B) –1 C) 0 D) 1 E) 2

4. $f(x)=3x^2(1-x)+\frac{2}{x} \Rightarrow f'(1)=?$

A) –1 B) –2 C) –3 D) –4 E) –5

5. $f(x)=\dfrac{3x^2+4x}{2x+6} \Rightarrow f'(0)=?$

A) $\frac{1}{3}$ B) $\frac{2}{3}$ C) 1 D) $\frac{4}{3}$ E) $\frac{5}{3}$

6. $f(x)=(2x+1)^3-\dfrac{1}{(x+1)^2} \Rightarrow f'(0)=?$

A) 2 B) 4 C) 6 D) 8 E) 10

7. $f(x)=\dfrac{(x-1)^2}{2x+1} \Rightarrow f'(0)=?$

A) 0 B) –1 C) –2 D) –3 E) – 4

8. $f(x)=\sqrt[3]{x^4\cdot\sqrt[3]{x^4\cdot\sqrt[3]{\dots}}}$,

$g(x)=\sqrt{x^2+x+\sqrt{x^2+x+\sqrt{\dots}}}$,

$h(x)=f(x)\cdot g(x) \Rightarrow h'(x)=?$

A) 2x + 1 B) $3x^2+2$ C) $3x^2+2x$ D) x^3+2 E) $\frac{x^3}{3}+\frac{x^2}{2}$

9. $f(x)=\sqrt[5]{x^3}+2\sqrt{x}+\frac{1}{2x} \Rightarrow \mathbf{f'(1)=?}$

A) $\frac{1}{5}$ B) $\frac{4}{5}$ C) $\frac{11}{10}$ D) $\frac{12}{5}$ E) $\frac{3}{5}$

17. $\left.\begin{aligned} f(x)&=3x^2-1,\\ g(2)&=4,\\ g'(2)&=3,\\ h(x)&=f(x)\cdot g(x)\end{aligned}\right\} \Rightarrow \mathbf{h'(2)=?}$

A) 54 B) 62 C) 74 D) 78 E) 81

11. $\mathbf{\frac{d}{dx}\left(\frac{2}{3-2x}\right)=?}$

A) $\frac{4}{3-2x}$ B) $\frac{2}{(3-2x)^2}$ C) $\frac{4}{(3-2x)^2}$

D) $2(3-2x)^2$ E) $\frac{1}{(3-2x)^2}$

12. $f(x)=\sqrt{2x^3+4} \Rightarrow \mathbf{f'(2)=?}$

A) $\frac{2}{\sqrt{3}}$ B) $\frac{3}{\sqrt{2}}$ C) $\frac{4}{\sqrt{5}}$ D) $\frac{5}{\sqrt{5}}$ E) $\frac{6}{\sqrt{5}}$

13. $f(x)=\sqrt{x\sqrt{x-1}} \Rightarrow \mathbf{f'(1)=?}$

A) $-\frac{5}{4}$ B) -1 C) 0 D) 1 E) $\frac{5}{4}$

14. $f(2x)=x^2-4x-6 \Rightarrow \mathbf{f'(1)=?}$

A) -2 B) $-\frac{3}{2}$ C) -1 D) 0 E) 2

15. $xy+\frac{1}{x}=y \Rightarrow \mathbf{f'(-1)=?}$

A) -1 B) $-\frac{3}{4}$ C) 0 D) 1 E) $\frac{3}{4}$

16. $f(x)=\frac{5}{\sqrt{x}}+x^2 \Rightarrow \mathbf{f'(1)=?}$

A) -1 B) $-\frac{1}{2}$ C) 0 D) $\frac{1}{2}$ E) 1

YANITLAR / ANSWERS

1	2	3	4	5	6	7	8
E	A	A	E	B	D	E	C
9	**10**	**11**	**12**	**13**	**14**	**15**	**16**
C	E	C	E	C	B	B	B

1. $f(x) = 4a^2 + 5b + 6$

$\Rightarrow f'(x) = \dfrac{df(x)}{dx} = ?$

A) 0 B) 8a + 5b C) 8a

D) 8a + 5 E) $4a^2 + 5b + c$

2. $f(x) = 3x^2 - 4x + 1$

$\Rightarrow f'(x) = \left.\dfrac{df(x)}{dx}\right|_{x=2} = ?$

A) 12 B) 10 C) 8 D) 6 E) 5

3. $y = \dfrac{2x^3 - 4x + 1}{5} \Rightarrow \left.\dfrac{dy}{dx}\right|_{x=2} = ?$

A) 20 B) 10 C) 5 D) 4 E) 0

4. $f(x) = 4x^3 - 3x + 1 \Rightarrow (f(125))' = ?$

A) 1245 B) 1205 C) 876 D) 650 E) 0

5. $f(x) = (x^4 + 1)\cdot(x^3 - 2x + 1) \Rightarrow f'(1) = ?$

A) 0 B) 2 C) 16 D) 32 E) 128

6. $f(x) = (x^2 + 1)\cdot(x^3 - x) \Rightarrow f'(x) = ?$

A) $5x^4 - 6x^2 - 1$ B) $5x^4 - 3x^2 - 1$

C) $5x^4 - 1$ D) $5x^4 + 1$

E) $5x^4 - x^2 - 1$

7. $f(x) = \dfrac{x^2 - 3x + 1}{2x - 1} \Rightarrow f'(x) = ?$

A) $\dfrac{2x^2 - 2x + 3}{(2x-1)^2}$ B) $\dfrac{2x^2 + 2x + 3}{(2x-1)^2}$ C) $\dfrac{2x^2 - 2x - 3}{(2x-1)^2}$

D) $\dfrac{2x^2 + 2x - 3}{(2x-1)^2}$ E) $\dfrac{2x^2 - 2x + 1}{(2x-1)^2}$

8. $f(x) = \dfrac{x^2 + x}{2x - 1} \Rightarrow f'(1) = ?$

A) −2 B) −1 C) 0 D) 1 E) 2

9. $f(x)=\dfrac{x^2+3x}{1+x^2} \Rightarrow \mathbf{f'(0) = ?}$

A) 2 B) 3 C) 4 D) 6 E) 9

10. $f(x)=\sqrt{x^2+2x} \Rightarrow \mathbf{f'(x) = ?}$

A) $\dfrac{2x+2}{\sqrt{x^2+2x}}$ B) $\dfrac{x+1}{2\sqrt{x^2+2x}}$ C) $\dfrac{x+1}{\sqrt{x^2+2x}}$

D) $\dfrac{x^2+2x}{2\sqrt{x^2+2x}}$ E) $\dfrac{x^2+2x}{\sqrt{x^2+2x}}$

11. $f(x)=\sqrt[3]{3x-1} \Rightarrow \mathbf{f'(3) = ?}$

A) $\frac{1}{12}$ B) $\frac{3}{4}$ C) $\frac{7}{12}$ D) $\frac{1}{2}$ E) $\frac{1}{4}$

12. $\left.\begin{array}{l} f(3)=4 \\ f'(3)=16 \\ g(x)=\sqrt{f(x)} \end{array}\right\} \Rightarrow \mathbf{g'(3) = ?}$

A) $\frac{1}{8}$ B) $\frac{1}{4}$ C) $\frac{1}{2}$ D) 2 E) 4

13. $f(x) = (x^2 - 3x + 1)^3 \Rightarrow \mathbf{f'(3) = ?}$

A) 3 B) 6 C) 9 D) 27 E) 36

14. $f(x) = (x^2 - 2x + 3)^4 + 3x^2 - 1 \Rightarrow \mathbf{f'(1) = ?}$

A) 0 B) 6 C) 74 D) 76 E) 78

15. $\left.\begin{array}{l} g(x)=f(2x^2+1) \\ f'(9)=4 \end{array}\right\} \Rightarrow \mathbf{g'(2) = ?}$

A) 32 B) 24 C) 16 D) 8 E) 4

16. $\left.\begin{array}{l} f(x)=3x^2+1 \\ g(x)=2x-1 \end{array}\right\} \Rightarrow \mathbf{(fog)'(x) = ?}$

A) 24x – 12 B) 12x – 6 C) 12x + 6

D) 24x + 12 E) 12x – 12

YANITLAR / ANSWERS

1	2	3	4	5	6	7	8
A	C	D	E	B	C	E	B
9	10	11	12	13	14	15	16
B	C	E	E	C	B	A	A

1. $f(x) = x^3 + ax^2 + 2x + c$

$f'(2) = 18 \Longrightarrow a = ?$

A) 1 B) 2 C) 3 D) 4 E) 5

2. $\left.\begin{array}{l} f(x) = ax^3 + 4 \\ g(x) = 2x^2 + b \\ f'(1) = g'(1) \end{array}\right\} \Rightarrow f(3) = ?$

A) 32 B) 36 C) 40 D) 44 E) 48

3. $f(x) = (2x - 3)^3 \Rightarrow f'(2) = ?$

A) 24 B) 18 C) 12 D) 8 E) 6

4. $f(x) = x^2 \cdot |x^2 - 12| \Rightarrow f'(3) = ?$

A) 36 B) 18 C) 0 D) –18 E) –36

5. $\left.\begin{array}{l} f(x) = x^2 + 2x \\ g(x) = x^4 \end{array}\right\} \Rightarrow (fog)'(1) = ?$

A) 12 B) 16 C) 20 D) 24 E) 30

6. $f(x) = \sqrt{2x+1} \cdot (x^2 - 1) \Rightarrow f'(4) = ?$

A) 24 B) 26 C) 27 D) 29 E) 32

7. $f(x) = \dfrac{3}{(3x-2)^3} \Rightarrow f'(1) = ?$

A) 36 B) 27 C) 9 D) –9 E) –27

8. $f(x) = (3x - 1)^5 \Rightarrow f'(1) = ?$

A) 80 B) 96 C) 120 D) 200 E) 240

9. $f(x) = (x-2)^{10} \cdot (x+1) \Rightarrow$ **$f'(2) = ?$**

A) 516 B) 512 C) 508 D) 502 E) 0

10. $f(x) = \sqrt{x + \sqrt{2x+1}} \Rightarrow$ **$f'(4) = ?$**

A) $\frac{\sqrt{7}}{7}$ B) $\frac{\sqrt{7}}{9}$ C) $\frac{\sqrt{7}}{12}$ D) $\frac{2\sqrt{7}}{21}$ E) $\frac{2\sqrt{7}}{9}$

11. $2x^2 + 3y - 8 = 0 \Rightarrow \left.\frac{dy}{dx}\right|_{x=6} = ?$

A) –4 B) –6 C) –8 D) –10 E) –12

12. $f(x) = x^3 + 1$

$(f''of')(a) = (f'of'')(a) \Rightarrow$ **$a = ?$**

A) –2 B) –1 C) 0 D) 1 E) 2

13. $f(x) = 3x^2 \cdot g(2x+1)$

$f'(2) = 12$

$g'(5) = 3 \Rightarrow$ **$g(5) = ?$**

A) –2 B) –3 C) –4 D) –5 E) –6

14. $f(x) = x + x^2 + x^3 + \ldots + x^{72}$

$\Rightarrow$ **$f'(-1) = ?$**

A) –32 B) –34 C) –36 D) –40 E) –72

15. $f(3x+1) = g^{-1}(6x-2)$

$\Rightarrow$ **$(gof)'(7) = ?$**

A) 1 B) 2 C) 7 D) 14 E) 21

YANITLAR / ANSWERS

1	2	3	4	5	6	7	8
A	C	E	E	B	D	E	E
9	10	11	12	13	14	15	16
E	D	C	C	D	C	B	

1. $y = \log_3(x^2 + 2x) \Rightarrow y' = \dfrac{dy}{dx} = ?$

A) $(x + 1)\cdot\log_3 e$ B) $(2x + 2)\cdot\log_3 e$ C) $\dfrac{x+1}{x+2}$ D) $\dfrac{2x+2}{x^2+2x}\cdot\log_3 e$ E) $\dfrac{2x+2}{x^2+2x}\cdot\ln 3$

2. $f(x) = \log_2(2x + 4) \Rightarrow f'(2) = ?$

A) $\frac{1}{4}\ln 2$ B) $\dfrac{1}{4\cdot\ln 2}$ C) $\dfrac{\log_2 e}{2}$ D) $4\cdot\ln 2$ E) $4\log_2 e$

3. $y = \ln(x^3 + 2x) \Rightarrow y' = ?$

A) $3x^2 + 2$ B) $\dfrac{3x^2+2}{x^3+2}$ C) $\dfrac{3x^2+2}{x^3+2x}\ln 2$ D) $\dfrac{3x^2+2x}{x^3+2x}\log_2 e$ E) $\dfrac{3x^2+2}{x^3+2x}$

4. $f(x) = \ln(x^2 - 2x - 3) \Rightarrow f'(1) = ?$

A) $-\frac{1}{4}$ B) $-\frac{1}{2}$ C) 0 D) $\frac{1}{2}$ E) $\frac{1}{4}$

5. $y = 2^{3x^2} + 4x \Rightarrow \dfrac{dy}{dx} = ?$

A) $6x\cdot 2^{3x^2} + 4x$ B) $2^{3x^2} + 4$ C) $2^{3x^2} + 4x$ D) $6x\cdot 2^{3x^2} + 4$ E) $2^{3x^2}\cdot 6x\cdot\ln 2 + 4$

6. $f(x) = 3^{x^2} + 5x^2 + 1 \Rightarrow f'(1) = ?$

A) $3\ln 3 + 10$ B) $6\ln 3 + 10$ C) $12\ln 3 + 10$ D) $3\ln 3 + 6$ E) $6\ln 3$

7. $y = e^{3x^2-1} \Rightarrow f'(1) = ?$

A) $12e$ B) $12e^2$ C) $6e$ D) $6e^2$ E) $6e^3$

8. $y = \sin x + \cos x \Rightarrow \dfrac{dy}{dx} = ?$

A) $\cos x + \sin x$ B) $-\cos x + \sin x$ C) $-\cos x - \sin x$ D) $\cos x - \sin x$ E) 0

9. $f(x) = \sin 2x\cdot\cos x \Rightarrow f'\left(\frac{\pi}{6}\right) = ?$

A) $-\frac{1}{2}$ B) $-\frac{\sqrt{3}}{2}$ C) $-\frac{\sqrt{3}}{4}$ D) $\frac{\sqrt{3}}{4}$ E) $\frac{\sqrt{3}}{2}$

10. $f(x)=2x\cdot\cos x \Rightarrow f'\left(\frac{\pi}{3}\right)=?$

A) $1-\sqrt{3}\pi$ B) π C) $\frac{\pi}{2}$

D) 0 E) $\frac{3-\sqrt{3}\pi}{3}$

11. $y=\tan(3x^2+1) \Rightarrow y'=?$

A) $6x+\tan^2(3x^2+1)$

B) $6x\cdot\cot(3x^2+1)$

C) $6x+x\cdot\tan^2(3x^2+1)$

D) $6x+6x\cdot\tan^2(3x^2+1)$

E) $6x+6x\cdot\tan(3x^2+1)$

12. $y=\log_2(\sin x) \Rightarrow y'=?$

A) $\tan x$ B) $\frac{\cot x}{\ln 2}$ C) $\tan x\cdot\log_2(\sin x)$

D) $\frac{\tan x}{\ln 2}$ E) $\tan x\cdot\ln 2$

13. $y=e^{\sin 3x} \Rightarrow \left.\frac{dy}{dx}\right|_{x=\frac{\pi}{6}}=?$

A) $\frac{\pi}{2}e^2$ B) $\frac{e}{2}$ C) $\frac{\sqrt{3}}{2}e^3$

D) $\frac{\sqrt{3}}{2}e^2$ E) 0

14. $y=\ln(\tan^3 x) \Rightarrow y'=?$

A) $3\cdot(\cot x+\tan x)$ B) $3\tan^2 x+1$ C) $3\tan^2 x$

D) $3\cdot(\tan^2 x+1)$ E) $\tan^3 x$

15. $y=\sin^2 3x \Rightarrow \left.\frac{dy}{dx}\right|_{\frac{\pi}{12}}=?$

A) $\frac{\sqrt{3}}{2}$ B) $\sqrt{3}+1$ C) $\frac{1}{2}$ D) $\frac{3}{2}$ E) 3

16. $y=\sin x\cdot e^{\cos x} \Rightarrow y'=?$

A) $e^{\cos x}(1-\sin 2x)$

B) $e^{\cos x}\left(\frac{2\cos x+\sin 2x}{2}\right)$

C) $e^{\cos x}\left(\frac{2\cos x-\sin 2x}{2}\right)$

D) $e^{\sin x}\left(\frac{2\cos x+\sin 2x}{2}\right)$

E) $e^{\cos x}\cdot(\cos x-\sin^2 x)$

17. $y=\sin^3(\cos x) \Rightarrow \frac{dy}{dx}=?$

A) $3\cdot\sin^2 x\cdot\cos(\cos x)$

B) $3\cdot\sin(\cos x)\cdot\cos(\cos 3x)$

C) $3\cdot\sin^2(\cos x)\cdot\cos(\cos x)\cdot\sin x$

D) $-3\sin^2(\cos x)\cdot\cos(\cos x)\cdot\sin x$

E) $-\sin^2(\cos x)\cdot\cos(\cos x)\cdot\sin x$

18. $f(x)=\sin^2 x\cdot\cos x \Rightarrow f'\left(\frac{\pi}{6}\right)=?$

A) 0 B) $\frac{1}{2}$ C) $\frac{3}{8}$ D) $\frac{5}{8}$ E) 1

YANITLAR / ANSWERS

1	2	3	4	5	6	7	8	9
D	B	E	C	E	B	D	D	D
10	11	12	13	14	15	16	17	18
E	D	B	E	A	E	E	D	D

1. $y = 5^{\cos x} \Rightarrow y' = \dfrac{dy}{dx} = ?$

A) $5^{\sin x}$ B) $5^{\cos x}$ C) $\sin x \cdot 5^{\cos x}$

D) $-\sin x \cdot 5^{\cos x}$ E) $-\sin x \cdot 5^{\cos x} \cdot \ln 5$

2. $f(x) = \sin^4 x \Rightarrow f'\left(\dfrac{\pi}{4}\right) = ?$

A) $\frac{1}{8}$ B) $\frac{1}{4}$ C) $\frac{1}{2}$ D) 1 E) 2

3. $y = \ln\sqrt{1-\cos x} \Rightarrow \dfrac{dy}{dx} = y' = ?$

A) $\dfrac{\sin x}{\sqrt{1-\cos x}}$ B) $\dfrac{\sin x}{2\sqrt{1-\cos x}}$ C) $\dfrac{\sin x}{2-2\cos x}$

D) $\frac{1}{2}\sin x$ E) $-\frac{1}{2}\cos x$

4. $f(x) = \ln(5^{\cos 3x}) \Rightarrow f'\left(\dfrac{\pi}{6}\right) = ?$

A) -3 B) 0 C) $\ln 5$ D) $-3\ln 5$ E) 15

5. $f(x) = 4^{e^{\cos x}} \Rightarrow f'\left(\dfrac{\pi}{2}\right) = ?$

A) $4 \cdot \ln 4$ B) $-4 \cdot \ln 4$ C) $e^4 \cdot \ln 4$

D) 4 E) $-e^4$

6. $f(x) = e^{\tan x} \Rightarrow f'\left(\dfrac{\pi}{4}\right) = ?$

A) $-e^{-\frac{3}{2}}$ B) $\frac{1}{3}e^{-1}$ C) $-e^{-1}$

D) $2e$ E) $3e^2$

7. $y = \ln(\sin x) \Rightarrow y' = \dfrac{dy}{dx} = ?$

A) cotx B) –cotx C) tanx

D) –tanx E) cosx

8. $f(x) = e^{\sin x} \Rightarrow f'(0) = ?$

A) e B) e^{-1} C) 0 D) 1 E) $\frac{1}{2}$

9. $y = e^x \cdot x^2 + 3x \cdot e^x \Rightarrow \mathbf{e^{-x} \cdot \left(\frac{dy}{dx}\right) = ?}$

A) x^2+3x B) $2x+3$ C) x^2+x-3

D) x^2+x+3 E) x^2+5x+3

10. $f(x) = \sin 2x \cdot \cos 2x \Rightarrow \mathbf{f'\left(\frac{\pi}{24}\right) = ?}$

A) $\frac{\sqrt{3}}{2}$ B) $\frac{1}{2}$ C) 1 D) $\sqrt{3}$ E) 2

11. $f(x) = \cos^2 x - \sin^2 x \Rightarrow \mathbf{f'\left(\frac{\pi}{12}\right) = ?}$

A) –2 B) $-\frac{3}{2}$ C) –1 D) $-\frac{1}{2}$ E) 0

12. $f(x) = \tan\left(\frac{\pi}{2}\cos x\right) \Rightarrow \mathbf{f'\left(\frac{\pi}{3}\right) = ?}$

A) $-\pi \cdot \sqrt{3}$ B) $-\frac{\pi \cdot \sqrt{3}}{2}$ C) $\frac{\pi \cdot \sqrt{3}}{2}$

D) $2\pi \cdot \sqrt{3}$ E) $\pi \cdot \sqrt{3}$

13. $f(x) = \sqrt{\cos^4 x - \sin^4 x + 1} \Rightarrow \mathbf{f'\left(\frac{\pi}{4}\right) = ?}$

A) –2 B) –1 C) $-\frac{1}{2}$ D) 1 E) 2

14. $y = e^{5x}$,

$$\frac{d^n y}{dx^n} = \left(\frac{1}{125}\right) \cdot \frac{d^6 y}{dx^6}$$

$\Rightarrow \mathbf{n^2 + 2^n = ?}$

A) 3 B) 8 C) 17 D) 32 E) 57

15. $y = x \cdot e^x$,

$\frac{d^n y}{dx^n} = e^x \cdot (12+x) \Rightarrow \mathbf{n = ?}$

A) 9 B) 10 C) 11 D) 12 E) 13

16. $f(x) = e^x + e^{2x} + e^{3x} + \ldots + e^{10x} \Rightarrow \mathbf{f'(0) = ?}$

A) 0 B) 10 C) 30 D) 55 E) 110

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1	2	3	4	5	6	7	8
E	D	C	D	B	D	A	D
9	**10**	**11**	**12**	**13**	**14**	**15**	**16**
E	D	C	B	B	C	D	D

1. $f(x) = (x^2 - 3x + 1)^{10} \Rightarrow f'(1) = ?$

A) 7 B) 8 C) 9 D) 10 E) 11

2. $f(x) = \sqrt{x^3 + \frac{1}{x}} \Rightarrow f'(1) = ?$

A) $\frac{\sqrt{2}}{2}$ B) $\frac{1}{3}$ C) $\frac{\sqrt{2}}{4}$ D) $\frac{\sqrt{2}}{3}$ E) $\frac{1}{4}$

3. $\left.\begin{aligned} f(3x-1) &= x \cdot g(x+1) \\ g(3) &= -4 \\ f'(5) &= 6 \end{aligned}\right\} \Rightarrow g'(3) = ?$

A) 9 B)10 C) 11 D) 12 E) 13

4. $\left.\begin{aligned} x &= 3t-1 \\ y &= t^2+4t+1 \end{aligned}\right\} \Rightarrow \left.\frac{dy}{dx}\right|_{x=5} = ?$

A) 1 B) $\frac{5}{3}$ C) 3 D) $\frac{8}{3}$ E) $\frac{14}{3}$

5. $f(x) = \arcsin^3(\sqrt{x}) \Rightarrow f'(\frac{1}{4}) = ?$

A) $\frac{\pi^2 \cdot \sqrt{3}}{15}$ B) $\frac{\pi \cdot \sqrt{3}}{8}$ C) $\frac{\pi^2 \cdot \sqrt{3}}{18}$

D) $\frac{\pi \cdot \sqrt{3}}{6}$ E) $\frac{\pi^2 \cdot \sqrt{3}}{6}$

6. $f(x) = e^x \cdot \sin x \cdot \cos x \Rightarrow f'(0) = ?$

A) $\frac{1}{2}$ B) 1 C) $\frac{1}{3}$ D) 2 E) $\frac{2}{3}$

7. $f(x) = \arctan^2(e^x) \Rightarrow f'(0) = ?$

A) π B) $\frac{\pi}{2}$ C) $\frac{\pi}{3}$ D) $\frac{\pi}{4}$ E) $\frac{\pi}{8}$

8. $f(x, y) = y - x + \operatorname{arccot} y = 0 \Rightarrow \frac{dy}{dx} = ?$

A) $\frac{1}{1+y^2}$ B) $\frac{2y}{1+y^2}$ C) $\frac{1+y^2}{y^2}$

D) $-\frac{2}{1+y^2}$ E) $-\frac{2y^2}{1+y^2}$

9. $\left.\begin{array}{l} x = \ln t \\ y = \sin t \end{array}\right\} \Rightarrow \left.\dfrac{d^2y}{dx^2}\right|_{x=\pi} = ?$

A) $\frac{1}{\pi}$ B) π C) -1 D) $-\pi$ E) $-\frac{1}{\pi}$

10. $\left.\begin{array}{l} y = 3v^2+2 \\ v = 2t+1 \\ t = x^2-1 \end{array}\right\} \Rightarrow \dfrac{dy}{dx} = ?$

A) $24x\cdot(2x^2-1)$ B) $18x\cdot(2x^2-1)$
C) $x\cdot(2x^2-1)$ D) $3x\cdot(x^2-1)$
E) $12x\cdot(2x^2-1)$

11. $y^2+y = x^3-x+2 \Rightarrow \left.\dfrac{dy}{dx}\right|_{(1,1)} = ?$

A) $\frac{2}{3}$ B) $\frac{3}{2}$ C) 0 D) $-\frac{3}{2}$ E) $-\frac{2}{3}$

12. $f(x) = x^x \Rightarrow f'(1) = ?$

A) 3 B) 2 C) 1 D) $\frac{1}{4}$ E) $\frac{1}{3}$

13. $f(x) = |x^2-4x-6| \Rightarrow f'(2) = ?$

A) -2 B) -1 C) 0 D) 1 E) 2

14. $\left.\begin{array}{l} x = \sin t \\ y = \sin t - \cos t \end{array}\right\} \Rightarrow \left.\dfrac{d^2y}{dx^2}\right|_{t=\frac{\pi}{3}} = ?$

A) 8 B) 6 C) $\frac{8\sqrt{3}}{9}$ D) $\frac{4\sqrt{3}}{3}$ E) 4

15. $\left.\begin{array}{l} x = \sin 3t \\ y = \cos 4t \end{array}\right\} \Rightarrow \left.\dfrac{dy}{dx}\right|_{t=\frac{\pi}{3}} = ?$

A) $-\frac{2\sqrt{3}}{3}$ B) $-\frac{4\sqrt{3}}{3}$ C) $-2\sqrt{3}$
D) $\frac{4\sqrt{3}}{3}$ E) $-3\sqrt{3}$

16. $y = \sin x \Rightarrow \dfrac{d^{73}y}{dx^{73}} = ?$

A) cosx B) –sinx C) 1
D) –cosx E) sinx

17. $xy + x = y^3 - 1 \Rightarrow \left.\dfrac{dx}{dy}\right|_{y=0} = ?$

A) –2 B) –1 C) 0 D) 1 E) 2

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1	2	3	4	5	6	7	8	9
D	A	C	D	C	B	D	C	D
10	11	12	13	14	15	16	17	
A	A	C	C	A	A	A	D	

1. $f(x)=x^4+3x^{-2}+\sqrt[3]{x^2} \Rightarrow$ **$f'(1)=?$**

A) $-\frac{4}{3}$ B) $-\frac{2}{3}$ C) $-\frac{1}{3}$ D) $\frac{1}{3}$ E) $\frac{4}{3}$

2. $f(x)=\dfrac{(4x-3)^3}{(2x-3)^2} \Rightarrow$ **$f'(1)=?$**

A) 8 B) 10 C) 12 D) 14 E) 16

3. $f(x)=3^{\cos x} \Rightarrow f'(\frac{\pi}{3})=?$

A) $-\ln 3\sqrt{3}$ B) $-\ln 3\sqrt{2}$ C) $-\ln\sqrt{3}$ D) $\ln\sqrt{3}$ E) $\ln 3\sqrt{3}$

4. $f(x)=\log_2(x^2-3x+4) \Rightarrow$ **$f'(2)=?$**

A) $\log_2 e$ B) $\dfrac{3\log_2 e}{2}$ C) $\dfrac{1}{\ln 6}$ D) $\dfrac{1}{2\ln 2}$ E) $\dfrac{\ln 2}{2}$

5. $f(x)=\ln[\ln(x^2+3)] \Rightarrow$ **$f'(3)=?$**

A) $\dfrac{1}{27\ln 3}$ B) $\dfrac{1}{9\ln 3}$ C) $\dfrac{1}{2\ln 12}$ D) $\dfrac{2}{9\ln 3}$ E) $\dfrac{2}{3\ln 3}$

6. $f(x)=x\cdot 2^{\tan 3x} \Rightarrow$ **$f'(\frac{\pi}{12})=?$**

A) $2+\ln 2$ B) $2+\ln 4$ C) $2+\pi\ln 2$ D) $2+6\pi\ln 2$ E) $2+12\pi\ln 2$

7. $f(x)=(4x-3)^{3x+1} \Rightarrow$ **$f'(1)=?$**

A) 2 B) 4 C) 8 D) 10 E) 16

8. $f(3x+1)=\dfrac{(x+1)^2}{g(2-x)}$

$f'(4)=2,\ g(1)=4 \Rightarrow$ **$g'(1)=?$**

A) 12 B) 15 C) 20 D) 24 E) 25

9. $f(x)=\sqrt{x}+x\cdot|x^2-2| \;\Rightarrow\; \mathbf{f'(1)+f(1)=?}$

A) $-\frac{3}{2}$ B) $-\frac{1}{2}$ C) $\frac{1}{2}$ D) 1 E) $\frac{3}{2}$

10. $f(x)=2^{\arctan x} \;\Rightarrow\; \mathbf{f'(1)=?}$

A) 0 B) $2^{\frac{\pi}{4}}$ C) ln2
D) $2^{\frac{\pi}{4}}\cdot\ln 2$ E) $2^{\frac{\pi}{4}}\cdot\ln\sqrt{2}$

11. $f(x)=x^{\cos x} \;\Rightarrow\; \mathbf{f'(\pi)=?}$

A) $-\frac{1}{\pi^2}$ B) $-\frac{1}{\pi}$ C) 0 D) $\frac{1}{\pi}$ E) $\frac{1}{\pi^2}$

12. $(f\circ g)(x)=\sin(\cos 2x)$,

$g\left(\frac{\pi}{4}\right)=3,\; g'\left(\frac{\pi}{4}\right)=2 \;\Rightarrow\; \mathbf{f(3)+f'(3)=?}$

A) –2 B) –1 C) 0 D) 1 E) 2

13. $f(x)=3x^2-4x+1$,
$g(x)=2x^2-x \;\Rightarrow\; \mathbf{(f\circ g)'(1)=?}$

A) 5 B) 6 C) 10 D) 12 E) 15

14. $f(x)=e^{-3x}-e^{2x}+2x^2 \;\Rightarrow\; \mathbf{f''(0)=?}$

A) 8 B) 9 C) 10 D) 12 E) 14

15. $y=\sqrt{x}$,
$x=t^2+1$,
$t=u^3-u+1 \;\Rightarrow\; \left.\mathbf{\frac{dy}{du}}\right|_{u=1}\mathbf{=?}$

A) $\frac{\sqrt{2}}{4}$ B) $\frac{\sqrt{2}}{2}$ C) $\frac{\sqrt{3}}{2}$ D) $\sqrt{2}$ E) 2

16. $f(x)=e^x\cdot x \Rightarrow \mathbf{f^{(7)}(1)=?}$

A) 7 B) 8 C) 6e D) 7e E) 8e

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1	2	3	4	5	6	7	8
A	E	A	D	C	C	E	C
9	**10**	**11**	**12**	**13**	**14**	**15**	**16**
E	E	A	B	B	B	D	E

1. $\left.\begin{array}{l} f(x) = (goh)(x) \\ f'(-1) = 14 \\ h(x) = 2x^2 - 3x \end{array}\right\} \Rightarrow$ **$g'(5) = ?$**

A) –7 B) –3 C) –2 D) –1 E) 7

2. $f(x) = e^{-2\ln x} + \arctan\frac{1}{2} \Rightarrow$ **$f''(1) = ?$**

A) –24 B) 24 C) –6 D) 6 E) 0

3. $\left.\begin{array}{l} x = 2t + \ln t \\ y = t^2 + \sin(t-1) \end{array}\right\} \Rightarrow \left.\frac{dy}{dx}\right|_{t=1} = ?$

A) $\frac{1}{3}$ B) $\frac{2}{3}$ C) 1 D) 2 E) 3

4. $f(x) = 3^{3\log_3 x} + \sin(\cos\pi) \Rightarrow$ **$f''(1) = ?$**

A) 6 B) 12 C) –12 D) –60 E) 60

5. $f(x) = x^2 - \frac{1}{x^2} - 3^x \cdot \ln x \Rightarrow$ **$f'(1) = ?$**

A) 4 B) 3 C) 2 D) 1 E) –1

6. $f(x) = x^3 + 2x \Rightarrow$ **$(f^{-1})'(3) = ?$**

A) –5 B) –3 C) $\frac{1}{3}$ D) $\frac{1}{5}$ E) 5

7. $f(x) = 2x^3 - 4x - \frac{1}{x^3} - 3\cdot\sqrt{4x+5} \Rightarrow$ **$f'(1) = ?$**

A) –3 B) –1 C) 1 D) 2 E) 3

8. $x^3y^2 - 4x = xy + 3y - 7 \Rightarrow \left.\frac{dy}{dx}\right|_{(1,1)} = ?$

A) 2 B) 1 C) 0 D) –1 E) –2

9. $2x^3 - \sin 4y = x^2y + 2 \Rightarrow \left.\dfrac{dy}{dx}\right|_{y=0} = ?$

A) $-\dfrac{6}{5}$ B) -2 C) -3 D) $\dfrac{6}{5}$ E) 2

10. $\left.\begin{array}{l} f(x) = x^3 + e^{-x} + 1 \\ h(x) = (gof)(x) \\ h'(0) = 4 \end{array}\right\} \Rightarrow g'(2) = ?$

A) -4 B) 4 C) 2 D) -2 E) 1

11. $f(x) = y = e^{-3x} \Rightarrow \left.\dfrac{d^{16}y}{dx^{16}}\right|_{x=0} = ?$

A) -3^{16} B) 3^{13} C) 3^{12} D) 3^{14} E) 3^{16}

12. $x^{\ln y} = 2 \Rightarrow \left.\dfrac{dy}{dx}\right|_{y=e} = ?$

A) $-\dfrac{e}{\ln 4}$ B) $\dfrac{1}{2e}$ C) $-\dfrac{e}{2}$

D) $-\dfrac{e^2}{\ln 2}$ E) $-e\ln 2$

13. $x \in [0, 2\pi]$

$y = e^{\sin x} \Rightarrow \min(y) = ?$

A) $\dfrac{1}{e}$ B) $-e$ C) -1 D) 1 E) e

14. $f(x) = \ln(x^2 + 4) - \arctan\dfrac{x}{2} \Rightarrow f'(2) = ?$

A) $\dfrac{\pi}{2}$ B) $-\dfrac{\pi}{4}$ C) $-\dfrac{\pi}{2}$ D) -1 E) $\dfrac{1}{4}$

15. $\left.\begin{array}{l} y = x^2 + 1 \\ x = u^2 - 1 \\ u = \ln t \end{array}\right\} \Rightarrow \left.\dfrac{dy}{dt}\right|_{t=e^2} = ?$

A) $24e^{-2}$ B) $12e^{-1}$ C) $18e^{-3}$

D) $12e^2$ E) $24e$

16. $\left.\begin{array}{l} y = 2t^2 - 1 \\ x = t^3 \end{array}\right\} \Rightarrow \left.\dfrac{d^2y}{dx^2}\right|_{t=1} = ?$

A) -9 B) -4 C) $-\dfrac{4}{9}$ D) $\dfrac{9}{4}$ E) 4

17. $\left.\begin{array}{l} y = t + e^t \\ x = e^{2t} \end{array}\right\} \Rightarrow \left.\dfrac{dy}{dx}\right|_{t=0} = ?$

A) -1 B) $-\dfrac{1}{2}$ C) 0 D) 1 E) 2

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1	2	3	4	5	6	7	8	9
C	D	C	A	D	D	E	D	D
10	**11**	**12**	**13**	**14**	**15**	**16**	**17**	
A	E	A	A	E	A	C	D	

1. $f: R \to R$,
$f(x) = x^2 + 2x - 6 \Rightarrow$ **$f'(2) = ?$**

A) 6 B) 4 C) 2 D) 0 E) 5

2. $f: R - \{-3\} \longrightarrow R - \{1\}$,
$f(x) = \dfrac{x}{x+3} \Rightarrow$ **$f'(2) = ?$**

A) $\frac{2}{5}$ B) $\frac{3}{5}$ C) $\frac{3}{10}$ D) $\frac{3}{25}$ E) $\frac{2}{25}$

3. $f: R \to R$,
$f(x) = \dfrac{x}{x^2+1} \Rightarrow$ **$f'(1) = ?$**

A) – 2 B) –1 C) 0 D) 1 E) 2

4. $f(x) = x^3 - 5x^2 + 7x + 8$
$g(x) = x^3 + 2x$
$\Rightarrow$ **$[(fog)(2)]' = ?$**

A) 7826 B) 7824 C) 6964 D) 6948 E) 0

5. $f(3x - 2) = x^3 + 6x - 8 \Rightarrow$ **$f'(4) = ?$**

A) 4 B) 6 C) 8 D) 12 E) 18

6. $f(x) = \sin^2 4x \Rightarrow$ **$f'(\frac{\pi}{24}) = ?$**

A) 4 B) $2\sqrt{3}$ C) $4\sqrt{3}$
D) –4 E) $-6\sqrt{3}$

7. $f(x) = \ln(\cos x) \Rightarrow$ **$f'\left(\frac{\pi}{4}\right) = ?$**

A) $\sqrt{3}$ B) $\frac{\sqrt{2}}{2}$ C) 1 D) 0 E) –1

8. $f(x) = xe^{-x} \cdot \dfrac{d}{dx}(e^x \cdot \ln x) \Rightarrow$ **$f'(e) = ?$**

A) $\frac{1}{e} + 1$ B) e + 1 C) $\frac{1-e}{e}$
D) 2 E) $\frac{e}{2}$

9. $f: R \to R$,
$f(x) = x^3 + 4x$
$\Rightarrow$ **$(f^{-1})'(16) = ?$**

A) $\frac{1}{20}$ B) $\frac{1}{10}$ C) $\frac{1}{16}$ D) 4 E) 8

10. $f : R \rightarrow R,$

$f(x) = x^3 + 8$

$\Rightarrow \mathbf{(f^{-1})'(0) = ?}$

A) $-\frac{1}{6}$ B) $-\frac{1}{18}$ C) $\frac{1}{4}$ D) $\frac{1}{12}$ E) $\frac{1}{16}$

11. $\left.\begin{matrix} x = t^2 - t \\ y = t^3 + t \end{matrix}\right\} \Rightarrow \left.\frac{d^2y}{dx^2}\right|_{t=2} = ?$

A) $-\frac{7}{9}$ B) $-\frac{14}{27}$ C) $\frac{7}{9}$ D) $\frac{10}{27}$ E) $\frac{3}{8}$

12. $\frac{dy}{dx} = f'(x)$

$\left.\begin{matrix} x = 5\sec t \\ y = 4\tan t \end{matrix}\right\} \Rightarrow f'\left(\frac{10}{\sqrt{3}}\right) = ?$

A) $\frac{4}{3}$ B) $\frac{5}{2}$ C) $\frac{4}{5}$ D) $\frac{12}{7}$ E) $\frac{8}{5}$

13. $\left.\begin{matrix} x = \cos 2t \\ y = \sin^2 2t \end{matrix}\right\} \Rightarrow \frac{d^2y}{d^2x} = ?$

A) $\tan^2 2t$ B) $1 + \sec^2 2t$ C) $1 + 4\cos 2t$

D) -2 E) -4

14. $x^3 + y^2 - 4y^3 + 3x - y - 4 = 0 \Rightarrow \mathbf{f'(1) = ?}$

A) -6 B) -1 C) 0 D) 4 E) 6

15. $f(x) = \arctan(\sin x)$

$\left.\frac{df(x)}{dx}\right|_{x = \frac{5\pi}{3}} = ?$

A) $-\frac{3}{4}$ B) $-\frac{1}{2}$ C) 0 D) $\frac{2}{7}$ E) $\frac{8}{7}$

16. $f(x) = 6^{3x^2} + 1$

$f'(a) = 6^{3a^2} \cdot \ln 6$

$\Rightarrow \mathbf{a = ?}$

A) 2 B) $\frac{1}{2}$ C) $\frac{1}{3}$ D) $\frac{2}{3}$ E) $\frac{1}{6}$

17. $f(x) = (1 + x^2)^{\cos x}$

$\Rightarrow \mathbf{f'\left(\frac{\pi}{2}\right) = ?}$

A) $\ln\left(\frac{4}{4 + \pi^2}\right)$ B) $\ln\left(\frac{4 + \pi^2}{4}\right)$ C) $4 + \pi^2$

D) $\frac{2 + \pi}{2}$ E) $-2\ln\left(\frac{1 + \pi^2}{2}\right)$

18. $\frac{dy}{dx} = f'(x)$

$\left.\begin{matrix} x = \sin 2t \\ y = \cos 2t \end{matrix}\right\} \Rightarrow \mathbf{f'\left(\frac{\sqrt{3}}{2}\right) = ?}$

A) $-\frac{1}{2}$ B) $-\sqrt{2}$ C) $-\sqrt{3}$ D) $2\sqrt{2}$ E) $\sqrt{3}$

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1	2	3	4	5	6	7	8	9
A	D	C	E	B	B	E	D	C
10	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**
D	D	E	D	E	D	E	A	C

1. $y = f(x)$

$$\frac{1}{x^2}+\frac{1}{y}=2$$

$\Rightarrow$ **$f'(2) = ?$**

A) $-\frac{4}{49}$ B) $\frac{4}{49}$ C) $\frac{12}{49}$ D) $\frac{15}{49}$ E) $\frac{3}{49}$

2. $x=\frac{2x\cdot f(x)+1}{x+1} \Rightarrow \frac{df(x)}{dx}=f'(x)=?$

A) $\frac{x^2}{x+1}$ B) $\frac{x^2+1}{2x^2}$ C) $\frac{x^2+2}{x^2}$ D) $\frac{x^2+2}{2x^2}$ E) $\frac{x^2+1}{4x^2}$

3. $y=x^3\cdot 3^x \Rightarrow \frac{1}{3^x\cdot x^2}\cdot\frac{dy}{dx}=?$

A) 1 + ln3 B) x + ln3 C) 1 + x·ln3 D) x + 3·ln3 E) 3 + x · ln3

4. $f(x)=\frac{2x+1}{2x-1} \Rightarrow f(2)+f'(2)=?$

A) $\frac{7}{9}$ B) $\frac{8}{9}$ C) $\frac{10}{9}$ D) $\frac{11}{9}$ E) $\frac{13}{9}$

5. $f(x)=3^{x^2-x} \Rightarrow f'(2)=?$

A) 3·ln3 B) 6·ln3 C) 9·ln3 D) 27·ln3 E) 81·ln3

6. $f(x)=e^{2x^2} \Rightarrow f'(\frac{1}{2})=?$

A) $\sqrt{e}$ B) $2\sqrt{e}$ C) $\frac{\sqrt{e}}{2}$ D) $4\sqrt{e}$ E) $\frac{2}{\sqrt{e}}$

7. $f(x)=x^e\cdot e^x \Rightarrow f'(2)=?$

A) $\frac{e^2\cdot 2^e}{2}$ B) $\frac{1}{2}e^2\cdot 2^e$ C) $e^2\cdot 2^e\left(\frac{e+2}{2}\right)$ D) $\frac{3}{2}e^2\cdot 2^e$ E) $\frac{5}{2}\cdot e^2\cdot 2^e$

8. $f(x)=\sin^2\left(\frac{\pi x}{48}\right) \Rightarrow f'(6)=?$

A) $\frac{\pi\cdot\sqrt{2}}{6}$ B) $\frac{\pi\cdot\sqrt{2}}{12}$ C) $\frac{\pi\cdot\sqrt{2}}{18}$ D) $\frac{\pi\cdot\sqrt{2}}{24}$ E) $\frac{\pi\cdot\sqrt{2}}{96}$

9. $f(x)=\sin\frac{\sqrt{x}}{4} \Rightarrow f'(16\pi^2)=?$

A) $-\frac{1}{8\pi}$ B) $-\frac{1}{16\pi}$ C) $-\frac{1}{32\pi}$ D) $-\frac{1}{64\pi}$ E) $-\frac{1}{128\pi}$

10. $f(x) = (2x^3+3x^2)\cdot e^{-2x} \Rightarrow \mathbf{e^{2x}\cdot\frac{dy}{dx} = ?}$

A) $3(x^2+x)$ B) x^3+x^2 C) $6x-4x^3$ D) $8(x^2+x)\cdot e^{-2x}$ E) $6(x+x^2)\cdot e^{-2x}$

11. $f(x)=\sin^2(3x)$

$\Rightarrow \mathbf{f'\left(\frac{\pi}{12}\right) = ?}$

A) 1 B) 2 C) 3 D) 4 E) 5

12. $f(x) = \sin^2\left(\frac{4x}{\pi}\right)$

$\Rightarrow \mathbf{f'\left(\frac{\pi^2}{16}\right) = ?}$

A) $\frac{2\sqrt{3}}{\pi}$ B) $\frac{2}{\pi}$ C) $\frac{3}{\pi}$ D) $\frac{4}{\pi}$ E) $\frac{6}{\pi}$

13. $f(x) = a\cdot\sin^2\left(\frac{x}{2a}\right)$

$\Rightarrow \mathbf{f'\left(\frac{\pi\cdot a}{3}\right) = ?}$

A) $\frac{1}{2}$ B) $\frac{1}{3}$ C) $\frac{\sqrt{3}}{3}$ D) $\frac{\sqrt{3}}{2}$ E) $\frac{\sqrt{3}}{4}$

14. $f(x) = \cos\cdot\left(\frac{\pi}{2}\cos x\right)$

$\Rightarrow \mathbf{f'\left(\frac{\pi}{3}\right) = ?}$

A) $\frac{\pi\cdot\sqrt{3}}{4}$ B) $\frac{\pi\cdot\sqrt{3}}{6}$ C) $\frac{\pi\cdot\sqrt{6}}{8}$ D) $\frac{\pi\cdot\sqrt{6}}{12}$ E) $\frac{\pi\cdot\sqrt{6}}{16}$

15. $f(x) = \cos^2(2x)$

$\Rightarrow \mathbf{f'\left(\frac{\pi}{24}\right) = ?}$

A) –2 B) –1 C) 0 D) 1 E) 2

16. $a \neq 0$

$f(x) = a\cdot\cos^2\left(\frac{x}{a}\right)$

$\Rightarrow \mathbf{f'\left(\frac{\pi\cdot a}{6}\right) = ?}$

A) $-\frac{\sqrt{3}}{2}$ B) $\frac{\sqrt{3}}{3}$ C) $-\frac{1}{2}$ D) $\frac{\sqrt{3}}{2}$ E) $\frac{\sqrt{2}}{2}$

17. $f(x) = x\cdot\cos\frac{1}{x} \Rightarrow \mathbf{f'\left(\frac{2}{\pi}\right) = ?}$

A) $-\frac{3}{\pi}$ B) $-\frac{4}{\pi}$ C) $-\frac{\pi}{2}$ D) $\frac{\pi}{4}$ E) $\frac{\pi}{2}$

YANITLAR / ANSWERS

1	2	3	4	5	6	7	8	9
A	B	E	D	D	B	C	E	C
10	11	12	13	14	15	16	17	
C	C	D	E	C	B	A	E	

1. $f(x) = \sqrt{x} + 3x - 4 \Rightarrow \lim\limits_{h \to 0} \dfrac{f(4+h) - f(4)}{h} = ?$

A) $\frac{13}{4}$ B) $\frac{13}{2}$ C) $\frac{9}{4}$ D) $\frac{9}{2}$ E) $\frac{3}{2}$

2. $f(x) = e^{\sin x} \Rightarrow \lim\limits_{x \to \frac{\pi}{6}} \dfrac{f(x) - f(\frac{\pi}{6})}{x - \frac{\pi}{6}} = ?$

A) $e\sqrt{3}$ B) e C) e^2

D) $\frac{\sqrt{3e}}{2}$ E) $e^2\sqrt{3}$

3. $f(x) = \dfrac{|3 - x^2| + |2x|}{|x - 1|} \Rightarrow f'(2) = ?$

A) –2 B) –1 C) 1 D) 2 E) 3

4. $f(x) - f(2x + 2) = a - 4x,$

$f'(-2) = a + 1 \Rightarrow a = ?$

A) 2 B) 3 C) 4 D) 5 E) 6

5. $f(x) = \arcsin(\sin x) \Rightarrow \left.\dfrac{dy}{dx}\right|_{x = \frac{\pi}{6}} = ?$

A) $\frac{\sqrt{3}}{4}$ B) $\frac{\sqrt{3}}{2}$ C) $\frac{1}{2}$ D) 1 E) $\sqrt{3}$

6. $f(x) = \sqrt[4]{x^3} - \dfrac{4}{\sqrt[3]{x}} \Rightarrow f'(1) = ?$

A) $-\frac{7}{12}$ B) $-\frac{1}{12}$ C) $\frac{7}{12}$ D) $\frac{13}{12}$ E) $\frac{25}{12}$

7. $f(x) = \sqrt{2 + \sqrt[3]{x}} \Rightarrow f'(8) = ?$

A) $\frac{1}{24}$ B) $\frac{1}{36}$ C) $\frac{1}{48}$ D) $\frac{1}{60}$ E) $\frac{1}{72}$

8. $x = t^4 + 4t^3 - 5$

$y = 2t^3 - 3t^2 + t \Rightarrow \left.\dfrac{dy}{dx}\right|_{t=1} = ?$

A) $\frac{1}{4}$ B) $\frac{1}{8}$ C) $\frac{1}{16}$ D) $\frac{1}{32}$ E) $\frac{1}{48}$

9. $f(x)=\left(\frac{x+1}{x}\right)^{x} \Rightarrow \mathbf{f'(1)=?}$

A) $\ln 2$ B) $\ln\frac{2}{e}$ C) $\ln 4$ D) $\ln\frac{4}{e}$ E) $\ln\frac{e}{4}$

10. $f(x)=\ln(3^{x}-2) \Rightarrow \mathbf{(f^{-1})'(0)=?}$

A) 1 B) $\frac{1}{\ln 3}$ C) $\frac{1}{2\cdot\ln 3}$ D) $\frac{1}{3\cdot\ln 3}$ E) $\frac{1}{4\cdot\ln 3}$

11. $f(x)=\sin\left[\arctan(x+\sqrt{3})\right] \Rightarrow \mathbf{f'(0)=?}$

A) $\frac{1}{2}$ B) $\frac{1}{4}$ C) $\frac{1}{8}$ D) $\frac{1}{16}$ E) $\frac{1}{32}$

12. $f(x)=(4x^{3}+3x^{4})\cdot e^{-4x}$

$$\Rightarrow \frac{1}{4}\cdot e^{8}\cdot\left.\frac{df(x)}{dx}\right|_{x=2}=?$$

A) −36 B) −44 C) −48 D) −54 E) −68

13. $f(x)=5^{\cos x}\cdot e^{\sin x} \Rightarrow \mathbf{f'\left(\frac{\pi}{2}\right)=?}$

A) −5eln5 B) −eln5 C) 0 D) eln5 E) 5eln5

14. $f(x)=(\tan x)^{x} \Rightarrow \mathbf{f'\left(\frac{\pi}{4}\right)=?}$

A) $\frac{1}{2}$ B) 1 C) $\frac{\pi}{4}$ D) $\frac{\pi}{3}$ E) $\frac{\pi}{2}$

15.

$$\left.\begin{array}{l} x=t^{2}-t \\ y=t^{3}+2t \end{array}\right\} \Rightarrow \left.\frac{d^{2}y}{dx^{2}}\right|_{t=1}=?$$

A) −8 B) −4 C) 0 D) 4 E) 8

YANITLAR / ANSWERS

1	2	3	4	5	6	7	8
A	D	C	B	D	E	C	C
9	**10**	**11**	**12**	**13**	**14**	**15**	**16**
D	D	C	B	B	E	B	

1. $f(x)=\dfrac{(x^2+1)^2}{(x^2-3)^2} \Rightarrow \mathbf{f'(2)=?}$

A) –200 B) –180 C) –160 D) –140 E) –120

2. $0<x<\dfrac{\pi}{2}$, $f(\sin 2x)=\cos 2x \Rightarrow \mathbf{f'\left(\dfrac{\sqrt{2}}{2}\right)=?}$

A) –2 B) –1 C) 0 D) 1 E) $\sqrt{2}$

3. $f(x)=\dfrac{e^x+1}{e^{-x}-1} \Rightarrow \mathbf{f'(\ln 2)=?}$

A) 1 B) 2 C) 3 D) 4 E) 5

4. $\left.\begin{array}{l} x=t^5-1 \\ y=t^3+4t^2+2t \end{array}\right\} \Rightarrow \left.\mathbf{\dfrac{dy}{dx}}\right|_{(x=31)} \mathbf{=?}$

A) $\frac{1}{8}$ B) $\frac{1}{4}$ C) $\frac{3}{8}$ D) $\frac{1}{2}$ E) $\frac{5}{8}$

5. $f(x) = ax^3 + 4x^2 - bx + 5$,

$f''(1) = 26 \Rightarrow \mathbf{a=?}$

A) 1 B) 2 C) 3 D) 4 E) 5

6. $x^2\sqrt{y}-y^2\sqrt{x}-xy+1=0 \Rightarrow \left.\mathbf{\dfrac{dy}{dx}}\right|_{(1,1)} \mathbf{=?}$

A) $\frac{1}{5}$ B) $\frac{1}{3}$ C) $\frac{1}{7}$ D) $\frac{2}{7}$ E) $\frac{3}{7}$

7. $f(x) = x^5 + x^4 - x^3 - x^2 - x + 1 \Rightarrow \mathbf{(f^{-1})'(1)=?}$

A) –2 B) –1 C) 0 D) 1 E) 2

8. $0° \le x \le 90°$,

$\cos x \cdot \dfrac{d}{dx}(2\cos x) = -\dfrac{1}{2} \Rightarrow \mathbf{x=?}$

A) 15° B) 30° C) 45° D) 60° E) 75°

9. $f(x) = 7^{\log_{49} x} + e^{2\ln x} \Rightarrow f'\left(\frac{1}{4}\right) = ?$

A) 0 B) 1 C) $\frac{3}{2}$ D) 2 E) $\frac{5}{2}$

10. $y = x^{\ln x} \Rightarrow \left.\frac{dy}{dx}\right|_{(x=e)} = ?$

A) e B) e^2 C) 1 D) 2 E) e^{-1}

11. $e^{-x} \cdot \frac{d}{dx}(x^2e^x) = ?$

A) $x^2 - 2x - 1$

B) $x^2 - 2x$

C) x^2

D) $x^2 + 2x$

E) $x^2 + 2x + 1$

12. $f'(2) = 2 \Rightarrow \lim\limits_{h \to 0} \frac{f(2+h) - f(2-2h)}{h} = ?$

A) 1 B) 2 C) 4 D) 6 E) 8

13. $f(x) = \frac{d}{dx}[\ln(2x)] \Rightarrow f'(2) = ?$

A) $-\frac{1}{8}$ B) $-\frac{1}{4}$ C) $-\frac{1}{2}$ D) $\frac{1}{2}$ E) $\frac{1}{4}$

14. $f(x) = |5 - x| + 2x - 4$

$\Rightarrow f(3) + f'(6) = ?$

A) −1 B) 0 C) 1 D) 5 E) 7

15. $y = \sin 2x \cdot \cos 2x \Rightarrow \frac{d^{10}y}{dx^{10}} = ?$

A) $-2^{20}\sin 4x$ B) $-2^{19}\cos 4x$ C) $-2^{19}\sin 4x$

D) $2^{20}\sin 4x$ E) $2^{20}\cos 4x$

YANITLAR / ANSWERS

1	2	3	4	5	6	7	8
C	B	B	C	C	A	B	A
9	10	11	12	13	14	15	
C	D	D	D	B	E	C	

1. $f(x) = x^2 \cdot e^{-x} \Rightarrow$ **max f(x) = ?**

A) $4e^{-2}$ B) e^2 C) e D) – 2 E) $2e^{-2}$

2. $f(x,y) = x \cdot \sin y + y \cdot \sin x - 5 = 0 \Rightarrow \left.\dfrac{dy}{dx}\right|_{x=\pi,\, y=0} = ?$

A) –2 B) –1 C) 0 D) $\frac{\pi}{2}$ E) π

3. $\left.\begin{aligned} y &= 3x \\ z &= y^2 + 4 \\ v &= 2z^3 + 1 \end{aligned}\right\} \Rightarrow \left.\dfrac{dv}{dx}\right|_{x=\frac{1}{3}} = ?$

A) 900 B) 450 C) 300 D) 64 E) 27

4. $f(x) = e^{-x} + e^{\frac{x}{2}} + \dfrac{1}{e^{2x}} \Rightarrow f''(0) = ?$

A) $-\frac{21}{2}$ B) $-\frac{21}{4}$ C) 0 D) $\frac{21}{2}$ E) $\frac{21}{4}$

5. $f(x) = \dfrac{x^2+2}{2-x^2} \Rightarrow f'(2) = ?$

A) –14 B) –10 C) 2 D) 4 E) 8

6. $f(x) = 3 - \dfrac{4}{x+1} \Rightarrow f'(1) = ?$

A) $\frac{1}{3}$ B) $\frac{1}{2}$ C) 1 D) 2 E) 3

7. $y = x \cdot e^x \Rightarrow \left.\dfrac{dy}{dx}\right|_{x=0} = ?$

A) 1 B) –1 C) –2 D) –3 E) – 4

8. $f(x) = \ln(e^{\sin x}) \Rightarrow f'\left(\frac{\pi}{4}\right) = ?$

A) –3 B) $-\frac{\sqrt{2}}{2}$ C) $\frac{1}{2}$ D) $\frac{\sqrt{2}}{2}$ E) 1

9. $4x^2-3xy+x^3y^2+5=0 \Rightarrow \left.\frac{dy}{dx}\right|_{(1,2)}=?$

A) –14 B) –7 C) –1 D) 7 E) 14

10. $f(x)=\sin(\cos 7x) \Rightarrow \left.\frac{dy}{dx}\right|_{x=\frac{\pi}{14}}=?$

A) –7 B) –6 C) –5 D) –3 E) 7

11. $\frac{d^2(\ln x^3)}{dx^2}=?$

A) $3x^2$ B) $\frac{3}{x^2}$ C) $3x$ D) $-\frac{3}{x^2}$ E) $-\frac{3}{x}$

12. $f(x)=\frac{3}{5-x}+\frac{x^2}{5} \Rightarrow f'(4)=?$

A) $\frac{23}{5}$ B) $\frac{28}{5}$ C) $\frac{31}{5}$ D) 6 E) 7

13. $f(x)=x\sqrt{x}-\sqrt{1-x} \Rightarrow \left.\frac{df(x)}{dx}\right|_{x=\frac{1}{2}}=?$

A) 0 B) $\frac{1}{2}$ C) $\frac{3\sqrt{2}}{2}$ D) $\sqrt{2}$ E) $\frac{5\sqrt{2}}{4}$

14. $y=\cos^3 4x \Rightarrow \left.\frac{d^2y}{dx^2}\right|_{x=\frac{\pi}{8}}=?$

A) $-\frac{1}{2}$ B) 0 C) 1 D) 4 E) 6

15. $f(x)=3^{2x^2-1} \Rightarrow f'(1)=?$

A) $4\ln 3$ B) $9\ln 3$ C) $\frac{9}{\ln 3}$ D) $12\ln 3$ E) $\frac{12}{\ln 3}$

16. $f(x)=\sin(2^x)+2^{\sin x} \Rightarrow f'(0)=?$

A) $\ln 2+\cos 1^\circ$ B) $\ln 2\cdot[1+\cos 1^\circ]$ C) $\frac{\cos 1^\circ}{\ln 2+1}$ D) $\frac{\ln 2}{\cos 1^\circ+1}$ E) 0

YANITLAR / ANSWERS

1	2	3	4	5	6	7	8
A	C	A	E	D	C	A	D
9	10	11	12	13	14	15	16
A	A	D	A	E	B	D	B

1. $y = f(x)$,

$3x^2 - 4xy + 2y - 5 = 0 \Rightarrow \mathbf{f'(1) = ?}$

A) 4 B) 5 C) 6 D) 7 E) 8

2. $f(x) = \frac{d}{dx}(\tan 2x) \;\Rightarrow\; \mathbf{f\left(\frac{\pi}{6}\right) = ?}$

A) 1 B) 2 C) 4 D) 6 E) 8

3. $y = 2u^3 - 1,\; u = \sin t,\; t = 3x + 3 \;\Rightarrow\; \left.\mathbf{\frac{dy}{dx}}\right|_{x=19} \mathbf{= ?}$

A) $\frac{27}{4}$ B) $\frac{27}{2}$ C) 27

D) $\frac{27\sqrt{3}}{4}$ E) $\frac{27\sqrt{3}}{2}$

4. $f(x) = \ln\frac{x^2+3}{x+2} \;\Rightarrow\; \mathbf{f'(1) = ?}$

A) $-\frac{1}{2}$ B) $-\frac{1}{6}$ C) 1 D) $\frac{1}{6}$ E) $\frac{1}{2}$

5. $\mathbf{\frac{2^x}{\ln 2} \cdot \frac{d^2(x \cdot 2^{-x})}{dx^2} = ?}$

A) $x \cdot \ln 2$ B) $2 - x \cdot \ln 2$ C) $-2 + x \cdot \ln 2$

D) $2 + x \cdot \ln 2$ E) $x + \ln 2$

6. $f(x, y) = x^2 - y^2 + 5 = 0 \Rightarrow \mathbf{f''(x, y) = ?}$

A) $-\frac{3}{y^3}$ B) $-\frac{y}{x}$ C) $\frac{x}{y^2}$ D) $\frac{y}{x}$ E) $\frac{y}{x^2}$

7. $x < 0,$

$f(x^2) = x^3 - 8x - 12 \Rightarrow \mathbf{f'(4) + f''(4) = ?}$

A) $-\frac{13}{8}$ B) $-\frac{5}{8}$ C) $-\frac{3}{8}$ D) $\frac{5}{8}$ E) $\frac{13}{8}$

8. $-\frac{3}{2} < x,$

$f(x) = \ln\sqrt[4]{2x+3} \Rightarrow \mathbf{(f^{-1})'(0) = ?}$

A) 1 B) 2 C) e D) e^2 E) e^4

9. $f(x) = \arctan(\cos x) \Rightarrow \mathbf{f'(\frac{\pi}{6}) = ?}$

A) $-\frac{2}{7}$ B) $-\frac{1}{7}$ C) 1 D) $\frac{1}{7}$ E) $\frac{2}{7}$

10. $f(x) = \ln x^2,$

$g(x) = 3^x \Rightarrow \mathbf{(gof)'(1) = ?}$

A) ln3 B) ln9 C) 3 D) 6 E) 9

11. $y = 5 + \cos 2t,$

$x = 3 - 4\cos 3t$

$\Rightarrow \mathbf{\left.\frac{dy}{dx}\right|_{t=\frac{\pi}{6}} = ?}$

A) $-\frac{\sqrt{3}}{6}$ B) $-\frac{\sqrt{3}}{9}$ C) $-\frac{\sqrt{3}}{12}$

D) $\frac{\sqrt{3}}{12}$ E) $\frac{\sqrt{3}}{9}$

12. $x \neq 1,\ f(x) = x^2 - 6x + 10 \Rightarrow \mathbf{(f^{-1})'(5) = ?}$

A) $-\frac{1}{2}$ B) $-\frac{1}{4}$ C) $\frac{1}{8}$ D) $\frac{1}{4}$ E) $\frac{1}{2}$

13. $f(x) = x^3 \cdot |x - 5| \Rightarrow \mathbf{f'(3) = ?}$

A) –35 B) –27 C) –9 D) 9 E) 27

YANITLAR / ANSWERS

1	2	3	4	5	6	7	8
B	E	A	D	C	D	A	B
9	**10**	**11**	**12**	**13**	**14**	**15**	**16**
A	B	C	D	E			

1. $f(x, y) = x^2 - 3xy + y^2 - 13 \Rightarrow \mathbf{f'(2, -1) = ?}$

A) $\frac{2}{3}$ B) $\frac{3}{4}$ C) $\frac{4}{5}$ D) $\frac{5}{6}$ E) $\frac{7}{8}$

2. $\left.\begin{array}{l} x = 3\sin t \\ y = 6\cos t \end{array}\right\} \Rightarrow \left.\frac{dy}{dx}\right|_{t=\frac{\pi}{4}} = ?$

A) –2 B) –1 C) 0 D) 1 E) 2

3. $f(x) = x^3 + 9x - 2 \Rightarrow \lim\limits_{x \to 2} \frac{f(x+1) - f(3)}{x^2 - 4} = ?$

A) 4 B) 5 C) 6 D) 8 E) 9

4. $f(x) = |\cos x| + 5x \Rightarrow \mathbf{f''\left(\frac{5\pi}{6}\right) = ?}$

A) $\frac{-\sqrt{3}}{2}$ B) $-\frac{1}{2}$ C) $\frac{1}{2}$ D) $\frac{\sqrt{2}}{2}$ E) $\frac{\sqrt{3}}{2}$

5. $f(x) = \sqrt[3]{x\sqrt{x^3}} \Rightarrow \mathbf{f'(64) = ?}$

A) $\frac{1}{12}$ B) $\frac{1}{6}$ C) $\frac{1}{4}$ D) $\frac{5}{12}$ E) $\frac{7}{12}$

6. $f(2x - 3) = x^3 + x + 1 \Rightarrow \mathbf{f'(1) + f(1) = ?}$

A) $\frac{11}{12}$ B) $\frac{17}{12}$ C) $\frac{23}{2}$ D) $\frac{27}{2}$ E) $\frac{35}{2}$

7. $f(x) = e^{\tan x} \Rightarrow \left.\dfrac{df^{-1}(x)}{dx}\right|_{x=1} = ?$

A) 0
B) 1
C) 2
D) 3
E) 4

8. $f(x) = \sin x - \cos x \Rightarrow \lim\limits_{x \to \frac{\pi}{4}} \dfrac{f(x) - f(\frac{\pi}{4})}{x - \frac{\pi}{4}} = ?$

A) $-\sqrt{2}$
B) $-\frac{1}{2}$
C) $-\frac{\sqrt{2}}{2}$
D) 1
E) $\sqrt{2}$

9. $f(x) = \dfrac{e^3}{2} \cdot \ln^2 x \Rightarrow f'(e) = ?$

A) $\frac{1}{e^2}$
B) $\frac{1}{e}$
C) e
D) e^2
E) e^3

10. $(xy)^2 + e^{xy} = \ln(x + y) + 1 \Rightarrow \left.\dfrac{dy}{dx}\right|_{(0,1)} = ?$

A) –2
B) –1
C) 0
D) 1
E) 2

11. $\lim\limits_{x \to \frac{\pi}{3}} \dfrac{\sin(2\cos x - 1)}{3x - \pi} = ?$

A) $-\frac{\sqrt{3}}{2}$
B) $-\frac{\sqrt{3}}{3}$
C) –1
D) $\frac{\sqrt{3}}{3}$
E) $\frac{\sqrt{3}}{2}$

12. $f(x) = \cos(\ln x^2) \Rightarrow f'(1) = ?$

A) –2
B) –1
C) 0
D) 1
E) 2

YANITLAR / ANSWERS

1	2	3	4	5	6
E	A	E	A	D	E
7	8	9	10	11	12
B	E	D	C	B	C

1. $f(x) = 1 + x + x^2 + x^3 + \ldots + x^n$

$\lim\limits_{x \to 1} \dfrac{f(x) - f(1)}{x - 1} = 120 \Rightarrow$ **$n = ?$**

A) 13 B) 14 C) 15 D) 16 E) 17

2. $f(2x) = g(x \cdot g(x))$

$g'(1) = g(1) = 1$

$\Rightarrow$ **$f'(2) = ?$**

A) 1 B) $\frac{3}{2}$ C) 2 D) $\frac{5}{2}$ E) 3

3. $f(x) = x \cdot (x-1) \cdot (x-2) \cdot \ldots \cdot (x-17)$

$\Rightarrow$ **$f'(17) = ?$**

A) 16! B) 17! C) 17·17! D) 18! E) 19!

4. $f(2x + m) = f(x) + f(m^3) + x \cdot m$

$f'(0) = 2 \Rightarrow$ **$f'(4) = ?$**

A) 12 B) 8 C) 6 D) 4 E) 3

5. $P(x) - P'(x) = 4x^2 - 2x - 2$

$\Rightarrow$ **$P'(1) = ?$**

A) 10 B) 12 C) 13 D) 14 E) 15

6. $f(x) = x^3 \cdot g(x)$

$\Rightarrow \sqrt{\dfrac{f'(2)}{g(2)} - \dfrac{g'(2) \cdot f(2)}{(g(2))^2}} = ?$

A) $\frac{\sqrt{3}}{2}$ B) $\sqrt{3}$ C) $2\sqrt{3}$ D) $\sqrt{3} + 2$ E) $\sqrt{3} - 2$

7. $f(x)=\dfrac{\sqrt{1+\sin 2x}}{\sin x+\cos x} \Rightarrow \lim\limits_{x\to\frac{\pi}{4}} \dfrac{f(x)-f(\frac{\pi}{4})}{x-\frac{\pi}{4}}=?$

A) 0 B) 1 C) 2 D) 3 E) 4

8. $f(x)=x^3+2x+4$

$\Rightarrow \lim\limits_{h\to 0}\dfrac{f(2h+2)-f(2-3h)}{h}=?$

A) 14 B) 28 C) 42 D) 56 E) 70

9. $f(x)=x^2+x+\dfrac{4}{x}$

$\Rightarrow \lim\limits_{x\to 2}\dfrac{f(x)-f(2)}{4x-8}=?$

A) 1 B) 2 C) 3 D) 4 E) 5

10. $f(x)=(x-1)^2+(x-2)^2+\ldots+(x-8)^2$

$f'(a)=0 \Rightarrow a=?$

A) $\frac{9}{4}$ B) $\frac{9}{2}$ C) 9 D) $\frac{17}{2}$ E) $\frac{17}{4}$

11. $y=\cos x+\sin x$

$\Rightarrow \left.\dfrac{d^{34}y}{dx^{34}}\right|_{x=\frac{\pi}{4}}=?$

A) $-\sqrt{2}$ B) $-\frac{\sqrt{2}}{2}$ C) 0 D) $\frac{\sqrt{2}}{2}$ E) $\sqrt{2}$

12. $\left.\begin{array}{l} y=\cos 2t \\ x=e^{3t} \end{array}\right\} \Rightarrow \dfrac{d^2y}{dx^2}=?$

A) $-\frac{2}{3}$ B) $\frac{2}{3}$ C) $\frac{4}{9}$ D) $-\frac{4}{9}$ E) $\frac{4}{3}$

13. $\left.\begin{array}{l} y=\sin 2^t \\ x=2^t \end{array}\right\} \Rightarrow \dfrac{dy}{dx}=?$

A) cost B) $2\cdot\cos 2^t$ C) $\sin 2^t$ D) sinx E) cosx

14. $f(x)=\sin\left(\dfrac{\pi}{3}\cdot\cos x\right)$

$\Rightarrow f'(\frac{\pi}{2})=?$

A) $\frac{\pi}{3}$ B) $-\frac{\pi\cdot\sqrt{3}}{3}$ C) $\frac{\pi\cdot\sqrt{3}}{3}$ D) $-\frac{\pi}{3}$ E) $-2\pi\cdot\sqrt{3}$

YANITLAR / ANSWERS

1	2	3	4	5	6	7
C	A	B	E	D	C	A
8	**9**	**10**	**11**	**12**	**13**	**14**
E	A	B	A	D	E	D

1. $\frac{d}{dx}(\cos 3x^2) = ?$

A) $-6x\sin 3x^2$ B) $-3x\sin 3x^2$ C) $-6x\sin 3x$ D) $3x\sin 3x$ E) $6x\sin 3x$

2. $f(x) = (2x-1)^2 \cdot (3x^2 - x + 1) \Rightarrow f'(1) = ?$

A) 13 B) 14 C) 15 D) 16 E) 17

3. $y = f(x)$,

$2xy + 3x - 5y - 6 = 0 \Rightarrow f'(3) = ?$

A) -3 B) -2 C) 2 D) 3 E) 5

4. $f(x) = \sin 3x \cdot \tan^2 2x \Rightarrow f'\left(\frac{\pi}{6}\right) = ?$

A) $2\sqrt{3}$ B) $4\sqrt{3}$ C) $8\sqrt{3}$ D) $16\sqrt{3}$ E) $32\sqrt{3}$

5. $f(x) = x^2 - 5x + 3$

$g(x) = \frac{f(x)}{x+3} \Rightarrow g'(2) = ?$

A) $-\frac{3}{25}$ B) $-\frac{2}{25}$ C) $-\frac{1}{25}$ D) $\frac{1}{25}$ E) $\frac{2}{25}$

6. $f(x) = \ln(x^2 + 3x) \Rightarrow f'(3) = ?$

A) $\frac{1}{3}$ B) $\frac{1}{2}$ C) $\frac{2}{3}$ D) $\frac{3}{4}$ E) $\frac{4}{5}$

7. $x^2 \cdot \dfrac{\mathbf{d^2(\ln x^2)}}{\mathbf{dx^2}} = ?$

A) $-2x$ B) $-\frac{1}{x^2}$ C) $\frac{2}{x^2}$ D) -2 E) -1

8. $f(x) = \cos\left(\frac{5\pi}{6}\cot x\right) \Rightarrow f'\left(\frac{\pi}{4}\right) = ?$

A) $\frac{5\pi}{6}$ B) $\frac{\pi}{3}$ C) $\frac{\pi}{6}$ D) $\frac{2\pi}{3}$ E) $\frac{\pi}{4}$

9. $f(x) = e^{ax} \cdot \arctan[(a+1)x]$

$f'(0) = 3 \Rightarrow$ **a = ?**

A) -3 B) -2 C) 1 D) 2 E) 3

10. $\left.\begin{array}{l} x = 3t+2 \\ y = t^2 \end{array}\right\} \Rightarrow \left.\dfrac{dy}{dx}\right|_{t=2} = ?$

A) $\frac{2}{3}$ B) 1 C) $\frac{4}{3}$ D) $\frac{5}{3}$ E) 2

11. $f : (0,1) \rightarrow R$

$f(x) = \ln|x| - |\ln x| \Rightarrow$ **f′(x) = ?**

A) $-\frac{2}{x}$ B) $-\frac{1}{x}$ C) 0 D) $\frac{1}{x}$ E) $\frac{2}{x}$

12. $f(x) = |x - x^2|$

$\Rightarrow f'\left(\frac{1}{2}\right) + f'(2) = ?$

A) 2 B) 3 C) 4 D) 5 E) 6

YANITLAR / ANSWERS

1	2	3	4	5	6
A	E	D	D	B	B
7	**8**	**9**	**10**	**11**	**12**
D	A	D	C	E	B

BÖLÜM
CHAPTER
29

TARİHSEL NOT / HISTORICAL NOTE

EMMY AMALIE NOETHER [1882 – 1935]

Soyut cebir ve kuramsal fiziğe çığır açıcı katkılarıyla bilinen bir Alman matematikçidir. Noether'in matematik çalışmaları üç döneme ayrılmıştır. İlkinde (1908-19), cebirsel değişmezler ve sayı alanları üzerine etkin katkılar sağlamıştır. Varyasyonlar hesabındaki diferansiyel sabitler üzerine çalışmaları, Noether teoremi, “modern fiziğin gelişmesine yol gösteren şimdiye kadar kanıtlanmış en önemli matematik teorileri” olarak sayılır. İkinci dönemde (1920–26), “soyut cebri değiştiren” çalışmasına başladı. Yaşamı boyunca ve hatta bugün bile Noether; Pavel Alexandrov, Hermann Weyl, ve Jean Dieudonné gibi matematikçiler tarafından tarihteki en iyi kadın matematikçi olarak gösterildi. Üçüncü döneminde (1927-1935), değişmez cebirler ve hiperkompleks sayılar hakkında çalışmalar yapmıştır ve grupların representation teorisi ve modüller ve idealler teorilerini birleştirmiştir.

She was a German mathematician known for her groundbreaking contributions to abstract algebra and theoretical physics. Noether's mathematical studies are divided into three "periods". In the first (1908-1919), she made effective contributions to algebraic constants and number spaces. Working on differential constants in the calculations of variants, Noether's theorem is counted as "the most important mathematical theories that have proven so far to lead to the development of modern physics". In the second period (1920-1926), she began her "abstract abstraction" study. Noether throughout his life and even today; Pavel Alexandrov, Hermann Weyl, and Jean Dieudonné as the best female mathematician in history. In the third period (1927-1935), she published major works on invariant algebra, hypercomplex numbers, groups representation theory, she also had united modular and ideal theories.

TÜREVİN UYGULAMALARI
APPLICATIONS of DERIVATIVE

Bu bölüm, 51 test sorusu içermektedir.
This chapter includes 51 test questions.

BÖLÜM / CHAPTER 29

TÜREVİN UYGULAMALARI / APPLICATIONS of DERIVATIVE

- Türevin Uygulamaları / Applications of Derivative .... 573 – 584

BÖLÜM

29

CHAPTER

TÜREVİN UYGULAMALARI
APPLICATIONS of DERIVATIVE

TÜREVİN UYGULAMALARI / APPLICATIONS of DERIVATIVE

L'HOSPITAL KURALI / L'HOSPITAL'S RULE

▼ $\lim\limits_{x\to x_0}\frac{f(x)}{g(x)}$ limiti bulunurken $\frac{0}{0}$ veya $\frac{\infty}{\infty}$ şeklinde belirsizlik oluşuyorsa limit, pay ve paydanın ayrı ayrı türevleri alınarak bulunabilir.

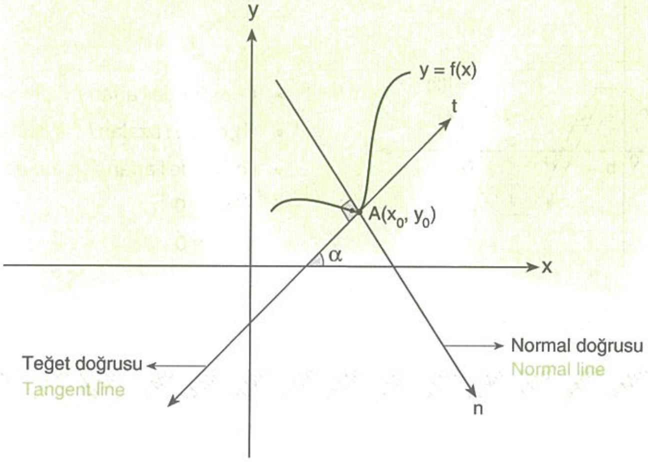
▼ If $\lim\limits_{x\to x_0}\frac{f(x)}{g(x)}$ is equal to $\frac{0}{0}$ or $\frac{\infty}{\infty}$ then the limit is found by taking the derivatives of the numerator and denominator seperately.

$$\lim_{x\to x_0}\frac{f(x)}{g(x)}=\frac{0}{0}\ \text{veya/or}\ \lim_{x\to x_0}\frac{f(x)}{g(x)}=\frac{\infty}{\infty}\Rightarrow\lim_{x\to x_0}\frac{f(x)}{g(x)}=\lim_{x\to x_0}\frac{f'(x)}{g'(x)}=\frac{f'(x_0)}{g'(x_0)}$$

▼ Tekrar belirsizlik devam ediyorsa kural yeniden uygulanır.

▼ If the indeterminate forms continue, then apply the same rule one more time.

TÜREVİN GEOMETRİK ANLAMI / GEOMETRIC MEANING of DERIVATIVE



▼ y = f(x) fonksiyonuna A(x_0, y_0) noktasından çizilen teğetin eğimi, ***fonksiyonun x_0 noktasındaki türevi***ne eşittir.

The function y = f(x) has a tangent at A (x_0, y_0) whose slope is the function's derivative in x_0.

$$m_t=\tan\alpha=f'(x_0)$$

▼ **Teğet doğrusunun denklemi / Equation of tangent line:**

A(x_0, y_0)

$y-y_0=f'(x_0)\cdot(x-x_0)$

▼ **Normal doğrusunun denklemi / Equation of normal line:**

A(x_0, y_0)

$y-y_0=\frac{-1}{f'(x_0)}\cdot(x-x_0)$

MAKSİMUM veya MİNİMUM DEĞERLERİN BULUNMASI / FINDING MAXIMUM or MINIMUM VALUES

▼ Bir fonksiyonun 1. türevi alınıp sonra eşitlenirse, bulunan x değeri, ***fonksiyonun maksimum veya minimum noktasının apsisi*** olabilir. Bulunan x değeri ana fonksiyonda yazılırsa, fonksiyonun maksimum veya minimum değeri (ordinatı) elde edilir.

▼ If we take first derivative of a function and equal it to zero, founded x is maximum and minimum point of the function. If finding x is written to the function, we obtain maximum or minimum value of function.

▼ Sorularda hangi ifadenin maksimum veya minimum değeri isteniyorsa, o ifade verilenler kullanılarak tek değişkene bağlı yazılır. Daha sonra türevi alınarak sıfıra eşitlenir.

▼ If the maximum or minimum value of any expression is intended in the questions, the expression is written in single variable form and then the derivative is taken and equal to zero.

ARTAN ve AZALAN FONKSİYONLAR / INCREASING and DECREASING FUNCTIONS

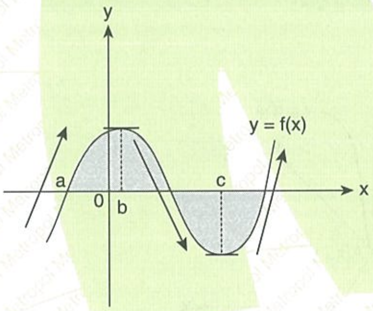
▼ f: R → R, $\forall x_1, x_2 \in R$ için/for

i. $x_1 < x_2$ iken $f(x_1) < f(x_2)$ ⇒ f, artan fonksiyon / increasing function

ii. $x_1 < x_2$ iken $f(x_1) > f(x_2)$ ⇒ f, azalan fonksiyon / decreasing function

▼ f, artan fonksiyon / increasing function ⇒ $f'(x) > 0$

f, azalan fonksiyon / decreasing function ⇒ $f'(x) < 0$



- $(-\infty, b)$ de f artan / f is incrensing in $(-\infty, b)$ → $f'(x) > 0$
- (b, c) de f azalan / f is decreasing in (b, c) → $f'(x) < 0$
- (c, ∞) de f artan / f is increasing in (c, ∞) → $f'(x) > 0$
- $f'(b) = 0$
- $f'(c) = 0$

Önemli / Important

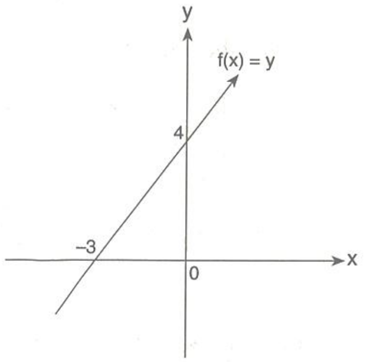
• f: R → R, $\forall x_1, x_2 \in R$
$x_1 < x_2$ iken / as $f(x_1) = f(x_2)$
⇒ ***f fonksiyonu, sabit fonksiyondur.***
The function f is a constant function.

FONKSİYONLARIN YEREL MAKSİMUM ve YEREL MİNİMUM NOKTALARI
LOCAL MAXIMUM and LOCAL MINIMUM POINTS of FUNCTIONS

▼ Bir fonksiyonun 1. türevini sıfıra eşit yapan noktalarda türev fonksiyonunun işareti değişiyorsa, bu noktalar ***fonksiyonun yerel maksimum veya yerel minimum noktaları***dır.

▼ If the sign of the function of first derivative of a function changes in the points in which the derivative equals to zero, these points are local maximum and local minimum points of the function.

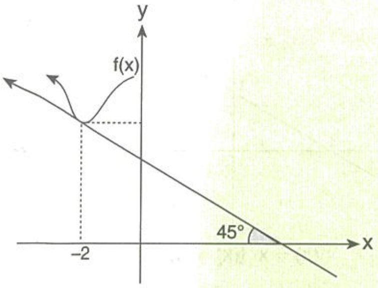
1.



$\Rightarrow$ **$f'(0) = ?$**

A) –1 B) 0 C) 4 D) $\frac{4}{3}$ E) 2

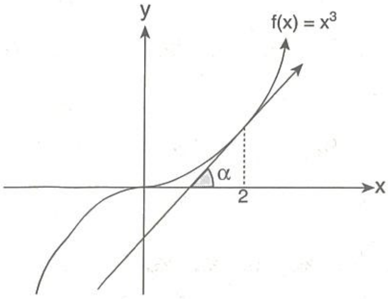
2.



$\Rightarrow$ **$f'(-2) = ?$**

A) $-\frac{\sqrt{3}}{2}$ B) $-\frac{\sqrt{2}}{2}$ C) –1 D) 1 E) $\frac{\sqrt{2}}{2}$

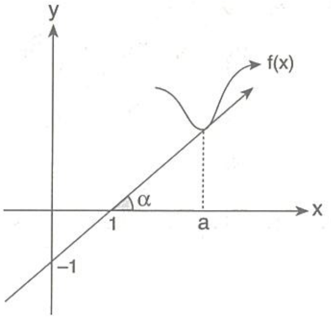
3.



$\Rightarrow$ **$\tan\alpha = ?$**

A) $\frac{1}{4}$ B) $\frac{1}{2}$ C) 1 D) 6 E) 12

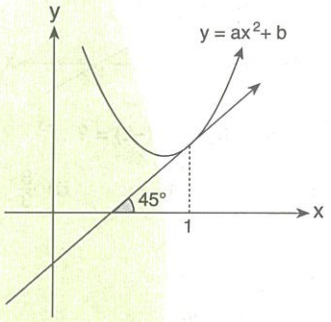
4.



$\Rightarrow$ **$f'(a) = ?$**

A) –1 B) $-\frac{1}{2}$ C) $\frac{1}{2}$ D) 1 E) $\frac{3}{2}$

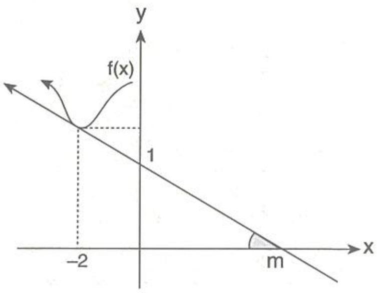
5.


y
y = ax² + b
45°
1
x

$\Rightarrow$ **$a = ?$**

A) $\frac{1}{4}$ B) $\frac{1}{3}$ C) $\frac{1}{2}$ D)1 E) 2

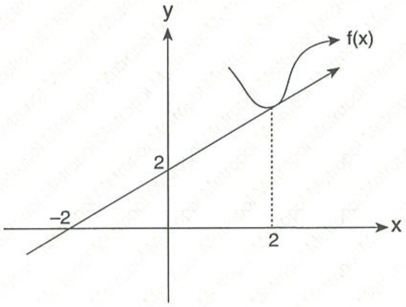
6.



$f'(-2) = -\frac{1}{3} \Rightarrow$ **$m = ?$**

A) 1 B) 2 C) 3 D) 4 E) 5

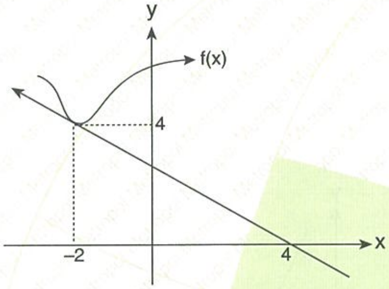
7.



$h(x) = x^2 \cdot f(x) \Rightarrow \mathbf{h'(2) = ?}$

A) 16 B) 18 C) 20 D) 22 E) 24

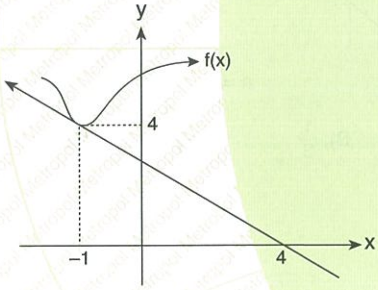
8.



$h(x) = x \cdot f(x) \Rightarrow \mathbf{h'(-2) = ?}$

A) $\frac{2}{3}$ B) $\frac{4}{3}$ C) 1 D) $\frac{8}{3}$ E) $\frac{16}{3}$

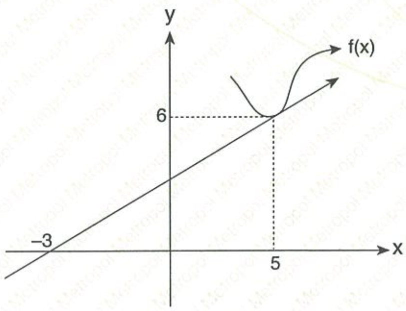
9.



$\Rightarrow \lim\limits_{x \to -1} \frac{f(x) - f(-1)}{x + 1} = ?$

A) $-\frac{4}{5}$ B) $-\frac{4}{3}$ C) –1 D) $\frac{5}{4}$ E) $\frac{4}{5}$

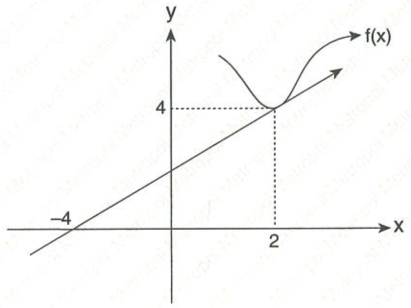
10.



$h(x) = (x + 3) \cdot f(x) \Rightarrow \mathbf{h'(5) = ?}$

A) –8 B) –4 C) –1 D) 8 E) 12

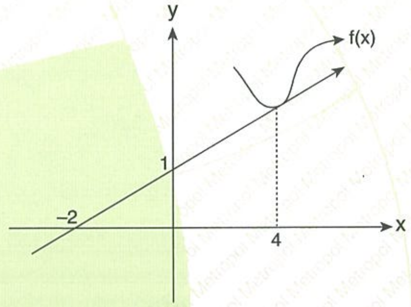
11.



$h(x) = \frac{f(x)}{x + 2} \Rightarrow \mathbf{h'(2) = ?}$

A) $-\frac{1}{12}$ B) $-\frac{1}{6}$ C) $-\frac{1}{3}$ D) $\frac{1}{3}$ E) $\frac{1}{6}$

12.

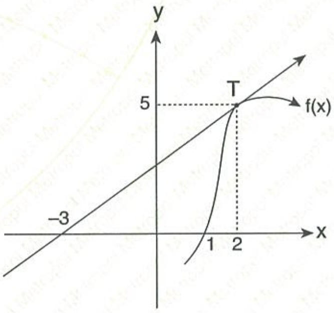


$h(x) = x \cdot f(x) \Rightarrow \mathbf{h'(4) = ?}$

A) 3 B) 4 C) 5 D) 6 E) 7

13. $g(x) = \frac{1}{4} \cdot x^2 \cdot f(x)$

$\Rightarrow \mathbf{g'(2) = ?}$

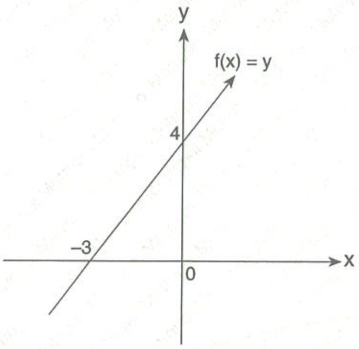


A) 3 B) 4 C) 5 D) 6 E) 8

YANITLAR / ANSWERS

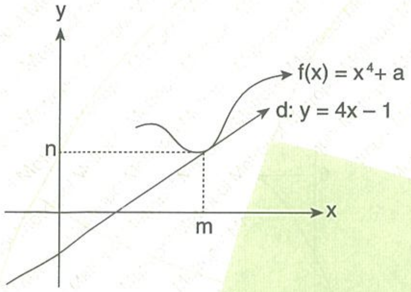
1	2	3	4	5	6	7
D	C	E	D	C	C	C
8	**9**	**10**	**11**	**12**	**13**	**14**
E	A	E	A	C	D	

1. $f'(1) + f(1) = ?$



A) 6 B) 8 C) 10 D) 11 E) 16

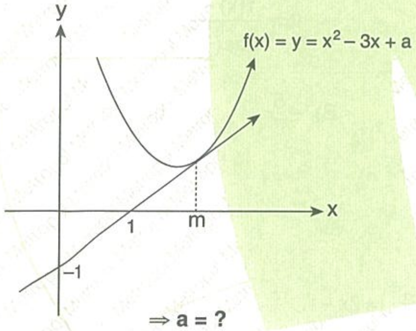
2.


y
f(x) = x⁴ + a
d: y = 4x – 1
n
m
x

$\Rightarrow n = ?$

A) 3 B) 4 C) 5 D) 6 E) 7

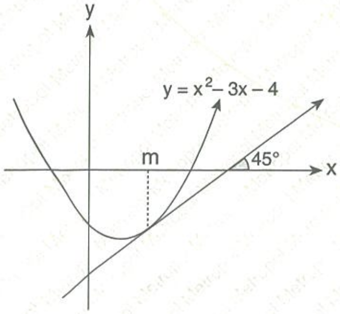
3.


y
f(x) = y = x² – 3x + a
1
m
–1
x

$\Rightarrow a = ?$

A) –2 B) 0 C) 2 D) 3 E) 5

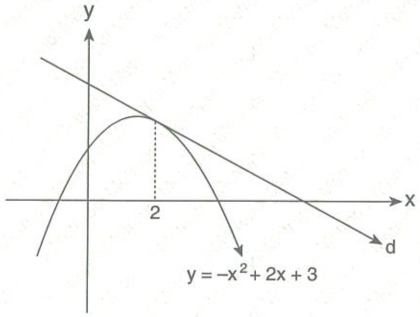
4.


y
y = x² – 3x – 4
m
45°
x

$\Rightarrow m = ?$

A) $\frac{1}{2}$ B) 1 C) $\frac{3}{2}$ D) 2 E) $\frac{5}{2}$

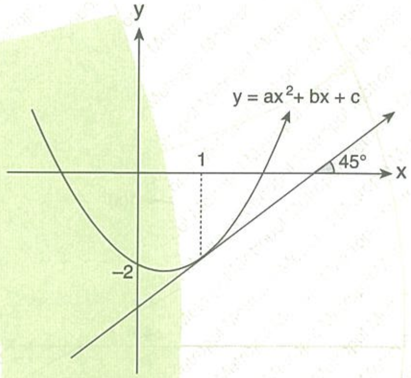
5.


y
2
x
d
y = –x² + 2x + 3

$\Rightarrow d : y = ?$

A) 2x – 3 B) 2x + 7 C) –2x + 3 D) –2x – 7 E) –2x + 7

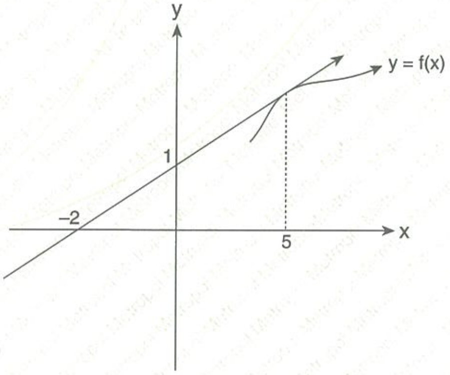
6.


y
y = ax² + bx + c
1
45°
x
–2

$\Rightarrow 2a + b + c = ?$

A) –1 B) 0 C) 1 D) 2 E) 3

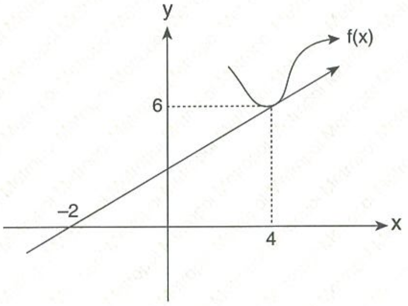
7.



$h(x) = \frac{f(3x-1)}{x^2} \Rightarrow h'(2) = ?$

A) $-\frac{1}{4}$ B) $-\frac{1}{3}$ C) $-\frac{1}{2}$ D) –1 E) 0

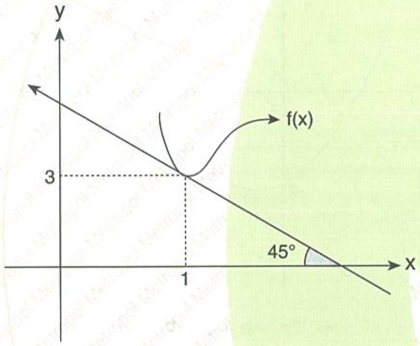
8.



$$\Rightarrow \lim_{x \to 4} \frac{f^2(x) - 36}{x^2 - 16} = ?$$

A) $\frac{1}{2}$ B) 1 C) $\frac{3}{2}$ D) 2 E) $\frac{5}{2}$

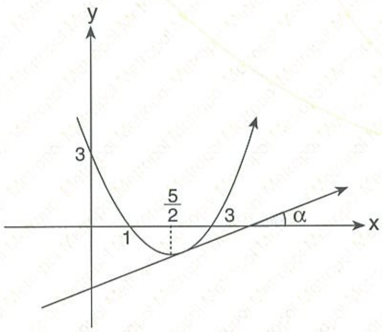
9.



$g(x) = 2x \cdot f(\frac{x}{3}) \Rightarrow g'(3) = ?$

A) 4 B) 6 C) 8 D) 9 E) 12

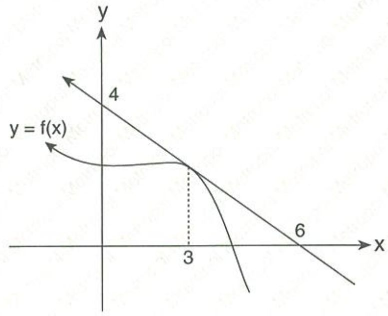
10.


y
3
$\frac{5}{2}$
3
α
1
x

$\Rightarrow \alpha = ?$

A) 0° B) 30° C) 45° D) 120° E) 135°

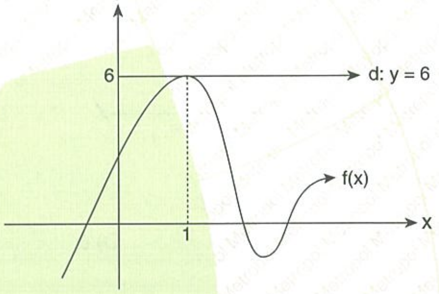
11.



$g(x) = x^2 - 1 \Rightarrow (fog)'(2) = ?$

A) $-\frac{8}{3}$ B) $-\frac{4}{3}$ C) $\frac{4}{3}$ D) $\frac{8}{3}$ E) 3

12.



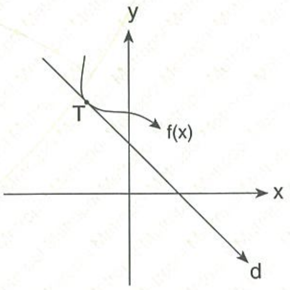
$$\Rightarrow \lim_{x \to 1} \frac{f(x) - 6x^2}{\cos(\frac{3\pi}{2}x)} = ?$$

A) $-\frac{8}{\pi}$ B) $-\frac{5}{\pi}$ C) -6π D) -4π E) $-\pi$

13. $f\left(\frac{2}{3x+1}\right) = 2x - 1,$

$d: y = mx + n$

$\Rightarrow$ **m = ?**



A) $-\frac{5}{3}$ B) $-\frac{4}{3}$ C) $-\frac{3}{4}$ D) $-\frac{3}{5}$ E) $-\frac{2}{3}$

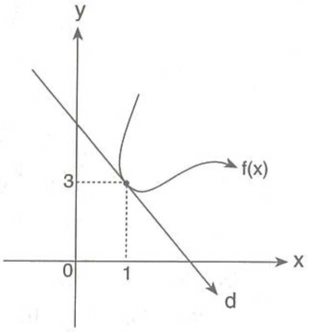
YANITLAR / ANSWERS

1	2	3	4	5	6	7
B	A	D	D	E	A	C
8	**9**	**10**	**11**	**12**	**13**	**14**
C	A	C	A	A	B	

1. $g(x) = x^3 \cdot f(x)$,

$g'(1) = 6$

$\Rightarrow$ **m_d = ?**



A) –3

B) –2

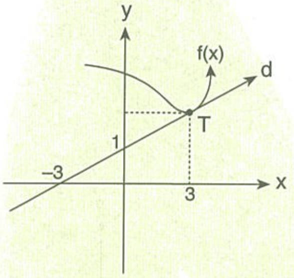
C) –1

D) 1

E) 2

2. $f(x) = x \cdot h(x)$

$\Rightarrow$ **$h'(3)$ = ?**

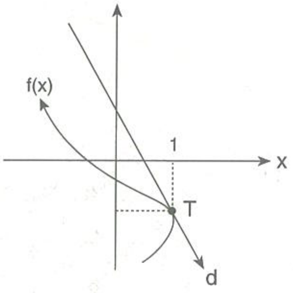


A) $-\frac{1}{9}$ B) $-\frac{1}{3}$ C) 0 D) $\frac{1}{3}$ E) $\frac{1}{9}$

3. $f(x) = x^3 - 3x^2 - 4x + 1$,

d: $y = mx + n$

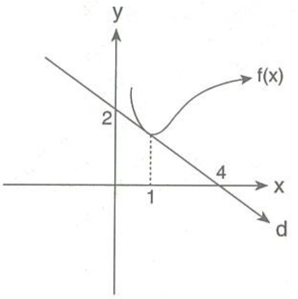
$\Rightarrow$ **$m + 3n$ = ?**



A) –2 B) –1 C) 1 D) 2 E) 3

4. $g(x) = \dfrac{x^2 \cdot f(x)}{x+1}$

$\Rightarrow$ **$g'(1)$ = ?**

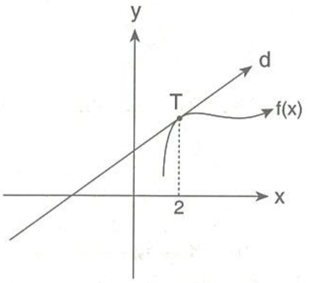


A) $\frac{3}{8}$ B) $\frac{5}{8}$ C) $\frac{7}{8}$ D) 1 E) $\frac{9}{8}$

5. d: $y = 2x + 1$,

$g(x) = x^2 \cdot f(x)$

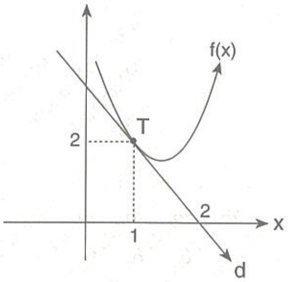
$\Rightarrow$ **$g'(2)$ = ?**



A) 14 B) 16 C) 20 D) 26 E) 28

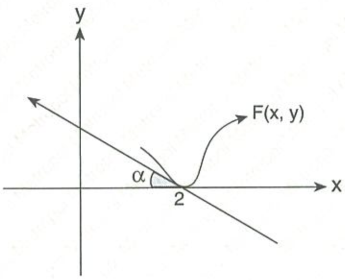
6. $x^2 \cdot g(x) = x^2 + f(x)$

$\Rightarrow$ **$g'(1)$ = ?**



A) –6 B) –2 C) 0 D) 2 E) 4

7.



$F(x, y) = x^2 + y^2 + \sin(xy) - 4 = 0$

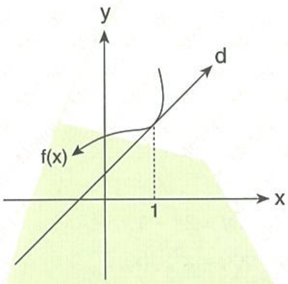
$\Rightarrow$ **$\tan\alpha$ = ?**

A) –2 B) –1 C) 0 D) 1 E) 2

8. $f(x) = x^3 - ax^2 - x + 1,$

d: $y = 4x + b$

$\Rightarrow$ **$a + b$ = ?**

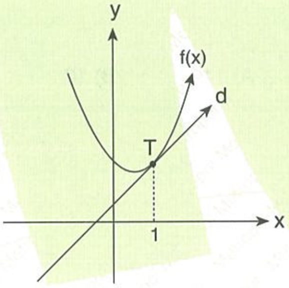


A) –3 B) –2 C) 0 D) 1 E) 2

9. $f(x) = 2x^2 + 1,$

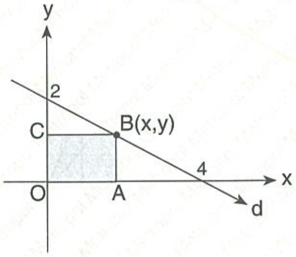
d: $y = mx + n$

$\Rightarrow$ **$m + n$ = ?**



A) 5 B) 4 C) 3 D) 2 E) 1

10.

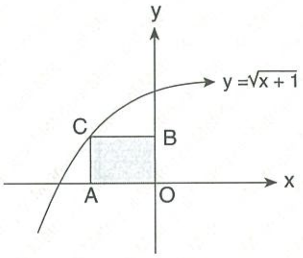


OABC bir dikdörtgendir. OABC is a rectangle.

$\Rightarrow$ **max(A(OABC)) = ? br^2 (u^2)**

A) 1 B) $\frac{3}{2}$ C) 2 D) $\frac{5}{2}$ E) 3

11.

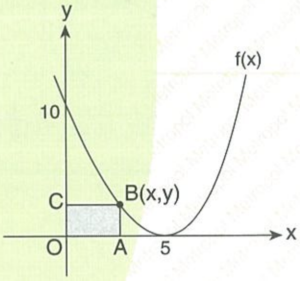


OABC bir dikdörtgendir. OABC is a rectangle.

$\Rightarrow$ **max(A(AOBC)) = ? br^2 (u^2)**

A) $\frac{\sqrt{3}}{9}$ B) $\frac{2\sqrt{3}}{9}$ C) $\frac{\sqrt{3}}{3}$ D) $\frac{2\sqrt{3}}{3}$ E) $\frac{4\sqrt{3}}{3}$

12.



OABC bir dikdörtgendir. OABC is a rectangle.

$\Rightarrow$ **max(A(OABC)) = ? br^2 (u^2)**

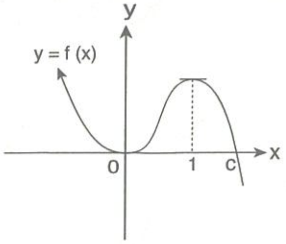
A) $\frac{200}{27}$ B) $\frac{100}{27}$ C) $\frac{100}{9}$

D) $\frac{200}{9}$ E) $\frac{50}{3}$

YANITLAR / ANSWERS

1	2	3	4	5	6
A	A	B	C	E	A
7	**8**	**9**	**10**	**11**	**12**
A	A	C	C	B	A

1.



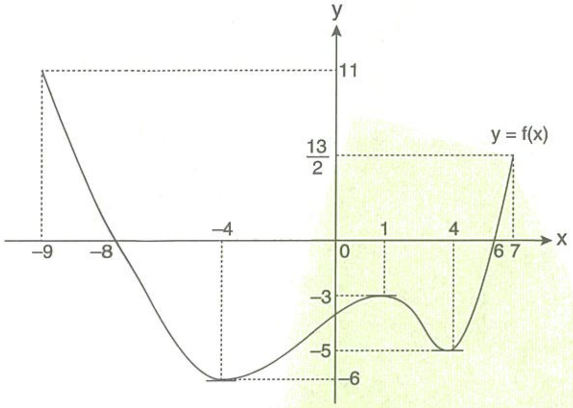
$y = f(x)$

$y = -x^3 + ax^2 + b$

$\Rightarrow$ **a = ?**

A) $\frac{3}{2}$ B) $\frac{5}{3}$ C) 2 D) $\frac{5}{2}$ E) 3

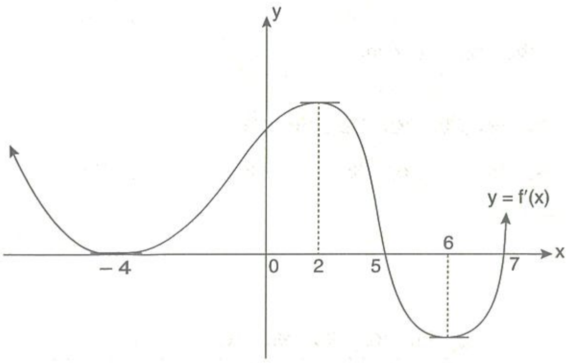
2.



$\Rightarrow$ **max(f(x)) + min(f(x)) = ?**

A) 1 B) 2 C) 3 D) 4 E) 5

3.



$\Rightarrow$ **min(f(x)) için (for) x = ?**

A) – 4 B) 2 C) 5 D) 6 E) 7

4. $f(x) = \frac{x^3}{3} - 2x^2 + 3x$

$\Rightarrow$ **f′(x) = 0 için (for) $\sum x$ = ?**

A) 4 B) 6 C) 8 D) 10 E) 12

5. $g(x) = \frac{2}{3}x^3 + 2x^2 - 1$

$\Rightarrow$ **g′(x) < 0 için (for) ÇK(SS) = ?**

A) (– ∞, – 2) B) (0, ∞) C) (0, 2) D) (– 2, 0) E) (– 2, 2)

6. $h(x) = -x^2 + 6x + 4$

$\Rightarrow$ **h′(x) > 0 için (for) ÇK(SS) = ?**

A) (– ∞, 6) B) (– ∞, 3) C) (3, 6) D) (3, ∞) E) (– ∞, – 3)

7. $f(x) = \frac{x^3}{3} - 2x^2 - 5x + 6$

$\Rightarrow$ **yerel (local) max(f(x)) = ?**

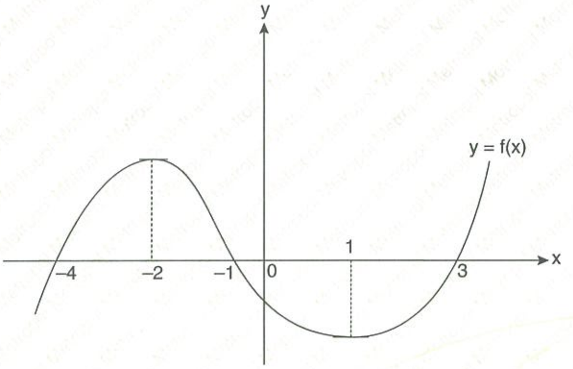
A) $\frac{29}{3}$ B) $\frac{26}{3}$ C) $\frac{25}{3}$ D) 7 E) 5

8. $f(x) = x^3 - 3x^2 + m$,

yerel (local) min(f(x)) = – 3 $\Rightarrow$ **m = ?**

A) 1 B) 2 C) 0 D) – 2 E) – 1

9.



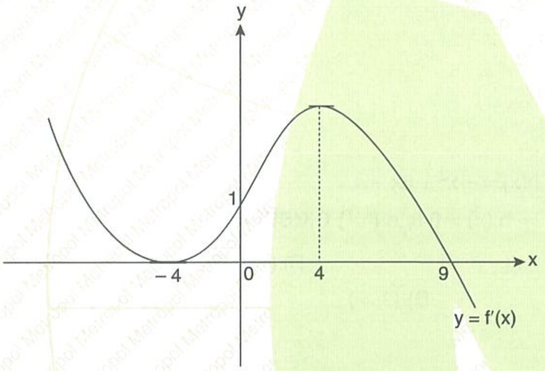
Grafiğe göre aşağıdakilerden hangisi yanlıştır?

Which of the following is false according to the graph?

A) $f'(-2) = 0$ B) $f'(1) = 0$ C) $f'(-3) > 0$

D) $f'(-1) < 0$ E) $f'\left(\frac{1}{2}\right) > f'\left(-\frac{5}{2}\right)$

10.



Grafiğe göre aşağıdakilerden hangisi yanlıştır?

Which of the following is false according to the graph?

A) (–4, 9) aralığında f(x) artan / f(x) is incrasing in the interval (–4, 9)

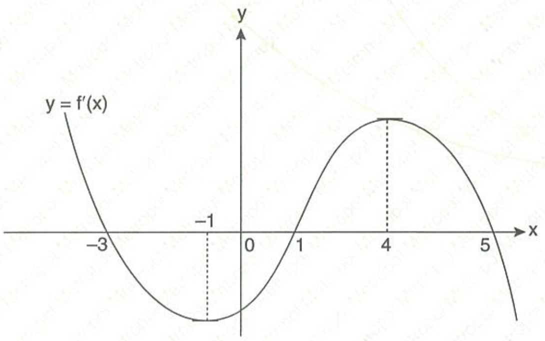
B) (– ∞, – 4) aralığında f(x) azalan / f(x) is decreasing the interval (9, ∞)

C) (9, ∞) aralığında f(x) azalan / f(x) is decreasing in the interval (9, ∞)

D) $f'(-4) = 0$

E) $f'(9) = 0$

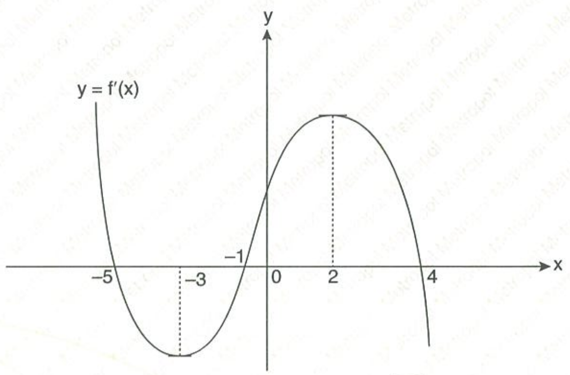
11.



⇒ **yerel (local) min (f(x)) için (for) x = ?**

A) –3 B) – 1 C) 1 D) 4 E) 5

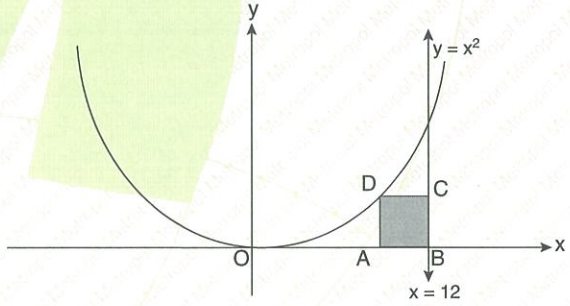
12.



⇒ **yerel (local) max(f(x)) için (for) $\sum x$ = ?**

A) 0 B) – 1 C) – 2 D) – 3 E) – 4

13.



ABCD bir dikdörtgendir.

ABCD is a rectangle.

⇒ **max(A(ABCD)) = ? br² (u²)**

A) 144 B) 196 C) 256 D) 324 E) 576

YANITLAR / ANSWERS

1	2	3	4	5	6	7
A	E	E	A	D	B	B
8	9	10	11	12	13	14
A	E	B	C	B	C	

BÖLÜM CHAPTER 30

TARİHSEL NOT / HISTORICAL NOTE

DAVID HILBERT [1862 - 1943]

Alman matematikçi. Geometriyi bir dizi aksiyoma indirgeyen ve matematiğin biçimsel temellerinin oluşturulmasına önemli katkıda bulunan Alman matematikçi David HİLBERT integralli denklemlere ilişkin çalışmalarıyla fonksiyonel analizin 20. yüzyıldaki gelişmesine öncülük etmiştir.

German mathematician. The German mathematician David HİLBERT, who reduced geometry to a number of axioms and contributed significantly to the formal foundations of mathematics, pioneered the development of functional analysis in the 20th century through his work on integral equations.

BELİRSİZ İNTEGRAL / INDEFINITE INTEGRAL

Bu bölüm, 149 test sorusu içermektedir.

This chapter includes 149 test questions.

BÖLÜM / CHAPTER 30

BELİRSİZ İNTEGRAL / INDEFINITE INTEGRAL

- Belirsiz İntegral / Indefinite Integral . . . . 585 – 610

Metropol

BÖLÜM
30
CHAPTER

BELİRSİZ İNTEGRAL
INDEFINITE INTEGRAL

BELİRSİZ İNTEGRAL / INDEFINITE INTEGRAL

Tanım: F(x) fonksiyonunun türevi f(x) ve diferansiyeli d(F(x)) = f(x)·dx olmak üzere, F(x) + C yi bulma işlemine ***belirsiz integral alma işlemi*** denir.

Definition: Let the derivative of F(x) be f(x) and the differantion of it be dF(x) = f(x)·dx. The operation of finding F(x) + C is called *indefinite integration.*

$$F'(x) = f(x) \Leftrightarrow \int f(x)dx = F(x) + C, \quad C \in R$$

Özellik / Property

★ $\frac{d}{dx}\int f(x)dx = \left(\int f(x)dx\right)' = \mathbf{f(x)} \quad \left[\left(\int f(x)dx\right)' = (F(x)+c)' = f(x)\right]$

Özellik / Property

★ $\int d(f(x)) = \mathbf{f(x)+C}, \ (C \in R)$

Özellik / Property

★ $\int k \cdot f(x)dx = \mathbf{k} \cdot \int \mathbf{f(x)dx}, \quad (k \in R)$

Özellik / Property

★ $\int (f(x) \pm g(x))dx = \left(\int \mathbf{f(x)dx}\right) \pm \left(\int \mathbf{g(x)dx}\right)$

TEMEL İNTEGRAL FORMÜLLERİ / BASIC INTEGRATION FORMULAS

- $\int a dx = a\int 1\cdot dx = \mathbf{a\cdot x + C}, \quad (C \in R)$
- $\int x^n dx = \frac{x^{n+1}}{n+1} + C, \quad (C \in R)$
- $\int \frac{1}{x} dx = \ln|x| + C, \quad (C \in R)$
- $\int \frac{dx}{ax+b} = \frac{1}{a}\cdot\ln|ax+b| + C, \quad (C \in R)$
- $\int e^x dx = e^x + C, \quad (C \in R)$
- $\int e^{ax+b} dx = \frac{1}{a}\cdot e^{ax+b}, \quad (C \in R)$
- $\int a^x dx = \frac{a^x}{\ln a} + C = a^x\cdot\log_a e + C, \quad (C \in R)$
- $\int a^{mx+n} dx = \frac{1}{m\cdot\ln a}\cdot a^{mx+n} + C, \quad (C \in R)$
- $\int \sin x\, dx = -\cos x + C, \quad (C \in R)$
- $\int \sin(ax+b) dx = \frac{-\cos(ax+b)}{a} + C, \quad (C \in R)$
- $\int \cos x\, dx = \sin x + C, \quad (C \in R)$
- $\int \cos(ax+b) dx = \frac{\sin(ax+b)}{a} + C, \quad (C \in R)$
- $\int \frac{1}{\sin^2 x} dx = \int \operatorname{cosec}^2 x\, dx = \int (1+\cot^2 x) dx = -\cot x + C, \quad (C \in R)$
- $\int (1+\cot^2(ax+b)) dx = \frac{-\cot(ax+b)}{a} + C, \quad (C \in R)$
- $\int \frac{1}{\cos^2 x} dx = \int \sec^2 x\, dx = \int (1+\tan^2 x) dx = \tan x + C, \quad (C \in R)$
- $\int (1+\tan^2(ax+b)) dx = \frac{\tan(ax+b)}{a} + C, \quad (C \in R)$
- $\int \frac{1}{1+x^2} dx = \arctan x + C = -\operatorname{arccot} x + C, \quad (C \in R)$
- $\int \frac{1}{a^2+x^2} dx = \frac{1}{a}\cdot\arctan\left(\frac{x}{a}\right) + C = -\frac{1}{a}\cdot\operatorname{arccot}\left(\frac{x}{a}\right) + C, \quad (C \in R)$
- $\int \frac{1}{\sqrt{1-x^2}} dx = \arcsin x + C = -\arccos x + C, \quad (C \in R)$
- $\int \frac{1}{\sqrt{a^2-x^2}} dx = \arcsin\frac{x}{a} + C = -\arccos\frac{x}{a} + C, \quad (C \in R)$
- $\int \sec x\, dx = \ln|\sec x + \tan x| + C, \quad (C \in R)$
- $\int \operatorname{cosec} x\, dx = \ln|\operatorname{cosec} x - \cot x| + C, \quad (C \in R)$

İNTEGRAL ALMA YÖNTEMLERİ / INTEGRATION METHODS

DEĞİŞKEN DEĞİŞTİRME YÖNTEMİ / VARIABLE CHANGING METHOD

▼ $\int f(x)dx$ integrali, temel integral alma formülleri ile hesaplanamıyorsa, değişken değiştirme yöntemi kullanılır.

▼ If the integral $\int f(x)dx$ could not be solved using basic integration formulas, the variable changing method is used.

▼ $\int f(x)dx$ integralinde, x = u(t) şeklinde bir değişken değiştirmesi yapıldığında, $x = u(t) \Rightarrow dx = u'(t)dt$ olur.

▼ When conversion x = u(t) is applied in $\int f(x)dx$, we have $x = u(t) \Rightarrow dx = u'(t)dt$

▼ $\int f(x)dx = \int f(u(t)) \cdot u'(t)dt$ integrali hesaplandıktan sonra sonuç tekrar x değişkeni cinsinden ifade edilir.

▼ $\int f(x)dx = \int f(u(t)) \cdot u'(t)dt$ after calculating this integral, we again convert the result to find the answer in terms of x.

BASİT KESİRLERE AYIRMA YÖNTEMİ / FACTORIZATION into SIMPLE FRACTIONS

▼ $\int \frac{P(x)}{Q(x)}dx$ şeklinde verilen integral hesaplanırken, aşağıdaki yöntemler kullanılır.

To calculate the integration $\int \frac{P(x)}{Q(x)}dx$, we can use the following methods.

i. **der (P(x)) ≥ der (Q(x)) ise,** polinom bölmesi yapılarak integral alınır.

If der (P(x)) ≥ der (Q(x)), we use polynomial division for integration.

ii. **der (P(x)) < der (Q(x)) ise,**

If der [P(x)] < der [Q(x)],

- Q(x) çarpanlara ayrılıyorsa ifade basit kesirlere ayrılarak integral alınır.

  If Q(x) is factorisable, we use the method of dividing into simple fractions.

- Q(x) çarpanlarına ayrılmıyorsa Q(x) tam kare ifadeye tamamlanarak integral alınır.

  If Q(x) is not factorisable, we use the perfect square expression for integration.

Önemli / Important

- $\int f^n(x) \cdot f'(x)dx = \frac{(f(x))^{n+1}}{n+1} + C, \quad C \in R$
- $\int \frac{f'(x)dx}{f(x)} = \ln|f(x)| + C, \quad C \in R$

Önemli / Important

- $a \neq 0$

$$\int \frac{dx}{x^2 - a^2} = \frac{1}{2a} \cdot \ln\left|\frac{x-a}{x+a}\right| + C, \quad C \in R$$

KISMİ İNTEGRASYON YÖNTEMİ / INTEGRATION BY PARTS

▼ u ile v, x değişkenine bağlı birer fonksiyon olmak üzere, u ile dv çarpımının integrali $\int u \cdot dv = u \cdot v - \int v \cdot du$ şeklinde hesaplanır.

Let u and v be a function x. The integration of u and dv is $\int u \cdot dv = u \cdot v - \int v \cdot du$.

$\int \sin^a x\,dx$, $\int \cos^a x\,dx$, $\int \sin^a x \cdot \cos^b x\,dx$'in İNTEGRALLERİ / INTEGRALS of $\int \sin^a x\,dx$, $\int \cos^a x\,dx$, $\int \sin^a x \cdot \cos^b x\,dx$

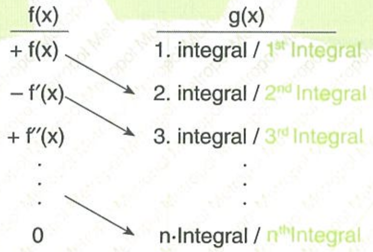
İntegral	Kuvvetler / Powers	Özdeşlik / Identity
$\int \sin^a x dx$	a çift sayı a is even number	$\cos 2x = 1 - 2 \cdot \sin^2 x$
$\int \sin^a x dx$	a tek sayı a is odd number	$\sin^2 x = 1 - \cos^2 x$ ve / and $\sin^a x = \sin^{(a-1)} x \cdot \sin x$
$\int \cos^a x dx$	a çift sayı a is even number	$\cos 2x = 2 \cdot \cos^2 x - 1$
$\int \cos^a x dx$	a tek sayı a is odd number	$\cos^2 x = 1 - \sin^2 x$ ve / and $\cos^a x = \cos^{(a-1)} x \cdot \cos x$
$\int \sin^a x \cdot \cos^b x dx$	a ve b tek sayı (a < b) a and b are odd numbers (a < b)	$\sin^a x = \sin^{(a-1)} x \cdot \sin x$ ve / and $\sin^2 x = 1 - \cos^2 x$
$\int \sin^a x \cdot \cos^b x dx$	a ve b çift sayı a and b are even numbers	$\sin 2x = 2\sin x \cdot \cos x$, $\cos 2x = 1 - 2 \cdot \sin^2 x$ $\cos 2x = 2 \cdot \cos^2 x - 1$
$\int \sin^a x \cdot \cos^b x dx$	a ve b'den biri tek (a tek ise) One of these is odd (a is odd)	$\sin^a x = \sin^{(a-1)} x \cdot \sin x$ ve / and $\sin^2 x = 1 - \cos^2 x$

Önemli / Important

- **L**ogaritma, **A**rc, **P**olinom, **T**rigonometrik, **Ü**stel fonksiyonlarının herhangi ikisinin çarpımı biçimindeki fonksiyonların integrali alınırken verilen sıra önceliği olan fonksiyona **u**, kalan kısma **dv** denilir.
- In the integration consists of Logarithm, Arc, Polynomial, Trigonometric, or Exponential functions; name the priorly ordered funcation **u**, and the remaining part **dv**.

Önemli / Important

- $\int f(x) \cdot g(x) dx$ integralindeki çarpanlardan f(x) in sonlu sayıdaki türevi sıfır ve g(x) in integrali de kolay alınabiliyorsa bu integral aşağıdaki gibi hesaplanır;
- In integral $\int f(x) \cdot g(x) dx$, if the finite order derivative of f(x) is 0 and the integral of g(x) is taken easily, we calculate as follows;


f(x)
g(x)
+ f(x)
1. integral / 1st Integral
− f′(x)
2. integral / 2nd Integral
+ f″(x)
3. integral / 3rd Integral
0
n·Integral / nthIntegral

$\int \cos ax \cdot \sin bx\,dx$, $\int \sin ax \cdot \sin bx\,dx$, $\int \cos ax \cdot \cos bx\,dx$'in İNTEGRALİ / INTEGRALS of $\int \cos ax \cdot \sin bx\,dx$, $\int \sin ax \cdot \sin bx\,dx$, $\int \cos ax \cdot \cos bx\,dx$

▼ Bu integralleri bulmak için ters dönüşüm formülleri kullanılır.

▼ We can use inverse conversion formulas to find this type of integral.

- $\sin x \cdot \cos y = \frac{1}{2} \cdot [\sin(x+y) + \sin(x-y)]$
- $\cos x \cdot \sin y = \frac{1}{2} \cdot [\sin(x+y) - \sin(x-y)]$
- $\cos x \cdot \cos y = \frac{1}{2} \cdot [\cos(x+y) + \cos(x-y)]$
- $\sin x \cdot \sin y = -\frac{1}{2} \cdot [\cos(x+y) - \cos(x-y)]$

1. $\int (3x^2 + 4x + 7)dx = ?$

A) $6x + C$ B) $3x^2 + 4x + C$ C) $x^3 + 2x^2 + C$
D) $x^3 + 2x^2 + 7x + C$ E) $x^3 + x^2 + 7x + C$

2. $\int x^3(10x + 4)dx = ?$

A) $4x^6 + 4x^5 + C$ B) $4x^4 + 4x^3 + C$
C) $x^5 + x^4 + C$ D) $2x^5 + x^3 + C$
E) $2x^5 + x^4 + C$

3. $\int \left(x^3 + \frac{4}{x}\right)dx = ?$

A) $\frac{x^4}{4} + \frac{4}{x^2} + C$ B) $\frac{x^4}{4} + \frac{2}{x^2} + C$
C) $x^4 + 4 \cdot \ln|x| + C$ D) $\frac{x^4}{4} + 2 \cdot \ln|x| + C$
E) $\frac{x^4}{4} + 4 \cdot \ln|x| + C$

4. $\int (\sin 2x + \cos x)dx = ?$

A) $\frac{\cos 2x}{2} - \sin x + C$ B) $-\frac{\cos 2x}{2} + \sin x + C$
C) $\cos 2x - \sin x + C$ D) $-\cos 2x + \sin x + C$
E) $\cos 2x + \sin x + C$

5. $\int \left(e^x + \frac{3}{x^2} + 4\right)dx = ?$

A) $e^x - \frac{3}{x} + 4x + C$ B) $e^x - \frac{1}{x} + 4x + C$
C) $e^x + 3\ln|x| + C$ D) $e^x + 6\ln|x| + C$
E) $\frac{e^{x^2}}{2} - \frac{3}{x} + C$

6. $\int (\sqrt{x} + 2^x)dx = ?$

A) $\frac{1}{2\sqrt{x}} + 2^x \cdot \ln 2 + C$ B) $\frac{2}{3} \cdot \sqrt{x^3} + 2^x \cdot \ln 2 + C$
C) $\frac{2x^3\sqrt{x}}{3} + 2^x \cdot \ln 2 + C$ D) $\frac{2\sqrt{x^3}}{3} + 2^x + C$
E) $\frac{2\sqrt{x^3}}{3} + \frac{2^x}{\ln 2} + C$

7. $\int \left(e^{3x} + \frac{1}{\sin^2 x}\right)dx = ?$

A) $\frac{e^{3x}}{3} + \frac{1}{\sin x} + C$ B) $\frac{e^{3x}}{3} - \frac{1}{\sin x} + C$
C) $\frac{e^{3x}}{3} + \cot x + C$ D) $\frac{e^{3x}}{3} + \tan x + C$
E) $\frac{e^{3x}}{3} - \cot x + C$

8. $\int \left(\frac{x^3 + 3x}{x^2}\right)dx = ?$

A) $x + \frac{3}{x^2} + C$ B) $\frac{x^2}{2} + \frac{3}{x^2} + C$
C) $\frac{x^2}{2} + 3 \cdot \ln|x| + C$ D) $x^2 + 3\ln|x| + C$
E) $\frac{x^2}{2} - 3\ln|x| + C$

9. $\int \frac{1}{1+x^2}dx = ?$

A) tanx + C
B) cotx + C
C) arctanx + C
D) arccotx + C
E) arcsinx + C

10. $\int \frac{1}{\sqrt{1-x^2}}dx = ?$

A) arcsinx + C
B) arctanx + C
C) arccosx + C
D) arccotx + C
E) –arccotx + C

11. $f'(x) = 4x^3 + 6x^2 + 4$

$f(1) = 12 \Rightarrow$ **f(2) = ?**

A) 43
B) 44
C) 45
D) 46
E) 47

12. $\int f(x)dx = 6x^2 + 7x + 4$

$\Rightarrow$ **f(3) = ?**

A) 42
B) 43
C) 44
D) 45
E) 46

13. $\int f(3x+1)dx = 4x^2 - 3x + 5$

$\Rightarrow$ **f(7) = ?**

A) 13
B) 25
C) 39
D) 53
E) 159

14. $\int \frac{1}{\sin^2 x}dx = ?$

A) tanx + C
B) cotx + C
C) $\frac{1}{\sin x} + C$
D) arctanx + C
E) –cotx + C

15. $\int (x^2 + 2\sqrt[3]{x})dx = ?$

A) $\frac{x^3}{3} + \frac{3}{2} \cdot x\sqrt[3]{x} + C$

B) $2x + \frac{2}{3\sqrt[3]{x}} + C$

C) $\frac{x^3}{3} + \frac{2}{3\sqrt[3]{x}} + C$

D) $\frac{x^3}{3} + \frac{3}{2} \cdot \sqrt{x} + C$

E) $\frac{x^3}{3} + \frac{3}{2} \cdot 3x\sqrt[3]{x} + C$

YANITLAR / ANSWERS

1	2	3	4	5	6	7	8
D	E	E	B	A	E	E	C
9	**10**	**11**	**12**	**13**	**14**	**15**	**16**
C	A	C	B	A	E	A	

1. $\int (x^2-3)^{10} \cdot x\,dx = ?$

A) $\frac{(x^2-3)^{11}}{11}+C$

B) $\frac{(x^2-3)^{10}}{20}+C$

C) $\frac{(x^2-3)^{11}}{22}+C$

D) $\frac{(2x-3)^{11}}{22}+C$

E) $(e^2-3)^{11}+C$

2. $\int \sin^3 x \cdot \cos x\,dx = ?$

A) $\frac{\sin^4 x}{4}+C$

B) $\sin^4 x+C$

C) $\frac{\sin 2x}{2}+C$

D) $\cos x+C$

E) $\frac{\cos^4 x}{4}+C$

3. $\int e^{x^2+1} \cdot 2x\,dx = ?$

A) $2e^{x^2+1}+C$

B) $2x \cdot e^{x^2+1}+C$

C) $\frac{e^{x^2+1}}{2x}+C$

D) $\frac{e^{x^2+1}}{2}+C$

E) $e^{x^2+1}+C$

4. $\int \frac{\ln^2 x}{x}\,dx = ?$

A) $\ln^2|x|+C$

B) $\frac{x \cdot \ln^3|x|}{3}+C$

C) $\frac{\ln^3|x|}{3}+C$

D) $\frac{1}{x^2}+C$

E) $\frac{1}{x^3}+C$

5. $\int \frac{e^{\tan x}}{\cos^2 x}\,dx = ?$

A) $e^{\cot x}+C$

B) $-e^{\tan x}+C$

C) $e^{\tan x}+C$

D) $\frac{e^{\tan x}}{\cos x}+C$

E) $\frac{e^{\tan x}}{\cos^2 x}+C$

6. $\int \tan x\,dx = ?$

A) $\frac{1}{\cos^2 x}+C$

B) $-\sin x+C$

C) $\frac{\sin^2 x}{2}+C$

D) $-\frac{\sin^2 x}{2}+C$

E) $-\ln|\cos x|+C$

7. $\int (2x-1)^{17}\,dx = ?$

A) $(2x-1)^{17}+C$

B) $\frac{(2x-1)^{18}}{18}+C$

C) $\frac{(2x-1)^{17}}{17}+C$

D) $\frac{(2x-1)^{18}}{36}+C$

E) $\frac{(2x-1)^{17}}{34}+C$

8. $\int \frac{6x^2+1}{2x^3+x}\,dx = ?$

A) $-(2x^3+x)+C$

B) $\frac{1}{2x^3+x}+C$

C) $\ln|6x^2+1|+C$

D) $2 \cdot \ln|2x^3+x|+C$

E) $\ln|2x^3+x|+C$

9. $\int \cos(4x-7)dx = ?$

A) $\frac{\sin(4x-7)}{4}+C$

B) $4\sin(4x-7)+C$

C) $-\frac{\sin(4x-7)}{4}+C$

D) $-4\sin(4x-7)+C$

E) $\frac{\cos(4x-7)}{4}+C$

10. $\int \frac{dx}{4+x^2} = ?$

A) $\arctan x + C$

B) $\frac{\arctan\left(\frac{x}{2}\right)}{2}+C$

C) $\frac{\arcsin\left(\frac{x}{2}\right)}{2}+C$

D) $2\cdot\arctan\left(\frac{x}{2}\right)+C$

E) $\arcsin\left(\frac{x}{2}\right)+C$

11. $\int \frac{dx}{\sqrt{9-4x^2}} = ?$

A) $\frac{\arctan 2x}{2}+C$

B) $\frac{3\arctan\left(\frac{2}{3}x\right)}{2}+C$

C) $\frac{\arcsin\left(\frac{2}{3}x\right)}{2}+C$

D) $\frac{2\arcsin\left(\frac{2}{3}x\right)}{3}+C$

E) $\frac{\arcsin\left(\frac{2}{3}x\right)}{6}+C$

12. $\int \frac{6x^2-8x}{x^3-2x^2+1}dx = ?$

A) $2(6x^2-8x)^2+C$

B) $(6x^2-8x)^2+C$

C) $\ln|x^3-2x^2+1|+C$

D) $\frac{\ln|x^3-2x^2+1|}{2}+C$

E) $\ln|x^3-2x^2+1|^2+C$

13. $\int (2^{\sin x}\cdot\cos x\cdot\ln 2)dx = ?$

A) $2^{\sin x}+C$

B) $\frac{2^{\sin x+1}}{\sin x+1}+C$

C) $2^{2\sin x}+C$

D) $\frac{2^{2\sin x}}{2}+C$

E) $\frac{2^{\sin x}}{2}+C$

14. $\int \frac{\sec^2 x}{1+2\tan x}dx = ?$

A) $\frac{1}{2}\cdot\ln|1+2\tan x|+C$

B) $2\ln|1+2\tan x|+C$

C) $\frac{1}{4}\cdot\ln|1+2\tan x|+C$

D) $4\cdot\ln|1+2\tan x|+C$

E) $\frac{\ln|\arctan x|}{2}+C$

15. $\int \frac{x}{\sqrt{2x^2+4}}dx = ?$

A) $\frac{\sqrt{2x^2+4}}{4}+C$

B) $\frac{\sqrt{2x^2+4}}{2}+C$

C) $\sqrt{2x^2+4}+C$

D) $2\sqrt{2x^2+4}+C$

E) $4\sqrt{2x^2+4}+C$

16. $\int \frac{\sin 2x}{\cos^2 x}dx = ?$

A) $2\cdot\ln|\cos x|+C$

B) $2\cdot\ln|\sin x|+C$

C) $\ln\left|\frac{1}{\cos x}\right|+C$

D) $2\cdot\ln\left|\frac{1}{\cos x}\right|+C$

E) $\ln|\cos^2 x|+C$

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1	2	3	4	5	6	7	8
C	A	E	C	C	E	D	E
9	10	11	12	13	14	15	16
A	B	C	E	A	A	B	D

1. $\int x^2 f(x)dx = 2x^4 - 4x^3 + x + C \Rightarrow \mathbf{f(x) = ?}$

A) $8x - 12$
B) $x^2 + 8x - 12$
C) $8x^2 - x^{-2} + 12$
D) $8x + x^{-2} - 12$
E) $x^2 - 8x + 12$

2. $\int (\tan x + \tan^3 x)dx = ?$

A) $3\tan x + C$
B) $2\tan^2 x + C$
C) $\tan x + C$
D) $\frac{\tan^3 x}{3} + C$
E) $\frac{\tan^2 x}{2} + C$

3. $f'(x) = 4x + 4$,
$f(1) = 7 \Rightarrow \mathbf{f(2) = ?}$

A) 16
B) 17
C) 18
D) 19
E) 20

4. $\int \frac{d(x^5)}{x^5 - 1} = ?$

A) $\ln|x^4 - 1| + C$
B) $\ln|x^4 + 1| + C$
C) $\ln|x^5 - 1| + C$
D) $\ln|x^5 + 1| + C$
E) $\ln|x^6 - 1| + C$

5. $\int (x-2)^2 dx = ?$

A) $\frac{1}{3}\cdot(x-2)^3 + C$

B) $\frac{1}{2}\cdot(x-2)^3 + C$

C) $\frac{1}{2}\cdot(x-2)^2 + C$

D) $\frac{1}{3}\cdot(x-2)^2 + C$

E) $\frac{1}{3}\cdot(x+2)^3 + C$

6. $\int x^2 \cdot e^{x^3+1} dx = ?$

A) $e^{x^3+1} + C$
B) $e^{x^2+1} + C$
C) $e^{x^2} + C$
D) $\frac{e^{x^2+1}}{2} + C$
E) $\frac{e^{x^3+1}}{3} + C$

7. $\int (5^x + \tan x)dx = ?$

A) $5^x \cdot \log_5 e + \tan x + C$
B) $5^x \cdot \log_5 e + C$
C) $5^x \cdot \log_5 e - \ln|\cos x| + C$
D) $5^x \cdot \log_5 e + \ln|\cos x| + C$
E) $5^x \cdot \log_5 e - \ln|\sin x| + C$

8. $\frac{df(x)}{dx} = 12x^3 - 3x^2 + 4x,\ f(2) = 37 \Rightarrow \mathbf{f(1) = ?}$

A) –7
B) –5
C) –3
D) 3
E) 7

9. $\int \frac{5 \cdot \ln x}{2x} dx = ?$

A) $\frac{5}{4} \cdot \ln x^3 + C$

B) $\frac{5}{2} \cdot \ln x + C$

C) $\frac{5}{4} \cdot \ln x + C$

D) $\frac{5}{4} \cdot \ln^2 x + C$

E) $\frac{5}{2} \cdot \ln^2 x + C$

10. $\int \cos^3 x \, dx = ?$

A) $\frac{\cos^3 x}{3} + \sin x + C$

B) $-\cos^3 x + \sin x + C$

C) $\sin^3 x - \cos x + C$

D) $\sin^3 x - \sin x + C$

E) $\frac{-\sin^3 x}{3} + \sin x + C$

11. $\int \frac{3x^2}{2x^3 + 3} dx = ?$

A) $\frac{1}{2} \ln|2x^3 + 3| + C$

B) $\frac{1}{3} \ln|2x^3 + 3| + C$

C) $\frac{1}{4} \ln|2x^3 + 3| + C$

D) $\frac{1}{6} \ln|2x^3 + 3| + C$

E) $\ln|2x^3 + 3| + C$

12. $f'(x) = 5x^4 + 3x^2 + 2,$
$f(1) = 0 \Rightarrow f(1) = ?$

A) 0 B) 2 C) 4 D) 6 E) 8

13. $\int \left(\sqrt[4]{x} + \sqrt[3]{x\sqrt{x}}\right) dx = ?$

A) $\frac{5}{4} x^{\frac{5}{4}} + \frac{3}{2} x^{\frac{3}{2}} + C$

B) $x^{\frac{5}{4}} + x^{\frac{3}{2}} + C$

C) $\frac{4}{5} x^{\frac{5}{4}} + \frac{2}{3} x^{\frac{3}{2}} + C$

D) $\frac{5}{2} x^{\frac{5}{4}} + \frac{3}{2} x^{\frac{3}{2}} + C$

E) $\frac{4}{5} x^{\frac{3}{2}} + \frac{2}{3} x^{\frac{5}{4}} + C$

14. $\int \frac{x^2 + 4}{x^2 + 1} dx = ?$

A) $x + \tan x + C$

B) $x - 2\arctan x + C$

C) $x - 3\arctan x + C$

D) $x + 3\arctan x + C$

E) $x + 5\arctan x + C$

15. $\int [2x + f(x)] dx = \frac{x^3}{3} + \frac{x^2}{3} + C \Rightarrow f(6) = ?$

A) 26 B) 28 C) 36 D) 48 E) 52

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1	2	3	4	5	6	7	8
D	E	B	C	A	E	C	A
9	10	11	12	13	14	15	16
D	E	A	E	C	D	B	

1. $\int \frac{3x^2+4x-1}{x+2}\,dx = ?$

A) $3x^2 - 2x + \ln|x+2| + C$

B) $\frac{x^2}{3} - x + \ln|x+2| + C$

C) $\frac{3x^2}{2} - x + \ln|x+2| + C$

D) $\frac{3x^2}{2} - 2x + 3\ln|x+2| + C$

E) $\frac{3x^2}{2} - 2x + \ln|x+2| + C$

2. $\int \frac{4x-1}{2x+1}\,dx = ?$

A) $2x + \frac{3}{2} \cdot \ln|2x+1| + C$

B) $2x - \frac{3}{2} \cdot \ln|2x+1| + C$

C) $2x + 2 \cdot \ln|2x+1| + C$

D) $2x + \ln|2x+1| + C$

E) $2x - \ln|2x+1| + C$

3. $\int \left(\frac{2x+1}{x^2+1}\right) dx = ?$

A) $\ln|x^2+1| + \arctan x + C$

B) $2\ln|x^2+1| + \arctan x + C$

C) $\ln|x^2+1| + \frac{x^2}{2} + C$

D) $\frac{\ln|x^2+1|}{2} + \frac{1}{x^2+1} + C$

E) $\frac{\ln|x^2+1|}{2} + \arctan x + C$

4. $\int \frac{6x^3+8x}{3x^2+2}\,dx = ?$

A) $\frac{x^2}{2} + \frac{2}{3} \cdot \ln|3x^2+2| + C$

B) $x^2 + \frac{1}{3} \cdot \ln|3x^2+2| + C$

C) $2x^2 + \frac{2}{3} \cdot \ln|3x^2+2| + C$

D) $x^2 + 2 \cdot \ln|3x^2+2| + C$

E) $x^2 + \frac{2}{3} \cdot \ln|3x^2+2| + C$

5. $\int \frac{3x+4}{x^2+x-6}\,dx = ?$

A) $\ln|x^2+x-6| + C$

B) $\ln|x-2| \cdot |x+3| + C$

C) $2 \cdot \ln|x-2| + \ln|x+3| + C$

D) $3 \cdot \ln|x-2| + \ln|x+3| + C$

E) $\ln|x-2| + \ln|x+3| + C$

6. $\int \frac{1}{x^2-1}\,dx = ?$

A) $\arctan x + C$

B) $\arcsin x + C$

C) $\ln|x^2-1| + C$

D) $\ln\left|\frac{x-1}{x+1}\right| + C$

E) $\frac{1}{2} \cdot \ln\left|\frac{x-1}{x+1}\right| + C$

7. $\int \frac{1}{x^2+4x+5}\,dx = ?$

A) $\ln|x+2| + C$

B) $2 \cdot \ln|x+2| + C$

C) $\ln|\arctan x| + C$

D) $\arctan x + C$

E) $\arctan(x+2) + C$

8. $\int \frac{1}{x^2-9}dx = ?$

A) $\frac{1}{3}\cdot \ln|x^2-9|+C$

B) $\frac{1}{6}\cdot \ln|x^2-9|+C$

C) $\frac{1}{6}\cdot \ln\left|\frac{x-3}{x+3}\right|+C$

D) $\frac{1}{6}\cdot \ln\left|\frac{x+3}{x-3}\right|+C$

E) $\frac{1}{3}\cdot \ln\left|\frac{x-3}{x+3}\right|+C$

9. $\int \frac{2x+4}{x^2+9}dx = ?$

A) $\ln|x^2+9|+\frac{4}{3}\cdot \arctan x+C$

B) $\ln|x^2+9|+\frac{4}{9}\cdot \arctan\left(\frac{x}{3}\right)+C$

C) $\ln|x^2+9|+\frac{4}{3}\cdot \arctan\left(\frac{x}{3}\right)+C$

D) $\ln|x^2+9|+\frac{4}{9}\cdot \arctan x+C$

E) $\ln|x^2+9|-\frac{4}{3}\cdot \arctan\left(\frac{x}{3}\right)+C$

10. $\int \frac{x+2}{x^2+7x+12}dx = ?$

A) $\ln\left|\frac{x+4}{x+3}\right|+C$

B) $\ln|x^2+7x+12|+C$

C) $\ln\left|\frac{x^2+8x+16}{x+3}\right|+C$

D) $2\ln\left|\frac{x+4}{x+3}\right|+C$

E) $\frac{1}{2}\cdot \ln\left|\frac{x+4}{x+3}\right|+C$

11. $\int x\cdot e^x\,dx = ?$

A) $x\cdot e^x + e^x + C$

B) $e^x - x\cdot e^x + C$

C) $x\cdot e^x + C$

D) $x\cdot e^x - e^x + C$

E) $x\cdot e^x - 2e^x + C$

12. $\int \ln x dx = ?$

A) $\ln x - x + C$

B) $x\cdot \ln x + x + C$

C) $\frac{x^2\cdot \ln x}{2}+x+C$

D) $x\cdot \ln x+\frac{x^2}{2}+C$

E) $x\cdot \ln x - x + C$

13. $\int (x^2+1)\cdot e^x dx = ?$

A) $e^x\cdot(x^2+2x+3)+C$

B) $e^x\cdot(x^2-2x-1)+C$

C) $e^x\cdot(x^2-2x+3)+C$

D) $e^x\cdot(x^2+2x+3)+C$

E) $e^x\cdot(x^2+2x-1)+C$

14. $\int \arctan x\cdot dx = ?$

A) $\arctan x - \ln|1+x^2| + C$

B) $\arctan x-\frac{1}{2}\cdot \ln|1+x^2|+C$

C) $x\cdot \arctan x-\frac{1}{2}\cdot \ln|1+x^2|+C$

D) $x\cdot \arctan x - \ln|1+x^2| + C$

E) $x\cdot \arctan x+\frac{1}{2}\cdot \ln|1+x^2|+C$

15. $\int x\cdot \sin x dx = ?$

A) $x\cdot\cos x + \sin x + C$

B) $-x\cdot\cos x + \sin x + C$

C) $-x\cdot\cos x - \sin x + C$

D) $-x\cdot\sin x + \cos x + C$

E) $x\cdot\sin x - \cos x + C$

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1	2	3	4	5	6	7	8
D	B	A	E	C	E	E	C
9	**10**	**11**	**12**	**13**	**14**	**15**	**16**
C	C	D	E	C	C	B	

1. $\int \frac{4x^3+4x+2}{x^2+1}dx = ?$

A) $x^2 + \arctan x + C$

B) $2x^2 + \arctan x + C$

C) $2x^2 + 2\arctan x + C$

D) $2x^2 + 2\arctan(x^2+1) + C$

E) $4x^2 + 2\arctan(x^2+1) + C$

2. $\int e^{3x^2+2}x\,dx = ?$

A) $\frac{1}{6}e^{3x^2+2}+C$

B) $\frac{1}{5}e^{3x^2}+C$

C) $\frac{1}{3}e^{3x^2+2}+C$

D) $6e^{3x^2+2}+C$

E) $5e^{3x+2}+C$

3. $\int \frac{3}{x^2+8x+17}dx = ?$

A) $\arctan(x+2) + C$

B) $3\cdot\arctan(x+2) + C$

C) $6\cdot\arctan(x+2) + C$

D) $\arctan(x+4) + C$

E) $3\arctan(x+4) + C$

4. $\int \tan(3x-2)dx = ?$

A) $\frac{1}{3}\ln|\tan(3x-2)|+C$

B) $-\frac{1}{3}\ln|\tan(3x-2)|+C$

C) $\frac{1}{3}\ln|\cos(3x-2)|+C$

D) $-\frac{1}{3}\ln|\cos(3x-2)|+C$

E) $\frac{1}{3}\ln|\sin(3x-2)|+C$

5. $0 < x < \pi$

$\Rightarrow \int \sqrt{1-\cos 2x}\,dx = ?$

A) $\sqrt{2}\sin x + C$

B) $\sqrt{2}\cos x + C$

C) $-\sqrt{2}\cos x + C$

D) $-2\cos x + C$

E) $-\sqrt{2}\sin x + C$

6. $\int (x^2+2x)e^x dx = ?$

A) $(x^2+2x)\cdot e^x + C$

B) $(x^2+2)\cdot e^x + C$

C) $(x^2-2)\cdot e^x + C$

D) $(x^2-2x)\cdot e^x + C$

E) $x^2\cdot e^x + C$

7. $\int \frac{3x+2}{x^2+x}dx = ?$

A) $2\cdot\ln|x| + \ln|x+1| + C$

B) $2\cdot\ln|x+1| + \ln|x| + C$

C) $\ln|x| + \ln|x+1| + C$

D) $2\cdot\ln|x+1| - \ln|x| + C$

E) $\ln|x+1| - \ln|x| + C$

8. $\int x\cdot f(x)dx = \frac{1}{3}x^3 + x^2 + 2x + C \Rightarrow f(2) = ?$

A) 4 B) 5 C) 6 D) 7 E) 8

9. $\displaystyle\int \frac{2x}{\sqrt{x^2-2}}dx = ?$

A) $\sqrt{x^2-2}+C$

B) $2\sqrt{x^2-2}+C$

C) $2\sqrt{2-x^2}+C$

D) $\frac{1}{2}\sqrt{x^2-2}+C$

E) $\frac{1}{2}\sqrt{2-x^2}+C$

10. $\displaystyle\int \frac{\sin(\tan x)}{\cos^2 x}dx = ?$

A) –sin(tanx) + C

B) –cos(tanx) + C

C) sin(cotx) + C

D) cos(cotx) + C

E) cos(tanx) + C

11. $\displaystyle\int e^x \cdot \cos x\ dx = ?$

A) $\frac{1}{2}e^x(\sin x+\cos x)+C$

B) $\frac{1}{2}e^x(\sin x-\cos x)+C$

C) $\frac{1}{3}e^x(\sin x+\cos x)+C$

D) $\frac{1}{3}e^x(\sin x-\cos x)+C$

E) $\frac{1}{3}e^x(\cos x-\sin x)+C$

12. $\displaystyle\int 2\cdot \sin 5x \cdot \sin x dx = ?$

A) $\frac{1}{8}\sin 4x-\frac{1}{2}\sin 6x+C$

B) $\frac{1}{6}\sin 6x-\frac{1}{4}\sin 4x+C$

C) $\frac{1}{4}\sin 4x-\frac{1}{6}\sin 6x+C$

D) sin4x + sin6x + C

E) sin4x – sin6x + C

13. $\displaystyle\int \sin 3x \cdot \cos x dx = ?$

A) $-\frac{1}{2}\cos 2x-\frac{1}{4}\cos 4x+C$

B) $-\frac{1}{4}\cos 2x-\frac{1}{8}\cos 4x+C$

C) $-\frac{1}{4}\cos 4x-\frac{1}{8}\cos 2x+C$

D) $\frac{1}{2}\cos 2x-\frac{1}{4}\cos 4x+C$

E) $\frac{1}{2}\cos 2x+\frac{1}{4}\cos 4x+C$

14. $\displaystyle\int \frac{x}{\cos^2 x}dx = ?$

A) tanx – ln|cosx| + C

B) tanx + ln|cosx| + C

C) x·tanx – ln|cosx| + C

D) x·tanx + ln|cosx| + C

E) x·tanx – ln|sinx| + C

YANITLAR / ANSWERS

1	2	3	4	5	6	7
C	A	E	D	C	E	A
8	9	10	11	12	13	14
B	B	B	A	C	B	D

1. $\int e^{\sin^2 x} \cdot \sin 2x \, dx = ?$

A) $e^{\sin 2x} + C$

B) $e^{\sin^2 x} \cdot \cos 2x + C$

C) $e^{\sin^2 x} + C$

D) $e^{\sin^2 x} \cdot \sin 2x + C$

E) $2e^{\sin^2 x} \cdot \sin 2x + C$

2. $\int \frac{\ln x}{x} \, dx = ?$

A) $\ln|x| + C$

B) $\ln|2x| + C$

C) $\frac{\ln^2|x|}{2} + C$

D) $-\ln|2x| + C$

E) $-\ln|x^2| + C$

3. $\int (2x+4)^7 \, dx = ?$

A) $\frac{(2x+4)^7}{14} + C$

B) $\frac{(2x+4)^8}{8} C$

C) $\frac{(2x+4)^8}{16} + C$

D) $\frac{(2x+4)^8}{4} + C$

E) $\frac{(2x+4)^8}{32} + C$

4. $\int \left(\frac{2x-1}{x+1}\right) dx = ?$

A) $2 - 3\ln|x+1| + C$

B) $2x + 3\ln|x+1| + C$

C) $2x + \ln|x+1| + C$

D) $2x + 3\ln|x| + C$

E) $2x - 3\ln|x+1| + C$

5. $\int \frac{\cos x}{\sin^3 x} \, dx = ?$

A) $-\frac{1}{2\sin^2 x} + C$

B) $\frac{1}{2\cos^2 x} + C$

C) $\frac{2}{\sin^2 x} + C$

D) $\frac{2}{\cos^2 x} + C$

E) $\frac{1}{\sin^2 x} + C$

6. $\int \frac{e^{\sqrt{x}}}{\sqrt{x}} \, dx = ?$

A) $\frac{e^x}{2} + C$

B) $2e^x + C$

C) $e^{\sqrt{x}} + C$

D) $2e^{\sqrt{x}} + C$

E) $\frac{e^{\sqrt{x}}}{2} + C$

7. $\int e^{x + \ln x} dx = ?$

A) $x \cdot e^x + C$

B) $x \cdot e^x + x + C$

C) $x \cdot e^x - x + C$

D) $x \cdot e^x - e^x + C$

E) $\ln x + x + C$

8. $\int \frac{5x+3}{x^2+2x-3} \, dx = ?$

A) $3 \cdot \ln|x+3| - 2 \cdot \ln|x-1| + C$

B) $2 \cdot \ln|x+3| - 3 \cdot \ln|x-1| + C$

C) $3 \cdot \ln|x+3| + 2 \cdot \ln|x-1| + C$

D) $3 \cdot \ln\left|\frac{x+3}{x-1}\right| + C$

E) $3 \cdot \ln|x^2 + 2x - 3| + C$

9. $\int e^x \cdot x^2 \, dx = ?$

A) $e^x \cdot (x^2 - 2x) + C$
B) $e^x \cdot (-2x + 2) + C$
C) $e^x \cdot (x^2 - 2x + 2) + C$
D) $e^x \cdot (x^2 + 2x - 2) + C$
E) $e^x \cdot (x^2 + 2x + 2) + C$

10. $\int x^2 \cdot \ln x \cdot dx = ?$

A) $\frac{\ln x}{3} - \frac{x^3}{9} + C$
B) $\frac{x^3 \cdot \ln x}{3} - \frac{x^3}{3} + C$
C) $\frac{x^3 \cdot \ln x}{3} + \frac{x^3}{9} + C$
D) $\frac{x^3 \cdot \ln x}{3} - \frac{x^3}{9} + C$
E) $\frac{x^3 \cdot \ln x}{3} + \frac{x^3}{3} + C$

11. $\int \sin^3 x \, dx = ?$

A) $-\cos x - \frac{\cos^3 x}{3} + C$
B) $-\sin x - \frac{\cos^3 x}{3} + C$
C) $\sin x - \frac{\cos^3 x}{3} + C$
D) $-\cos x + \frac{\sin^3 x}{3} + C$
E) $-\cos x + \frac{\cos^3 x}{3} + C$

12. $\int \sin^4 x \cdot \cos^3 x \, dx = ?$

A) $\frac{\sin^4 x}{4} - \frac{\sin^5 x}{5} + C$
B) $\frac{\sin^5 x}{5} - \frac{\sin^7 x}{7} + C$
C) $\frac{\sin^5 x}{5} - \frac{\sin^6 x}{6} + C$
D) $\frac{\sin^4 x}{4} + \frac{\sin^5 x}{5} + C$
E) $\frac{\sin^5 x}{5} - \frac{\cos^4 x}{4} + C$

13. $\int \sin 3x \cdot \cos 5x \, dx = ?$

A) $\sin 8x - \sin 2x + C$
B) $\frac{\sin 8x}{8} - \frac{\sin 2x}{2} + C$
C) $\cos 8x - \cos 2x + C$
D) $\frac{\cos 8x}{8} - \frac{\cos 2x}{2} + C$
E) $-\frac{\cos 8x}{16} + \frac{\cos 2x}{4} + C$

14. $\int \sin^2 x \, dx = ?$

A) $\frac{x}{2} - \frac{\sin 2x}{4} + C$
B) $x - \frac{\sin 2x}{2} + C$
C) $\frac{x}{2} + \frac{\sin 2x}{2} + C$
D) $\frac{x}{2} + \frac{\sin 2x}{4} + C$
E) $\frac{x}{4} - \frac{\sin 2x}{2} + C$

15. $\int \tan^2 x \, dx = ?$

A) $\frac{\tan^3 x}{3} + \frac{x^2}{2} + C$
B) $\frac{\tan^2 x}{2} + x + C$
C) $\frac{\tan^3 x}{3} + x + C$
D) $\frac{\tan^3 x}{3} - x + C$
E) $\tan x - x + C$

YANITLAR / ANSWERS

1	2	3	4	5	6	7	8
C	C	C	E	A	D	D	C
9	10	11	12	13	14	15	16
C	D	E	B	E	A	E	

1. $f'(x) = 2x - 3$

$f(2) = 4 \Rightarrow \mathbf{f(3) = ?}$

A) 6 B) 8 C) 12 D) 14 E) 16

2. $\left.\begin{array}{l} f'(x) = 4 \\ (fog)(x) = 4x + 6 \\ f(4) = 2 \end{array}\right\} \Rightarrow \mathbf{g(x) = ?}$

A) x + 2 B) x + 3 C) x + 4

D) x + 5 E) x + 6

3. $\int \sqrt{2x-3}\,dx = ?$

A) $\dfrac{2x-3}{2} + C$

B) $\dfrac{(2x-3)^{\frac{3}{2}}}{2} + C$

C) $\dfrac{(2x-3)^2}{2} + C$

D) $\dfrac{(2x-3)^3}{2} + C$

E) $\dfrac{(2x-3)^{\frac{3}{2}}}{3} + C$

4. $\int x\sqrt{x-2}\,dx = ?$

A) $\frac{3}{4}(x-2)^{\frac{5}{2}} + (x-2)^2 + C$

B) $\frac{2}{5}(x-2)^{\frac{5}{2}} + \frac{4}{3}(x-2)^{\frac{3}{2}} + C$

C) $\frac{1}{5}(x-2)^{\frac{5}{2}} + \frac{1}{3}(x-2)^2 + C$

D) $\frac{2}{5}(x-2)^{\frac{5}{2}} + (x-2)^2 + C$

E) $\frac{2}{5}(x-2)^{\frac{5}{2}} + \frac{1}{2}(x-2)^2 + C$

5. $\int \dfrac{1+\sqrt{x}}{\sqrt[3]{x}}\,dx = ?$

A) $\frac{3}{2}x^{\frac{1}{3}} + \frac{1}{7}x^{\frac{7}{6}} + C$

B) $\frac{3}{2}x^{\frac{2}{3}} + \frac{6}{7}x^{\frac{7}{6}} + C$

C) $\frac{3}{2}x^{\frac{2}{3}} + \frac{1}{5}x^{\frac{5}{6}} + C$

D) $\frac{3}{2}x^{\frac{1}{3}} + \frac{1}{7}x^{\frac{1}{7}} + C$

E) $\frac{1}{2}x^{\frac{1}{2}} + \frac{1}{7}x^{\frac{7}{6}} + C$

6. $\int x(x+2)^2\,dx = ?$

A) $\dfrac{(x+2)^5}{5} - \dfrac{(x+2)^4}{2} + C$

B) $\dfrac{(x+2)^5}{5} + \dfrac{(x+2)^3}{2} + C$

C) $\dfrac{(x+2)^4}{4} - \dfrac{(x+2)^3}{3} + C$

D) $\dfrac{(x+2)^5}{5} - \dfrac{1}{2}(x+2)^2 + C$

E) $\dfrac{(x+2)^4}{4} - \dfrac{2}{3}(x+2)^3 + C$

7. $\int x \cdot \sqrt{x-3}\,dx = ?$

A) $(x-3)^{\frac{5}{3}} + (x-3)^{\frac{3}{2}} + C$

B) $\dfrac{(x-3)^{\frac{5}{3}}}{5} + (x-3)^{\frac{3}{2}} + C$

C) $\frac{3}{5}(x-3)^{\frac{5}{2}} + (x-3)^{\frac{3}{2}} + C$

D) $\frac{2}{5}(x-3)^{\frac{5}{2}} + 2\cdot(x-3)^{\frac{3}{2}} + C$

E) $\frac{2}{5}(x-3)^{\frac{5}{2}} + (x-3)^{\frac{3}{2}} + C$

8. $\displaystyle\int \frac{x}{\sqrt{x+3}}\,dx = ?$

A) $\frac{2}{3}(x+3)^{\frac{3}{2}} - 6\cdot(x+3)^{\frac{1}{2}} + C$

B) $\frac{1}{3}(x+3)^{\frac{3}{2}} - (x+3)^{\frac{1}{2}} + C$

C) $(x+3)^{\frac{3}{2}} - \frac{1}{3}\cdot(x+3)^{\frac{1}{2}} + C$

D) $\dfrac{(x+3)^{\frac{3}{2}}}{2} - \dfrac{(x+3)^{\frac{1}{2}}}{3} + C$

E) $(x+3)^{\frac{3}{2}} - (x+3)^{\frac{1}{2}} + C$

9. $\displaystyle\int (4x-2)^5\,dx = ?$

A) $\frac{1}{18}(4x-2)^6 + C$

B) $\frac{1}{24}(4x-2)^6 + C$

C) $\frac{1}{24}(2x-4)^6 + C$

D) $\frac{1}{18}(2x-4)^6 + C$

E) $\frac{2}{3}(4x-2)^6 + C$

10. $\displaystyle\int x^2(x^3+1)^2\,dx = ?$

A) $\dfrac{(x^3+1)^3}{3} + C$

B) $\dfrac{(x^3+1)^3}{9} + C$

C) $\dfrac{(x^3+1)^2}{3} + C$

D) $\dfrac{(x^3+1)^2}{9} + C$

E) $3x + 1 + C$

11. $\displaystyle\int \frac{2x+1}{x}\,dx = ?$

A) $2x + \ln|x| + C$

B) $2 + \ln|x| + C$

C) $\dfrac{2}{x^2} + C$

D) $\dfrac{x^2}{2} + \ln|x| + C$

E) $2x + \ln x^2 + C$

12. $\displaystyle\int \frac{2^x}{2^{x+1} + 4^x + 1}\,dx = ?$

A) $\dfrac{\arctan 2^x}{\ln 2} + C$

B) $-\dfrac{1}{\ln 2\cdot(2^x+1)} + C$

C) $\dfrac{\arctan(2^x+1)}{\ln 2} + C$

D) $\dfrac{1}{\ln 2\cdot(2^x+1)} + C$

E) $-\dfrac{1}{\ln 2\cdot 2^x} + C$

13. $\displaystyle\int \frac{x\,dx}{1-x^2} = ?$

A) $-\frac{1}{2}\ln|1-x^2| + C$

B) $\frac{1}{2}\ln|1-x^2| + C$

C) $\frac{1}{4}\ln|1-x| - \frac{1}{4}\ln|1+x| + C$

D) $\ln|x^2-1| + C$

E) $2\ln|x-1| + C$

14. $\displaystyle\int (\ln x)^2\,\frac{dx}{x} = ?$

A) $\frac{1}{2}\ln x^2 + C$

B) $\frac{1}{3}(\ln x)^3 + C$

C) $\frac{1}{3}\ln x^3 + C$

D) $\frac{1}{6}(\ln x)^2 + C$

E) $3\ln x + C$

YANITLAR / ANSWERS

1	2	3	4	5	6	7
A	D	E	B	B	E	D
8	**9**	**10**	**11**	**12**	**13**	**14**
A	B	B	A	B	A	B

1. $\int \frac{\sin x}{2-\cos x}dx$

A) $2 + \cos x + C$
B) $\ln(2 - \cos x) + C$
C) $\ln(2 - \sin x) + C$
D) $2 - \ln(\cos x) + C$
E) $\ln(4 - \tan x) + C$

2. $\int \frac{(x+1)}{x^2+2x+2}dx = ?$

A) $\ln(x^2+2x+2)+C$
B) $\frac{1}{2}(x^3+x^2+2x)+C$
C) $x^4+x^3+2x^2+x+C$
D) $\frac{1}{2}\ln(x^2+2x+2)+C$
E) $\arctan(x^2+2x+2)+C$

3. $\int \frac{x}{2x-1}dx = ?$

A) $\frac{1}{2}x^2+\frac{1}{2}\ln|2x-1|+C$
B) $\frac{1}{2}x+\frac{1}{2}\ln|2x-1|+C$
C) $\frac{1}{2}x^3+\frac{1}{4}\ln|2x-1|+C$
D) $\frac{1}{2}x+\frac{1}{4}\ln|2x-1|+C$
E) $\frac{1}{4}x+\frac{1}{4}\ln|2x-1|+C$

4. $\int \frac{x\,dx}{(x^2+1)^2} = ?$

A) $\frac{x^2+1}{2}+C$
B) $\frac{-1}{2(x^2+1)}+C$
C) $\frac{1}{x^2+1}+C$
D) $\frac{1}{2(x^2+1)}+C$
E) $\frac{-1}{(x^2+1)^3}+C$

5. $\int \frac{2x^2+6x-2}{x^2+2x-2}dx = ?$

A) $2x-\ln|x^2+2x-2|+C$
B) $x^2+4x-8+C$
C) $2x+\ln|x^2+2x-2|+C$
D) $x-\ln|x^2-2x-2|+C$
E) $2x+\frac{1}{2}\ln|x^2+2x-2|+C$

6. $\int \frac{x^2+2x-1}{3-x}dx = ?$

A) $-\frac{1}{2}x^2-5x-14\ln|3-x|+C$
B) $\frac{1}{2}x^2-5x+14\ln|x-3|+C$
C) $-x^2+5x-12\ln|x-3|+C$
D) $\frac{1}{2}x^2-5x-14\ln|3-x|+C$
E) $-x+8\ln|3-x|+C$

7. $\int \frac{1+e^{\arctan x}}{1+x^2}dx = ?$

A) $\arctan x + e^{\arctan x} + C$
B) $x + e^{\arctan x} + C$
C) $x + \arctan x + C$
D) $2e^{\arctan x} + C$
E) $x - e^{\arctan x} + C$

8. $f(x) = \int \frac{1}{x\ln x}dx$ ve / and $f(e) = 6 \Rightarrow f(x) = ?$

A) $\ln x + 6$
B) $\ln^2 x + 6$
C) $2\ln x + 3$
D) $\ln(\ln x) + 6$
E) $x^2\ln x + 9$

9. $\int \frac{x-2}{(x^2-4x)^2}dx = ?$

A) $-\frac{1}{x^2-4x}+C$

B) $-\frac{1}{2(x^2-4x)}+C$

C) $-\frac{1}{2(x^2-4x)^2}+C$

D) $2(x^2-4x)+C$

E) $\frac{1}{2}(x^2-4x)+C$

10. $\int \frac{e^{\tan x}}{\cos^2 x}dx = ?$

A) $\arctan(\cos x)+C$

B) $\tan(\cos x)+C$

C) $e^{\sin^2 x}+C$

D) $e^{\cos^2 x}+C$

E) $e^{\tan x}+C$

11. $\int x \cdot \log_5 e^{x^2} dx = ?$

A) $\frac{x^4}{4\ln 5}+C$

B) $\frac{x^3}{3}\log_5 e+C$

C) $\frac{x^4}{4}\cdot\log_5 e^{x^2}+C$

D) $2\log_5 e^{x^2}+C$

E) $\frac{2\cdot\log_5 e^{x^2}}{\ln 5}+C$

12. $\int \frac{x^2+3x+1}{x^2+x}dx = ?$

A) $\arctan|x^2|+C$

B) $\ln|x^2+x|+C$

C) $x+\ln|x^2+x|+C$

D) $\frac{x^2+\ln|x^2+x|}{x}+C$

E) $(2x+1)\cdot\ln|x|+C$

13. $\int \frac{dx}{\sqrt{x}+x} = ?$

A) $2\sqrt{x}-\ln(1+\sqrt{x})+C$

B) $-2\ln(1+\sqrt{x})+C$

C) $x-\ln(1+\sqrt{x})+C$

D) $\frac{\ln(1+\sqrt{x})}{2}+C$

E) $2\ln(1+\sqrt{x})+C$

14. $\int \frac{x^2+2x+1}{x^2+1}dx = ?$

A) $2x+\arctan x+C$

B) $x+\ln(x^2+1)+C$

C) $x-\ln(x^2+1)+C$

D) $\frac{1}{2}x^2+\ln(x^2+1)+C$

E) $\frac{1}{2}x+\ln(x^2+1)^2+C$

15. $\int (x-4)f(x)dx = \frac{1}{3}x^3-\frac{9}{2}x^2+20x+3$

$\Rightarrow$ **f(x) = ?**

A) $x-3$

B) $x-4$

C) $x-5$

D) $x+4$

E) $x+5$

YANITLAR / ANSWERS

1	2	3	4	5	6	7	8
B	D	D	B	C	A	A	D
9	10	11	12	13	14	15	16
B	E	A	C	E	B	C	

1. $\int \sin x \cdot f(x)dx = \sin^2 x - \cos^2 x + x \Rightarrow f\left(\frac{\pi}{6}\right) = ?$

A) 2
B) $\frac{2\sqrt{3}}{3}$
C) $\frac{4\sqrt{3}}{3}$
D) $\frac{2\sqrt{3}+1}{3}$
E) $2\sqrt{3}+2$

2. $\int x \cdot f(x)dx = x^3 + 3ax^2 - 2x + 4$

$f(2) = 9 \Rightarrow$ **a =?**

A) 2
B) $\frac{1}{2}$
C) $\frac{1}{4}$
D) $\frac{2}{3}$
E) $\frac{3}{4}$

3. $\int x f(x)dx = x^3 + 3x^2 + 4x + 6 \Rightarrow$ **$f(x) = ?$**

A) $3x + \frac{4}{x} + 6$
B) $3x^2 + 6x + 4$
C) $3x^2 + \frac{4}{x} + 2$
D) $x^2 + 3x + \frac{6}{x} + 4$
E) $6x + \frac{2}{x} + 3$

4. $\int \frac{1}{x^2 + 5x + 6} dx = ?$

A) $\ln\left|\frac{x+1}{x+2}\right| + C$
B) $\ln\left|\frac{x+3}{x+2}\right| + x + C$
C) $\ln\left|\frac{x+2}{x+3}\right| + C$
D) $\ln\left|\frac{x^3}{3} + \frac{5x^2}{2} + 6x\right| + C$
E) $\ln|x^2 + 5x + 6| + C$

5. $\int \frac{x \cdot dx}{x^2 + 4x - 5} = ?$

A) $\frac{1}{6}\ln\left|\frac{x-1}{(x+5)^5}\right| + C$
B) $\frac{1}{3}\ln\left|\frac{x+5}{x-1}\right| + C$
C) $\frac{1}{6}\ln\left|(x+5)^5 \cdot (x-1)\right| + C$
D) $\frac{1}{2}\ln\left|(x-5)^3 \cdot (x-1)\right| + C$
E) $\frac{1}{6}\ln\left|(x-5) \cdot (x+1)\right| + C$

6. $\int \cos^3 x dx = ?$

A) $\cos x - \frac{1}{3}\cos^3 x + C$
B) $\sin x - \frac{1}{3}\sin^3 x + C$
C) $\sin^2 x - \frac{1}{4}\sin^4 x + C$
D) $\cos^2 x - \frac{1}{3}\sin^3 x + C$
E) $\cos^2 x - \frac{1}{3}\cos^3 x + \sin^2 x + C$

7. $\int \sin^3 4x \cdot \cos^2 4x dx = ?$

A) $\frac{1}{20}\cos^5 4x - \frac{1}{12}\cos^3 4x + C$
B) $\frac{1}{5}\cos^5 4x - \frac{1}{6}\cos^3 4x + C$
C) $\frac{1}{10}\sin^4 4x - \frac{1}{5}\sin^3 4x + C$
D) $\frac{1}{20}\cos^5 4x - \frac{1}{12}\cos^3 4x + \frac{1}{4}x + C$
E) $\frac{1}{10}\cos^5 4x \cdot \sin 4x - \frac{1}{12}\cos^2 4x + C$

8. $\int \tan^2 4x\,dx = ?$

A) $\frac{1}{4}\tan 4x - x + C$

B) $\frac{1}{8}\tan 4x + x + C$

C) $\frac{1}{4}\tan 4x - \sin 4x + C$

D) $\frac{1}{4}\tan^3 4x + x^2 + C$

E) $\frac{1}{4}\tan 4x + x + C$

9. $\int \frac{\cos x}{2+\sin x}dx = ?$

A) $\frac{1}{(2+\sin x)^2} + C$

B) $\ln(2 + \sin x) + C$

C) $2\ln(2 + \sin x) + C$

D) $\frac{1}{\ln(2+\sin 2x)} + C$

E) $\ln(\cos x)^2 + C$

10. $\int \frac{\ln(\sin x)\cdot\cos x}{\sin x}dx = ?$

A) $\frac{\ln|\cos x|}{2} + C$

B) $\frac{\ln|\sin x|}{2} + C$

C) $\frac{\ln^2|\sin x|}{2} + C$

D) $\frac{\ln|\arcsin x|}{2} + C$

E) $\frac{\ln|\sin x| + \cos x}{2} + C$

11. $\int \frac{x^2 \arctan x^3}{1+x^6}dx = ?$

A) $\frac{(\arctan x^3)}{3} + C$

B) $\frac{\arctan x^2}{6} + C$

C) $\frac{(\arctan x^3)^2}{3} + C$

D) $\frac{(\arctan x^3)^2}{6} + C$

E) $\frac{(\arctan x^3)^2}{2} + C$

12. $\int \frac{\cot x}{\ln|\sin x|}dx = ?$

A) $\ln|\sin x| + C$

B) $\ln|\ln|\sin x|| + C$

C) $\ln|\cos x| + C$

D) $\frac{1}{\ln|\sin x|} + C$

E) $1 + \ln|\sin x| + C$

13. $\int e^{\sin x}\cos x\,dx = ?$

A) $e^{\cos x} + C$

B) $e^{\tan x} + C$

C) $e^{\sin x} + x + C$

D) $e^{\cos x} + 1 + C$

E) $e^{\sin x} + C$

14. $\int \frac{\sec^2 x}{1+\tan x}dx = ?$

A) $\ln|\tan x| + C$

B) $\ln|1+\tan x| + C$

C) $e^{1+\tan x} + C$

D) $\arctan(1 + \tan x) + C$

E) $\frac{1}{2}\ln|1-\tan x| + C$

YANITLAR / ANSWERS

1	2	3	4	5	6	7
E	D	A	C	C	B	A
8	**9**	**10**	**11**	**12**	**13**	**14**
A	B	C	D	B	E	B

1. $\int \frac{1}{x^2+6x+10}dx = ?$

A) $\arcsin(x+3)+C$
B) $2\arctan(x+3)+C$
C) $\frac{\arctan(x+3)}{2}+C$
D) $\frac{\arctan(x+3)}{4}+C$
E) $\arctan(x+3)+C$

2. $\int \frac{1}{4+64x^2}dx = ?$

A) $\frac{\arcsin 16x}{16}+C$
B) $\frac{\arctan 4x}{16}+C$
C) $\frac{\arctan 64x}{16}+C$
D) $\frac{\arctan 16x}{64}+C$
E) $\frac{\arcsin 4x}{4}+C$

3. $\int \frac{1}{x^2+8x+17}dx = ?$

A) $\arctan(x+4)+C$
B) $\operatorname{arccot}(x+4)+C$
C) $\ln(x+4)+C$
D) $\arcsin(x+4)+C$
E) $\ln(x+4)^2+C$

4. $\int \frac{dx}{\sqrt{4-(x-1)^2}} = ?$

A) $\arcsin\left(\frac{x-1}{2}\right)+C$
B) $\arccos\left(\frac{x-1}{2}\right)+C$
C) $\operatorname{arcsec}\left(\frac{x-1}{2}\right)+C$
D) $\arctan\left(\frac{x-1}{4}\right)+C$
E) $\operatorname{arccosec}\left(\frac{x-1}{4}\right)+C$

5. $\int \frac{\cos x}{1+\sin^2 x}dx = ?$

A) $\arctan x+C$
B) $\operatorname{arccot} x+C$
C) $\arctan(\sin x)+C$
D) $\arctan(\cos x)+C$
E) $\arcsin x+C$

6. $\int \frac{1}{\sqrt{81-9x^2}}dx = ?$

A) $\arctan\frac{x}{3}+C$
B) $\frac{\arcsin\frac{x}{3}}{9}+C$
C) $\frac{\arccos\frac{x}{3}}{2}+C$
D) $\frac{\arctan\frac{x}{3}}{3}+C$
E) $\frac{1}{3}\arcsin\frac{x}{3}+C$

7. $\int \frac{-\sin x}{\sqrt{2-\cos^2 x}}dx = ?$

A) $\sqrt{2}\arctan\left(\frac{\cos x}{\sqrt{2}}\right)+C$
B) $\sqrt{2}\arcsin\left(\frac{\cos x}{\sqrt{2}}\right)+C$
C) $\arcsin\left(\frac{\cos x}{\sqrt{2}}\right)+C$
D) $\frac{\arcsin\left(\frac{\cos x}{\sqrt{2}}\right)}{\sqrt{2}}+C$
E) $-\sqrt{2}\arcsin\left(\frac{\cos x}{\sqrt{2}}\right)+C$

8. $\int \frac{1}{\sqrt{(x-x^2)}}dx = ?$

A) $\arcsin\frac{x}{2}+C$
B) $2\arcsin\frac{3x}{2}+C$
C) $\arctan\sqrt{x}+C$
D) $2\arcsin\sqrt{x}+C$
E) $\arcsin(2x-1)+C$

9. $\displaystyle\int \frac{\sin 3x}{4+\cos^2 3x}dx = ?$

A) $\frac{1}{12}\arctan(\cos^2 3x)+C$

B) $\frac{1}{6}\ln(\cos 3x)+C$

C) $\frac{1}{6}\text{arccot}(\frac{1}{2}\cos 3x)+C$

D) $-\frac{1}{2}\arctan(\cos 3x)+C$

E) $-\frac{1}{6}\arctan(\frac{1}{2}\cos 3x)+C$

10. $\displaystyle\int \ln x dx = ?$

A) $x \cdot \ln x + C$

B) $x \cdot \ln x - x + C$

C) $\ln x - x = C$

D) $x^2 \cdot \ln x - 2x + C$

E) $\frac{x}{2} + x \cdot \ln x + C$

11. $\displaystyle\int \arctan x \, dx = ?$

A) $\arctan x - \ln|1+x^2| + C$

B) $\arctan x + \ln|1+x^2| + C$

C) $x \cdot \arctan x - \frac{1}{2}\ln|1+x^2| + C$

D) $x \cdot \arctan x + \ln|1+x^2| + C$

E) $x \cdot \arctan x - \frac{1}{2}x + C$

12. $\displaystyle\int x \cdot \cos x \, dx = ?$

A) $x \cdot \sin x + \cos x + C$

B) $x \cdot \cos x + \sin x + C$

C) $x \cdot \sin x + \sin x + C$

D) $x \cdot \cos x - \frac{1}{2}\sin x + C$

E) $\sin x - \cos x + C$

13. $\displaystyle\int x \cdot e^x dx = ?$

A) $x \cdot e^x + e^x + C$

B) $x \cdot e^x - x + C$

C) $x \cdot e^x + x + C$

D) $x \cdot e^x - e^x + C$

E) $x \cdot e^x + C$

14. $\displaystyle\int \left(\frac{f'(x)\cdot g(x) - f(x)\cdot g'(x)}{g^2(x)}\right)dx = 2x+1,$

$$\left.\begin{array}{l} f(1)=4, \\ g(1)=2, \\ g(2)=8 \end{array}\right\} \Rightarrow \mathbf{f(2) = ?}$$

A) 16 B) 20 C) 24 D) 32 E) 36

15. $\displaystyle\int \frac{f'(x)}{2\sqrt{f(x)}}dx = x^2+2x, \ f(2)=9 \Rightarrow \mathbf{f(x) = ?}$

A) $(x^2 + 2x - 2)^2$

B) $(x^2 + 2x - 3)^2$

C) $(x^2 + 2x - 4)^2$

D) $(x^2 + 2x - 5)^2$

E) $(x^2 + 2x - 6)^2$

16. $$\left.\begin{array}{l} \int g'[f(x)]\cdot f'(x)dx = 4x+3 \\ f(x)=2x-1 \\ g(2)=7 \end{array}\right\} \Rightarrow \mathbf{g(x) = ?}$$

A) 2x + 1 B) 2x + 2 C) 2x + 3

D) 2x + 4 E) 2x + 5

YANITLAR / ANSWERS

1	2	3	4	5	6	7	8
E	B	A	A	C	E	C	E
9	10	11	12	13	14	15	16
E	B	C	A	D	D	D	C

BÖLÜM CHAPTER 31

TARİHSEL NOT / HISTORICAL NOTE

AUGUSTİN LOUIS CAUCHY [1789 - 1857]

Fransız matematikçi. Analizin temellerine kesinlik kazandıran çalışmaları, karmaşık değişkenli fonksiyonlar teorisine katkılarıyla tanınır. 27 yaşına geldiğinde Cauchy çağının en önde gelen matematikçilerinden biri sayılıyordu.1814'te belirli integrallere ilişkin teoremlerini yayımladı. Kendi adıyla anılan ünlü integral kuramını ortaya attı.

French mathematician. His work, which gives certainty to the fundamentals of analysis, is known for his contributions to the theory of functions of complex variables. At the age of 27, Cauchy was considered to be one of the leading mathematicians of his time. In 1814, he published his theorems on specific integrals. He also introduced the famous integral theory referred to in his name.

BELİRLİ İNTEGRAL / DEFINITE INTEGRAL

Bu bölüm, 132 test sorusu içermektedir.
This chapter includes 132 test questions.

BÖLÜM / CHAPTER 31

BELİRLİ İNTEGRAL / DEFINITE INTEGRAL

- Belirli İntegral / Definite Integral .... 611 – 630

BÖLÜM 31 CHAPTER

BELİRLİ İNTEGRAL
DEFINITE INTEGRAL

BELİRLİ İNTEGRAL / DEFINITE INTEGRAL

- ▼ Belirli integraller aşağıdaki gibi ifade edilir.
- ▼ I definite integrals are expressed as follows.
- ▼ $f:[a, b] \rightarrow R,\ \ y = f(x)$

f(x), [a, b] aralığında integrallenebilir bir fonksiyon olmak üzere,

If the function f(x)'s integral can be calculated in [a, b],

$$\int f(x)dx = F(x)+C \Rightarrow \int_a^b f(x)dx = F(x)\Big|_a^b = F(b)-f(a)$$

ifadesine f fonksiyonunun [a, b] aralığındaki ***belirli integrali*** denir.

is called the ***definite integral*** of the function f in the interval [a, b].

Özellik / Property

★ $\int_a^b k\cdot f(x)\,dx = \mathbf{k}\cdot\int_a^b \mathbf{f(x)dx},\ \ (\mathbf{k}\in \mathbf{R})$

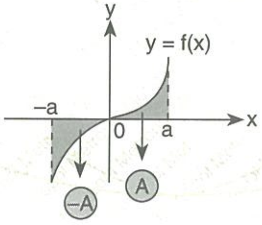
Özellik / Property

★ $\int_a^b (f(x)\pm g(x))\,dx = \left(\int_a^b f(x)dx\right)\pm\left(\int_a^b g(x)dx\right)$

Özellik / Property

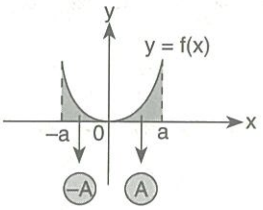
★ **i)** f(x) tek fonksiyon / is an odd function

$\Rightarrow \int_{-a}^{a} f(x)dx = 0$ ⇒



ii) f(x) çift fonksiyon / is an ever function

$\Rightarrow \int_{-a}^{a} f(x)dx = 2\cdot\int_{0}^{a} f(x)dx$ ⇒



Özellik / Property

★ $\int_a^b f(x)\,dx = -\int_b^a \mathbf{f(x)\,dx}$

Özellik / Property

★ $a < c < b$

$\Rightarrow \int_a^b f(x)\,dx = \int_a^c \mathbf{f(x)\,dx} + \int_c^b \mathbf{f(x)\,dx}$

İNTEGRALİN TÜREVİ / DERIVATIVE of an INTEGRAL

Özellik / Property

★ $a \in R,\ F(x) = \int_a^x f(t)\,dt \Rightarrow F'(x) = f(x)$

Özellik / Property

★ $a \in R,\ F(x) = \int_a^{u(x)} f(t)\,dt \Rightarrow F'(x) = u'(x) \cdot f(u(x))$

Özellik / Property

★ $F(x) = \int_{v(x)}^{u(x)} f(t)\,dt \Rightarrow F'(x) = u'(x) \cdot f(u(x)) - v'(x) \cdot f(v(x))$

Önemli / Important

- $a, b \in R$

  $F(x) = \int_a^b f(t)\,dt \Rightarrow \mathbf{F'(x) = 0}$

MUTLAK DEĞER FONKSİYONUNUN İNTEGRALİ / INTEGRAL of THE ABSOLUTE VALUE FUNCTION

▼ $\int_a^b |f(x)|\,dx$ ntegrali hesaplanırken f(x) = 0 denkleminin kökleri bulunur. f(x) fonksiyonunun sıfırdan büyük veya sıfırdan küçük değerler almasına göre, integral sınırları köklere göre parçalanarak hesaplanır.

▼ While the integral $\int_a^b |f(x)|\,dx$ is found, firstly the roots of the equation f(x) = 0 and critical points should be found. Integral is found by slicing to intervals with end points of these points.

1. $\int_{1}^{2} (2x+5)\,dx = ?$

A) 6
B) 8
C) 9
D) 10
E) 12

2. $\int_{0}^{1} (x^2-2x+2)\,dx = ?$

A) $\frac{4}{3}$
B) $\frac{5}{2}$
C) 3
D) $\frac{9}{2}$
E) 5

3. $\int_{-1}^{1} (x+1)^2\,dx = ?$

A) $\frac{8}{3}$
B) 3
C) 4
D) $\frac{17}{4}$
E) $\frac{13}{3}$

4. $\int_{0}^{2} \sqrt{4x+1}\,dx = ?$

A) 3
B) $\frac{7}{2}$
C) 4
D) $\frac{17}{4}$
E) $\frac{13}{3}$

5. $\int_{0}^{1} \frac{dx}{(2x+1)^3} = ?$

A) $\frac{1}{3}$
B) $\frac{4}{9}$
C) $\frac{2}{9}$
D) $\frac{1}{12}$
E) $\frac{1}{15}$

6. $\int_{0}^{1} (x^2+3)x\,dx = ?$

A) $\frac{7}{4}$
B) $\frac{11}{2}$
C) $\frac{13}{2}$
D) $\frac{17}{4}$
E) 6

7. $\int_{0}^{4} \frac{x\,dx}{\sqrt{3x^2+2}} = ?$

A) $\frac{\sqrt{3}}{3}$
B) $\frac{2\sqrt{3}}{3}$
C) $\frac{4\sqrt{2}}{3}$
D) $\frac{4\sqrt{3}}{3}$
E) $2\sqrt{2}$

8. $\int_{0}^{2} \frac{x\,dx}{x^2+4} = ?$

A) $\ln 4$
B) 2
C) $\ln 2$
D) $\ln\sqrt{2}$
E) $\ln 2+\ln\sqrt{2}$

9. $\displaystyle\int_4^5 \frac{2}{(x-3)^3}dx = ?$

A) 2 B) $\frac{3}{2}$ C) 1 D) $\frac{2}{3}$ E) $\frac{3}{4}$

10. $\displaystyle\int_0^1 (x^2-2x)\cdot(x-1)dx = ?$

A) $-\frac{1}{4}$ B) $-\frac{1}{2}$ C) 0 D) $\frac{1}{2}$ E) $\frac{1}{4}$

11. $\displaystyle\int_0^{e-1} \frac{x}{x+1}dx = ?$

A) e + 1 B) e C) e – 1 D) e – 2 E) e – 3

12. $\displaystyle f(x) = \int_0^x \sqrt{1+t^2}\,dt \Rightarrow f'(2\sqrt{2}) = ?$

A) 1 B) 2 C) 3 D) 4 E) 6

13. $\displaystyle f(x) = \int_3^{\ln x} \frac{t^2}{1+2t}dt \Rightarrow f'(e) = ?$

A) $\frac{1}{2}$ B) $\frac{1}{3}$ C) $\frac{1}{e}$ D) $\frac{1}{2e}$ E) $\frac{1}{3e}$

14. $\displaystyle f(x) = \int_1^{2x} \cos t^2\,dt \Rightarrow f'(x) = ?$

A) $\cos x^2$ B) $\cos 4x^2$ C) $2\cos 4x$ D) $2\cos 4x^2$ E) $2x\cos 4x^2$

15. $\displaystyle f(x) = \int_{\sin x}^{0} \frac{dt}{1+t} \Rightarrow f'\left(\frac{\pi}{6}\right) = ?$

A) $\frac{3}{2}$ B) $\frac{2}{3}$ C) $-\frac{\sqrt{3}}{6}$ D) $-\frac{1}{3}$ E) $-\frac{\sqrt{3}}{3}$

16. $\displaystyle F(x) = \int_x^{x^2} \ln t\,dt \Rightarrow F'(e) = ?$

A) 4e –1 B) 2e – 1 C) 3 D) 2 E) 4e + 1

17. $\displaystyle F(x) = \int_x^{\ln x} e^t\,dt \Rightarrow F'(1) = ?$

A) 1 + e B) 1 – e C) e D) 0 E) 1

YANITLAR / ANSWERS

1	2	3	4	5	6	7	8	9
B	A	A	E	C	A	C	D	E
10	11	12	13	14	15	16	17	
E	D	C	E	D	E	A	B	

1. $f(x)=\int\limits_{2}^{4x}\sin t^2\,dt \;\Rightarrow\; f'(x)=?$

A) $4\sin(16x^2)$
B) $4\sin\left(\frac{x^2}{4}\right)$
C) $2\cos(16x^2)$
D) $8\cos\left(\frac{x^2}{4}\right)$
E) $4\sin^2(4x^2)$

2. $f(x)=\int\limits_{\sin x}^{0}\frac{dt}{2+t^2} \;\Rightarrow\; f'\left(\frac{\pi}{3}\right)=?$

A) $-\frac{1}{8}$
B) $-\frac{2}{11}$
C) $\frac{3}{11}$
D) $\frac{2}{7}$
E) $\frac{2}{13}$

3. $f(x)=\int\limits_{x}^{x^2}\frac{2+t}{t}\,dt \;\Rightarrow\; f'(x)=?$

A) $2x+\frac{2}{x}-1$
B) $x+\frac{1}{x}-1$
C) $\frac{x^2+x+1}{x}$
D) $2x^2-\frac{1}{x}$
E) $\frac{2x^2+6x-2}{x}$

4. $\int\limits_{-2}^{2}(x^2-4x)\,dx=?$

A) 24
B) 8
C) $\frac{16}{3}$
D) 0
E) –8

5. $\int\limits_{1}^{3}(x^2-3x+2)\,dx=?$

A) 3
B) $\frac{7}{3}$
C) $\frac{4}{3}$
D) 1
E) $\frac{2}{3}$

6. $\int\limits_{2}^{3}\left(\frac{3x^3-2x^2+x}{x}\right)dx=?$

A) 12
B) 15
C) 16
D) 18
E) 25

7. $\int\limits_{1}^{4}\left(\sqrt{x}+\frac{1}{\sqrt{x}}\right)dx=?$

A) 9
B) 8
C) $\frac{32}{3}$
D) $\frac{22}{3}$
E) $\frac{20}{3}$

8. $\int\limits_{3}^{6}t^2\,dt=?$

A) 27
B) 48
C) 54
D) 63
E) 72

9. $\int_{0}^{2}(x+1)(2x+6)dx = ?$

A) 33 B) $\frac{100}{3}$ C) 32 D) $\frac{97}{3}$ E) $\frac{50}{3}$

10. $\int_{\sqrt{5}}^{2\sqrt{5}}\frac{x}{x^2-4}dx = ?$

A) 4 B) ln2 C) ln4 D) 1 + ln2 E) $4\log_2 e$

11. $\int_{1}^{2}\frac{(x^2+2)dx}{(x^3+6x-2)^2} = ?$

A) $\frac{7}{36}$ B) $\frac{11}{72}$ C) $\frac{13}{144}$ D) $\frac{13}{270}$ E) $\frac{17}{288}$

12. $\int_{0}^{1}(x^3-3x^2+6)(x^2-2x)dx = ?$

A) $-\frac{20}{3}$ B) $-\frac{10}{3}$ C) $\frac{1}{2}$ D) $\frac{4}{9}$ E) $\frac{13}{6}$

13. $\int_{\frac{e}{2}}^{\frac{e^3}{2}}\left(\frac{\ln^3(2x)}{x}\right)dx = ?$

A) $\frac{40}{3}$ B) 20 C) 24 D) $\frac{76}{3}$ E) 26

14. $\int_{0}^{1}e^{x^2+x-2}\cdot(2x+1)dx = ?$

A) $\frac{1}{e}$ B) $\frac{1-e}{e^2}$ C) $1+\frac{e}{3}$ D) $\frac{2-e^2}{e^2}$ E) $\frac{e^2-1}{e^2}$

15. $\int_{0}^{\frac{\pi}{4}}\left(\frac{\tan x+1}{\cos x}\right)^2 dx = ?$

A) $\frac{7}{3}$ B) 2 C) $\frac{3}{2}$ D) $\frac{2}{3}$ E) $\frac{1}{3}$

16. $\int_{\frac{\sqrt{3}}{2}}^{\frac{3}{2}}\frac{dx}{9+4x^2} = ?$

A) $\frac{\pi}{12}$ B) $\frac{\pi}{24}$ C) $\frac{\pi}{36}$ D) $\frac{\pi}{54}$ E) $\frac{\pi}{72}$

17. $\int_{0}^{\frac{\pi}{12}}\sin^2 4x\cos 2x\,dx = ?$

A) $\frac{1}{16}$ B) $\frac{11}{120}$ C) $\frac{17}{240}$ D) $\frac{23}{120}$ E) $\frac{19}{80}$

YANITLAR / ANSWERS

1	2	3	4	5	6	7	8	9
A	B	A	C	E	B	E	D	B
10	11	12	13	14	15	16	17	
C	D	B	B	E	A	E	C	

1. $\displaystyle\int_1^e \frac{\sin(\ln x)}{x}dx = ?$

A) $e-\cos 1$
B) $1-\cos 1$
C) $e+\cos 1$
D) $1+\cos 1$
E) 1

2. $\displaystyle\int_{\frac{\pi}{2}}^{\frac{\pi}{4}} \frac{\sqrt[3]{\cot x}}{\sin^2 x}dx = ?$

A) $-\frac{3}{2}$
B) -1
C) $-\frac{3}{4}$
D) $\frac{3}{4}$
E) $\frac{3}{2}$

3. $\displaystyle\int_a^b (2x-3)dx = 16, a-b=4 \Rightarrow a+b = ?$

A) -2
B) -1
C) 0
D) 1
E) 2

4. $\displaystyle f(x)=\int_x^{x^2}(t^2+t)dt \Rightarrow f'(1) = ?$

A) -3
B) -1
C) 0
D) 1
E) 2

5. $\displaystyle f(x)=\int_2^{\ln(x^5+3)} e^t dt \Rightarrow f'(x) = ?$

A) $\ln x^4$
B) x^5+3
C) $5x^4$
D) 1
E) 3

6. $\displaystyle\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \sqrt{1+\sin 2x}\,dx = ?$

A) $\frac{\sqrt{3}-1}{2}$
B) $\frac{\sqrt{3}+1}{2}$
C) 0
D) $\sqrt{3}-1$
E) $\sqrt{3}+1$

7. $\displaystyle\int_0^3 \left[\sqrt{9-x^2}-(3-x)\right]dx = ?$

A) π
B) 3π
C) $\frac{9\pi}{4}$
D) $\frac{9\pi-2}{2}$
E) $\frac{9\pi-18}{4}$

8. $\displaystyle\int_0^1 \frac{\sqrt{x}+1}{\sqrt[3]{x}}dx = ?$

A) $\frac{8}{7}$
B) $\frac{21}{20}$
C) $\frac{28}{9}$
D) $\frac{32}{15}$
E) $\frac{33}{14}$

9. $\displaystyle\int_{0}^{\frac{\pi}{2}} \frac{1+\cos x+\sin x}{1+\sin x}dx = ?$

A) $\pi+\ln 2$ B) $\pi+\ln 2+1$ C) $\ln 2$ D) $\frac{\pi}{2}+\ln 2-1$ E) $\frac{\pi}{2}+\ln 2$

10. $\displaystyle\int_{\frac{\pi}{8}}^{\frac{\pi}{4}} (\tan x-\cot x)dx = ?$

A) $-\ln 2$ B) $\frac{-\ln 2}{2}$ C) 0 D) $\ln 2$ E) $\frac{\ln 2}{2}$

11. $\displaystyle\int_{2}^{3} \frac{6x^2-4x}{x^3-x^2+m}dx=\ln 9 \Rightarrow m = ?$

A) 3 B) 4 C) 5 D) 6 E) 7

12. $\displaystyle f(x)=\int_{3}^{e^x} (5-t^2)\cdot\ln t\cdot dt \Rightarrow f'(\ln 2) = ?$

A) $\ln 2$ B) $\ln 4$ C) $\ln 8$ D) 1 E) 2

13. $\displaystyle\int_{4}^{e+3} \left(\frac{x+5}{x-3}\right)dx = ?$

A) $e+3$ B) $e+4$ C) $e+5$ D) $e+6$ E) $e+7$

14. $\displaystyle\int_{0}^{3} (|2x-4|+2)\,dx = ?$

A) 8 B) 9 C) 10 D) 11 E) 12

15. $\displaystyle\int_{\frac{\pi}{2}}^{\pi} \frac{\sin 2x}{2-\cos^2 x}dx = ?$

A) $-2\ln 2$ B) $-\ln 2$ C) $\ln 2$ D) $2\ln 2$ E) $4\ln 2$

16. $\displaystyle 0<\alpha<\frac{\pi}{3},\ \int_{0}^{\alpha} (\tan^3 x+\tan x)\,dx=\frac{1}{6} \Rightarrow \alpha = ?$

A) $\frac{\pi}{6}$ B) $\frac{\pi}{4}$ C) $\frac{\pi}{3}$ D) $\frac{\pi}{2}$ E) π

YANITLAR / ANSWERS

1	2	3	4	5	6	7	8
B	C	B	E	C	D	E	E
9	10	11	12	13	14	15	16
E	B	A	B	E	D	B	A

1. $\displaystyle\int_{e}^{e^2} \frac{dx}{x\ln x} = ?$

A) $\frac{1}{2}$ B) 2 C) $-\ln2$ D) $\ln2$ E) $2\ln2$

2. $\displaystyle\int_{0}^{\ln2} e^{-2x}\,dx = ?$

A) $\frac{3}{4}$ B) $\frac{3}{8}$ C) $\frac{3}{16}$ D) $\frac{5}{6}$ E) $\frac{5}{8}$

3. $\displaystyle\int_{0}^{1} 3x^2\sqrt{x^3+1}\,dx = ?$

A) $\frac{2\sqrt{3}-2}{3}$ B) $\frac{4\sqrt{2}-2}{3}$ C) $\frac{5\sqrt{2}-2}{3}$

D) $3\sqrt{3}-2$ E) $4\sqrt{3}-2$

4. $\displaystyle\int x^2 f(x)dx = x^4+x^3+C \Rightarrow \int_{1}^{2} f(x)\,dx = ?$

A) 6 B) 7 C) 8 D) 9 E) 10

5. $\displaystyle\int_{1}^{3} \frac{x+3}{x+2}dx = ?$

A) $1+\ln\frac{3}{2}$ B) $2+\ln\frac{5}{3}$ C) $1+\ln\frac{4}{3}$

D) $1+\ln\frac{3}{4}$ E) $2\ln2$

6. $\displaystyle\int_{0}^{1} \frac{x^2}{x+2}dx = ?$

A) $-\frac{1}{2}+2\ln2$ B) $-\frac{1}{2}+4\ln2$ C) $-\frac{3}{2}+2\ln2$

D) $-\frac{3}{2}+4\ln\frac{3}{2}$ E) $-1+4\ln\frac{3}{2}$

7. $\displaystyle\int_{\ln2}^{\ln3} \frac{e^{3x}+1}{e^x}dx = ?$

A) $\frac{8}{3}$ B) $\frac{10}{3}$ C) $\frac{14}{3}$ D) $\frac{16}{3}$ E) $\frac{20}{3}$

8. $\displaystyle\int_{1}^{4} \frac{dx}{1+\sqrt{x}} = ?$

A) $2+2\ln6$ B) $1+2\ln\frac{1}{3}$ C) $2+2\ln\frac{2}{3}$

D) $2+\ln\frac{4}{3}$ E) $1+\ln\frac{4}{9}$

9. $\displaystyle\int_{e}^{e^2} \frac{dx}{x(\ln x)^2} = ?$

A) 2 B) $\frac{3}{2}$ C) $\frac{4}{3}$ D) 1 E) $\frac{1}{2}$

10. $\displaystyle\int_{3}^{6} x \cdot d(2x+3) = ?$

A) 27 B) 54 C) 63 D) 72 E) 87

11. $\displaystyle\int_{a}^{a+3} (x+1)dx = 12 \Rightarrow a = ?$

A) $\frac{1}{2}$ B) 1 C) $\frac{3}{2}$ D) 2 E) 3

12. $\displaystyle\int_{1-a}^{e-a} \frac{x}{x+a}dx = e-6 \Rightarrow a = ?$

A) 3 B) 4 C) 5 D) 6 E) 9

13. $\displaystyle\int_{0}^{1} \frac{x-1}{x^2-2x+a}dx = \ln\sqrt{\frac{2}{3}} \Rightarrow a = ?$

A) 2 B) 3 C) 4 D) 6 E) 9

14. $\displaystyle\int_{a}^{b} (4x+1)dx = 30$ ve / and $b-a=6 \Rightarrow a+b = ?$

A) 12 B) 8 C) 5 D) 4 E) 2

15. $\displaystyle\int_{a}^{b} (4x+5)dx = 150, a+b=10 \Rightarrow a \cdot b = ?$

A) 24 B) 21 C) 16 D) 9 E) 8

16. $f: R-\{4\} \rightarrow R-\{-2\}$

$\displaystyle f(x) = \frac{-2x+3}{x-4} \Rightarrow \int_{1}^{2} d\left(f^{-1}(x)\right) = ?$

A) $\frac{5}{12}$ B) $\frac{3}{8}$ C) $\frac{7}{16}$ D) $\frac{1}{4}$ E) $\frac{2}{3}$

YANITLAR / ANSWERS

1	2	3	4	5	6	7	8
D	B	B	D	B	D	A	C
9	**10**	**11**	**12**	**13**	**14**	**15**	**16**
E	A	C	C	B	E	C	A

1. $\displaystyle\int_{1}^{2} \frac{x^2+1}{x^2}\,dx = ?$

A) $\frac{3}{2}$ B) $\frac{4}{3}$ C) $\frac{5}{2}$ D) $\frac{5}{3}$ E) $\frac{5}{4}$

2. $\displaystyle\int_{e^{-1}}^{1} \frac{\sqrt{1+\ln x}}{x}\,dx = ?$

A) $-\frac{3}{2}$ B) $-\frac{2}{3}$ C) $\frac{1}{2}$ D) $\frac{2}{3}$ E) $\frac{3}{2}$

3. $\displaystyle\int_{0}^{1} (3x^2-1)\cdot e^x\,dx = ?$

A) $2e-6$ B) $2e-5$ C) $2e-4$ D) $e-6$ E) $e-5$

4. $\displaystyle\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \frac{\cos x}{1-\cos^2 x}\,dx = ?$

A) -2 B) -1 C) 0 D) 1 E) 2

5. $\displaystyle\int_{0}^{\frac{\pi}{2}} x^2\cdot\sin x\,dx = ?$

A) π^2-2 B) π^2-1 C) $\pi-2$ D) $\pi+2$ E) $\pi-1$

6. $\displaystyle\int_{0}^{1} \frac{e^x}{e^x+e^{-x}}\,dx = ?$

A) $\frac{1}{2}\ln\frac{e^2+1}{2}$ B) $\frac{1}{2}\ln\frac{e^2+2}{2}$

C) $\frac{1}{2}\ln\frac{e^2-1}{2}$ D) $\frac{1}{2}\ln\frac{e^2-2}{2}$

E) $\frac{1}{4}\ln\frac{e^2+1}{2}$

7. $\displaystyle\int_{0}^{\frac{\pi}{4}} \frac{\cos^3 x+3}{\cos^2 x}\,dx = ?$

A) $\frac{\sqrt{2}+3}{2}$ B) $\frac{\sqrt{2}+6}{2}$ C) $\frac{\sqrt{2}-3}{2}$ D) $\frac{\sqrt{2}-6}{2}$ E) $\frac{\sqrt{2}-8}{2}$

8. $\displaystyle\int_{0}^{4} (x-2)\cdot(x^2-4x+1)^3\,dx = ?$

A) $-\frac{1}{4}$ B) $-\frac{1}{2}$ C) 0 D) $\frac{1}{2}$ E) $\frac{1}{4}$

9. $\displaystyle\int_{-1}^{1} \frac{x^2}{x^2+1}\,dx = ?$

A) $-\frac{\pi}{2}$ B) $\frac{\pi-4}{2}$ C) $\frac{-\pi-4}{2}$ D) $\frac{\pi+4}{2}$ E) $\frac{4-\pi}{2}$

10. $\displaystyle\int_{e^3}^{e^4} \frac{dx}{x\cdot \ln x} = ?$

A) $\ln\frac{2}{3}$ B) $\ln\frac{3}{2}$ C) $\ln\frac{3}{4}$ D) $\ln\frac{4}{3}$ E) $\ln\frac{1}{2}$

11. $f(x)=\begin{cases} x^2-2 & x\geq 1 \\ x+3 & x<1 \end{cases} \Rightarrow \displaystyle\int_{-2}^{3} f(x)\,dx = ?$

A) $\frac{43}{5}$ B) $\frac{53}{6}$ C) $\frac{63}{5}$ D) $\frac{73}{6}$ E) $\frac{83}{5}$

12. $\displaystyle\int_{2}^{5} |2x-8|\,dx = ?$

A) 5 B) 7 C) 9 D) 11 E) 13

13. $\displaystyle\frac{d}{dx}\left[\left(\int_{-3}^{5}(x^4-3x^3+2)\,dx\right)\right] = ?$

A) –975 B) –162 C) 0 D) 162 E) 975

14. $\displaystyle\int_{\frac{\pi}{2}}^{\pi} \sin x\cdot\cos^5 x\,dx = ?$

A) $-\frac{1}{12}$ B) $-\frac{1}{6}$ C) $-\frac{1}{4}$ D) $-\frac{1}{3}$ E) $-\frac{1}{2}$

15. $\displaystyle\int_{0}^{\frac{\pi}{2}} e^{\cos^2 x}\cdot \sin 2x\,dx = ?$

A) e – 2 B) e – 1 C) e D) e + 1 E) e + 2

16. $\displaystyle\int_{-2}^{1} 3x\cdot|x+1|\,dx = ?$

A) $-\frac{1}{6}$ B) $-\frac{1}{4}$ C) $-\frac{1}{3}$ D) $-\frac{1}{2}$ E) – 1

YANITLAR / ANSWERS

1	2	3	4	5	6	7	8
A	D	B	D	C	A	B	C
9	10	11	12	13	14	15	16
E	D	D	A	C	B	B	D

1. $\displaystyle\int_{1}^{e^{-1}} \frac{x^2-1}{x^3}dx = ?$

A) $\frac{e^2+1}{2}$ B) $\frac{e^2+2}{2}$ C) $\frac{e^2-3}{2}$

D) $\frac{e^2-1}{2}$ E) $\frac{e^2-2}{2}$

2. $f(x)=\displaystyle\int_{x}^{x^2}(2t-1)dx \Rightarrow f'(2)=?$

A) 20 B) 22 C) 25 D) 27 E) 30

3. $\displaystyle\frac{d}{dx}\left(\int_{1}^{4}\frac{x^2+x+1}{x^2-1}dx\right)=?$

A) –4 B) –3 C) 0 D) 2 E) 4

4. $\displaystyle\int_{e^{-2}}^{e^{2}}\frac{3}{t}dt = ?$

A) –12 B) –6 C) 0 D) 6 E) 12

5. $\displaystyle\int_{1}^{3}(3x^2+m)dx=28 \Rightarrow m=?$

A) –2 B) – 1 C) 0 D) 1 E) 2

6. $\displaystyle\int_{-3}^{0}(x^5+x^3)dx+\int_{0}^{2}(x^5+x^3)dx+\int_{2}^{3}(x^5+x^3)dx=?$

A) –3 B) –2 C) 0 D) 2 E) 3

7. $\displaystyle\int_{\pi/4}^{\pi}\tan^2x\,dx = ?$

A) $\frac{-3\pi-4}{4}$ B) $\frac{-2\pi+3}{4}$ C) $\frac{4-3\pi}{4}$

D) $\frac{4+3\pi}{4}$ E) $\frac{3\pi-4}{4}$

8. $f(x)=\begin{cases}2x^2-3, & x\geq 0\\ 3x-1\ , & x<0\end{cases} \Rightarrow \displaystyle\int_{-1}^{2}f(x)dx = ?$

A) $-\frac{5}{3}$ B) $-\frac{19}{6}$ C) 2 D) $\frac{13}{6}$ E) $\frac{7}{3}$

9. $\int_{-3}^{2} |x-1|\,dx = ?$

A) 7 B) $\frac{15}{2}$ C) 8 D) $\frac{17}{2}$ E) 9

10. $\int_{-1}^{1} \frac{2x}{\sqrt{1-x^4}}\,dx = ?$

A) $-\frac{\pi}{4}$ B) $-\frac{\pi}{2}$ C) 0 D) $\frac{\pi}{2}$ E) $\frac{\pi}{4}$

11. $\int_{0}^{\pi} \sin^2 x\,dx = ?$

A) $-\frac{\pi}{2}$ B) 0 C) $\frac{\pi}{2}$ D) $\frac{\pi}{4}$ E) $\frac{\pi}{6}$

12. $\int_{\ln 2}^{\ln 4} x \cdot e^x\,dx = ?$

A) ln2 – 4 B) 3ln2 – 2 C) 5ln2 – 4
D) 6ln2 – 4 E) 6ln2 – 2

13. $\int_{0}^{2} \frac{dx}{x^2-2x+2} = ?$

A) $-\frac{3\pi}{2}$ B) $-\frac{\pi}{2}$ C) 0 D) $\frac{\pi}{2}$ E) $\frac{3\pi}{2}$

14. $\int_{\ln 2}^{\ln 3} \frac{e^{2x}}{1+e^{2x}}\,dx = ?$

A) 0 B) $\ln\frac{1}{2}$ C) $\ln\sqrt{2}$
D) ln4 E) ln6

15. $\int_{\pi/4}^{\pi/3} (\tan^5 x + \tan^3 x)\,dx = ?$

A) – 2 B) –1 C) 0 D) 1 E) 2

16. $\int_{-1}^{0} (x^3+4)^2 \cdot x^2\,dx = ?$

A) $\frac{11}{3}$ B) $\frac{34}{9}$ C) $\frac{35}{9}$ D) 4 E) $\frac{37}{9}$

YANITLAR / ANSWERS

1	2	3	4	5	6	7	8
C	C	C	E	D	C	A	B
9	**10**	**11**	**12**	**13**	**14**	**15**	**16**
D	C	C	E	A	C	E	E

1. $\int_{-1}^{0} x(x-a)\,dx = \frac{4}{3} \Rightarrow$ **a = ?**

A) 2 B) 1 C) 0 D) –1 E) –2

2. $\int_{-2}^{2} x^{\frac{5}{3}}\,dx =$ **?**

A) $\frac{3\sqrt[2]{2}}{8}$ B) $\frac{3\sqrt[2]{2}}{4}$ C) 0 D) 2 E) $\frac{3}{8}$

3. $\int_{0}^{b} 24x^2\,dx = \int_{0}^{b} 6x^5\,dx \Rightarrow$ **b = ?**

A) $3\sqrt{4}$ B) $\sqrt{2}$ C) 2 D) –4 E) –2

4. $\int_{0}^{2} \frac{x}{2+x^2}\,dx = ?$

A) 3 B) $\frac{7}{2}$ C) $\ln\sqrt{3}$ D) ln2 E) 4

5. $\int_{1}^{4} \frac{1+\sqrt{x}}{\sqrt{x}}\,dx = ?$

A) 5 B) 4 C) 3 D) 2 E) 1

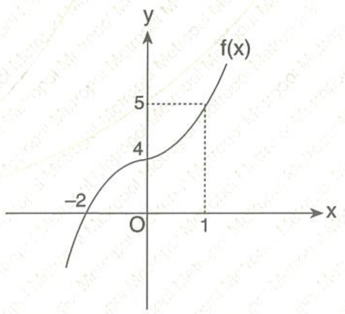
6. $f(x) = \begin{cases} 2x+3, & x<3 \\ 3x^2+1, & x \geq 3 \end{cases} \Rightarrow \int_{0}^{4} f(x)\,dx = ?$

A) 38 B) 44 C) 56 D) 62 E) 74

7. $f(x) = \begin{cases} 2x+a, & x<1 \\ 3x^2+4, & x \geq 1 \end{cases}, \int_{0}^{2} f(x)\,dx = 20 \Rightarrow$ **a = ?**

A) 4 B) 5 C) 6 D) 7 E) 8

8.



$\Rightarrow \int_{-2}^{0} (f(x))^3 \cdot f'(x)\,dx = ?$

A) 32 B) 40 C) 48 D) 56 E) 64

9. $\int_{1}^{4} |x-2|\,dx = ?$

A) $\frac{3}{2}$ B) $\frac{5}{2}$ C) $\frac{7}{2}$ D) $\frac{9}{2}$ E) $\frac{11}{2}$

10. $\int_{-1}^{1} |x^2+x|\,dx = ?$

A) $\frac{1}{2}$ B) 1 C) $\frac{3}{2}$ D) 2 E) $\frac{5}{2}$

11. $\int_{0}^{\pi} (\cos x + |\cos x|)\,dx = ?$

A) 1 B) 2 C) 3 D) 4 E) 5

12. $\int_{0}^{\frac{\pi}{3}} (2\sin 2x - 1)\,dx = ?$

A) $3+2\sqrt{3}$ B) $-\frac{\sqrt{3}}{4}$ C) $\frac{3\sqrt{3}}{4}$ D) $4-2\sqrt{3}$ E) $4+2\sqrt{3}$

13. $A = \int_{-3}^{3} |x^2-4|\,dx \Rightarrow A = ?$

A) 21 B) $\frac{62}{3}$ C) $\frac{46}{3}$ D) 15 E) $\frac{32}{3}$

14. $\int_{0}^{1} x^3 e^x\,dx = ?$

A) 3 + e B) 3 − e C) $\frac{1}{2}(3-2e)$ D) 2(3 − e) E) 2(e + 3)

15. $\int_{1}^{e} \ln x\,dx = ?$

A) e B) 1 C) 1 − e D) 2 + e E) 2 − e

16. $\int_{0}^{1} \arctan x\,dx = ?$

A) $\frac{\pi - \ln 4}{4}$ B) $\frac{\pi - 1}{2}$ C) $\frac{\pi + \ln 2}{2}$ D) ln8 E) $\frac{\pi}{4}$

17. $\int_{4}^{6} \frac{dx}{x^2-4} = ?$

A) $\ln\sqrt{2}$ B) $\ln\sqrt[4]{3}$ C) $\frac{1}{4}\ln\frac{3}{2}$ D) $\ln\sqrt[4]{6}$ E) 1 − ln2

YANITLAR / ANSWERS

1	2	3	4	5	6	7	8	9
A	C	C	C	A	C	E	E	B
10	11	12	13	14	15	16	17	
B	B	B	C	D	B	A	C	

1. $\displaystyle\int_0^{\pi} \sin x \cdot e^{\cos x}\,dx = ?$

A) $e-1$ B) $\frac{e-1}{e}$ C) $\frac{e^2-1}{e}$ D) $\frac{e+1}{e}$ E) $\frac{1-e}{2}$

2. $\displaystyle\int_2^{a} (-2t+5)\,dt = -2 \Rightarrow a = ?$

A) 3 B) 4 C) 5 D) 6 E) 8

3. $\displaystyle\int_{\frac{\pi}{18}}^{\frac{\pi}{6}} \sin^2 3x \cdot \cos 3x\,dx = ?$

A) $\frac{3}{8}$ B) $\frac{5}{16}$ C) $\frac{9}{32}$ D) $\frac{11}{72}$ E) $\frac{7}{72}$

4. $\displaystyle\int_0^{\frac{\pi}{2}} (1+\sin x)^3 \cdot \cos x\,dx = ?$

A) $\frac{1}{2}$ B) $\frac{3}{4}$ C) $\frac{5}{16}$ D) $\frac{15}{4}$ E) 3

5. $\displaystyle\int_0^{\frac{\pi}{3}} \cos^2 x \cdot \sin x\,dx = ?$

A) $-\frac{3}{8}$ B) $-\frac{3}{16}$ C) $\frac{3}{16}$ D) $\frac{5}{16}$ E) $\frac{7}{24}$

6. $\displaystyle\int_0^{\frac{\pi}{6}} (\sin 3x - \cos 3x)^2\,dx = ?$

A) $\frac{\pi+2}{6}$ B) $\frac{\pi-2}{6}$ C) $\frac{\pi-2}{3}$ D) $\frac{\pi+2}{3}$ E) 0

7. $\displaystyle\int_0^{\frac{3}{2}} \frac{dx}{4x^2+9} = ?$

A) $\frac{\pi-4}{6}$ B) $\frac{\pi}{21}$ C) $\frac{\pi}{24}$ D) $\frac{\pi+1}{24}$ E) $\frac{\pi}{12}$

8. $\frac{\pi}{4} < a < \frac{\pi}{2}$

$\displaystyle\int_{\frac{\pi}{4}}^{a} (\tan^5 x + \tan^3 x)\,dx = 2 \Rightarrow a = ?$

A) $\frac{\pi}{12}$ B) $\frac{4\pi}{9}$ C) $\frac{\pi}{3}$ D) $\frac{2\pi}{9}$ E) $\frac{\pi}{6}$

9. $\displaystyle\int_{0}^{\frac{\pi}{2}} \sqrt{1+\cos x}\,dx = ?$

A) 1
B) $\sqrt{2}+1$
C) $\sqrt{3}-1$
D) 2
E) $2\sqrt{2}$

10. $\displaystyle\int_{0}^{\frac{\pi}{2}} \cos 2x \cdot \cos 3x\,dx = ?$

A) $\frac{1}{2}$
B) $\frac{3}{4}$
C) $\frac{5}{8}$
D) $\frac{3}{5}$
E) $\frac{2}{3}$

11. $\displaystyle\int_{0}^{\frac{\pi}{3}} \sec^3 x \cdot \tan x\,dx = ?$

A) $\frac{7}{3}$
B) 2
C) $\frac{5}{3}$
D) $\frac{3}{2}$
E) 1

12. $\displaystyle\int_{0}^{\frac{\pi}{4}} \tan^2 x\,dx = ?$

A) $\frac{1}{4}$
B) $\frac{1+\pi}{3}$
C) $\frac{4-\pi}{8}$
D) $2-\pi$
E) $\frac{4-\pi}{4}$

13. $\displaystyle\int_{\frac{\pi}{2}}^{\pi} \frac{dx}{1-\cos x} = ?$

A) -1
B) $-\frac{1}{2}$
C) $\frac{1}{2}$
D) 1
E) 2

14. $\displaystyle\int_{0}^{\frac{\pi}{2}} \cos 3x \cdot \sin 2x\,dx = ?$

A) -1
B) $-\frac{1}{3}$
C) $-\frac{2}{5}$
D) $-\frac{3}{5}$
E) $\frac{3}{5}$

15. $\displaystyle\int_{1}^{\frac{\pi}{4}} \cos^2 x\,dx = ?$

A) $\frac{\pi}{8}$
B) $\frac{\pi+2}{8}$
C) $\frac{\pi-2}{8}$
D) $\frac{\pi+4}{16}$
E) $\frac{\pi-4}{16}$

16. $\displaystyle\int_{1}^{4} x\ln x\,dx = ?$

A) $\ln 128 - e^3$
B) $8\ln 2 - e$
C) $8\ln 2 + 3$
D) $16\ln 2 - 1$
E) $8\ln 4 - \frac{15}{4}$

17. $\displaystyle\int_{0}^{\frac{\pi}{2}} e^x \sin x\,dx = ?$

A) $\sqrt{e^\pi}-1$
B) $\frac{\sqrt{e^\pi}}{2}-1$
C) $\frac{e^\pi}{2}+1$
D) $\frac{1}{2}(\sqrt{e^\pi}+1)$
E) $\frac{\sqrt{e}-1}{2}$

YANITLAR / ANSWERS

1	2	3	4	5	6	7	8	9
C	B	E	D	E	B	C	C	D
10	11	12	13	14	15	16	17	
D	A	E	D	C	B	E	D	

BÖLÜM CHAPTER 32

TARİHSEL NOT / HISTORICAL NOTE

GEORG FRİEDRİCH BERNHARD RİEMANN [1826 – 1866]

Alman matematikçi. Analiz, sayı teorisi ve diferansiyel geometriye katkıda bulunan, gerçek analiz alanında, integrallerin ilk titiz formülasyonu, Riemann integrali ve Fourier serileri üzerine çalışmaları bilinir. Riemann hipotezinin orijinal ifadesini içeren asal sayma işleviyle ilgili ünlü 1859 tarihli makalesi, analitik sayı teorisindeki en etkili makalelerden biri olarak kabul edilmektedir. Geometriye öncülük eden katkılarıyla Riemann, genel göreliliğin matematiğinin temellerini attı.

German mathematician. He is known for his works in the fields of real analysis, which contributes to analysis, number theory and differential geometry, the first rigorous formulation of integrals, Riemann integral and Fourier series are. His famous 1859 paper on prime counting, which includes the original expression of the Riemann hypothesis, is considered one of the most effective papers in analytic number theory. With his contributions leading to geometry, Riemann laid the foundations of mathematics of general relativity.

İNTEGRAL UYGULAMALARI
APPLICATIONS of INTEGRAL

Bu bölüm, 36 test sorusu içermektedir.
This chapter includes 36 test questions.

BÖLÜM / CHAPTER 32

İNTEGRAL UYGULAMALARI / APPLICATIONS of INTEGRAL

- İntegral Uygulamaları / Applications of Integral .... 631 – 642

BÖLÜM
32
CHAPTER

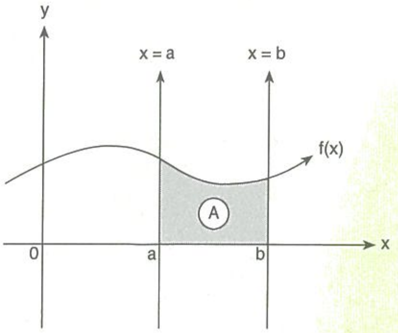
İNTEGRAL UYGULAMALARI
APPLICATIONS of INTEGRAL

BELİRLİ İNTEGRALİN UYGULAMALARI / APPLICATIONS of THE DEFINITE INTEGRAL

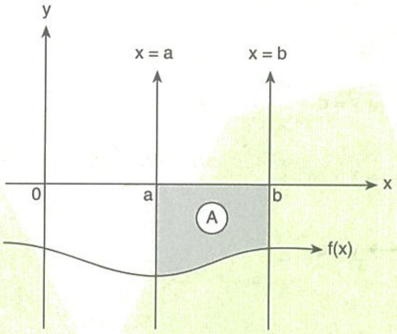
ALAN HESABI / AREA CALCULATION

▼ $y = f(x)$ fonksiyonu, x – ekseni, $x = a$ ve $x = b$ doğruları tarafından sınırlandırılan bölgenin alanı, aşağıdaki gibi hesaplanır.

▼ The area of the region bounded by the function $y = f(x)$, the lines $x = a$, $x = b$ and the x-axis is calculated as follows.

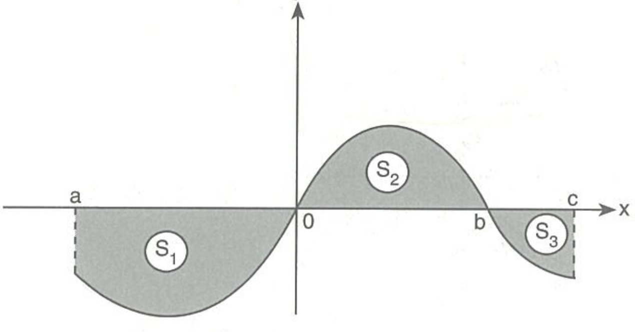


$$\Rightarrow A = \int_a^b f(x)\,dx$$



$$\Rightarrow A = -\int_a^b f(x)\,dx$$

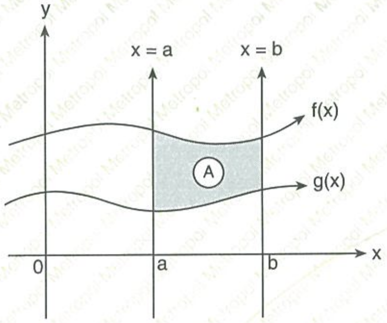
▼



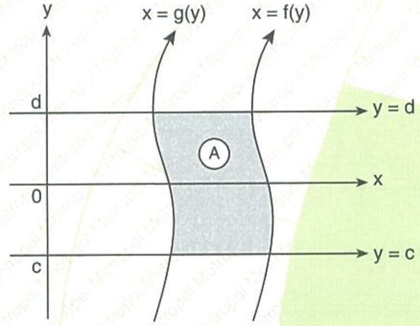
$$\Rightarrow \int_a^c f(x)\,dx = -S_1 + S_2 - S_3$$

▼ y = f(x), y = g(x) fonksiyonları ile x = a, x = b doğruları tarafından sınırlanan bölgenin alanı aşağıdaki gibi bulunur.

The area of the region bounded by the functions y = f(x), y = g(x), the lines x = a and x = b is calculated as follow.



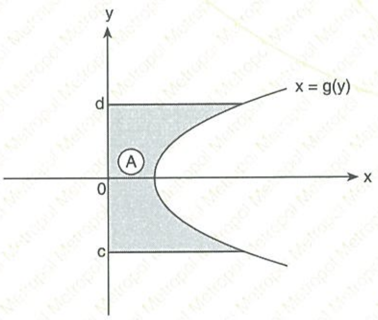
$$\Rightarrow A = \int_a^b |f(x) - g(x)|\,dx$$

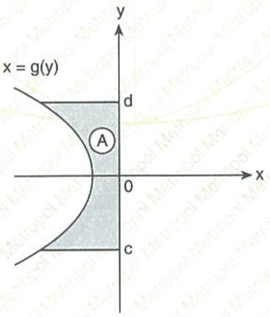


$$\Rightarrow A = \int_c^d |f(y) - g(y)|\,dy$$

▼ y ekseni, x = g(y) eğrisi ve y = c ile y = d doğruları arasında kalan bölgenin alanı:

Area of a region bounded by the y-axis, curve x = g(y), and lying between lines y = c and y = d.



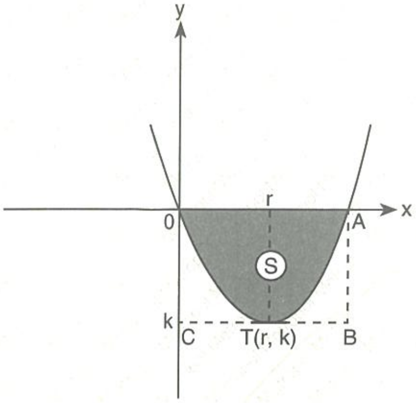


$$\Rightarrow A = \int_c^d g(y)\,dy$$

$$\Rightarrow A = -\int_c^d g(y)\,dy$$

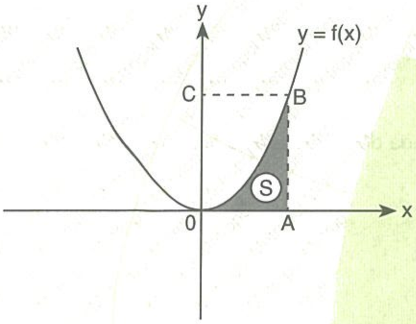
- $a \neq 0$,
  $y = f(x) = ax^2 + bx + c$

i.


y
x
r
0
A
S
k
C
T(r, k)
B

$\Rightarrow S = \frac{2}{3} \cdot$ *Alan/Area*(*OABC*)

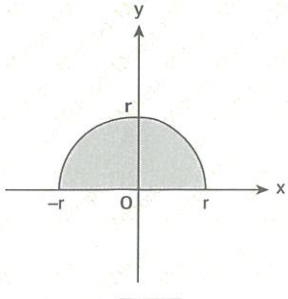
ii.


y
y = f(x)
C
B
S
x
0
A

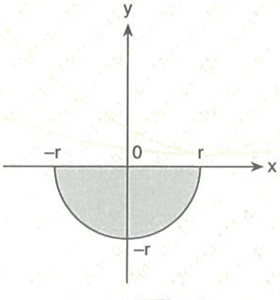
$\Rightarrow S = \frac{1}{3} \cdot$ *Alan/Area*(*OABC*)

ALAN YARDIMIYLA İNTEGRAL BULMA / FINDING INTEGRAL USING AREA

- $x^2 + y^2 = r^2 \Rightarrow \mathbf{y = \pm\sqrt{r^2 - x^2}}$



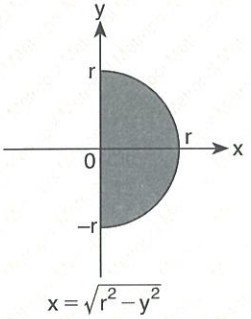
$y = \sqrt{r^2 - x^2}$

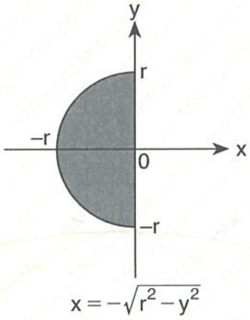


$y = -\sqrt{r^2 - x^2}$

$$\Rightarrow \int_{-r}^{r} f(x)\,dx = \frac{\pi \cdot r^2}{2}$$

▼ $x^2+y^2=r^2 \Rightarrow x=\pm\sqrt{r^2-y^2}$



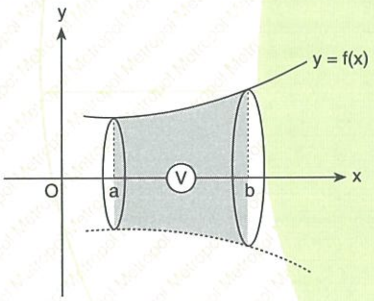


$$\Rightarrow \int_{-r}^{r} x\cdot dy=\frac{\pi\cdot r^2}{2}$$

HACİM HESABI / VOLUME CALCULATION

▼ y = f(x) fonksiyonu, x ekseni ve x = a, x = b doğruları arasında kalan bölgenin x – ekseni etrafında döndürülmesiyle oluşan cismin hacmi aşağıdaki gibi bulunur.

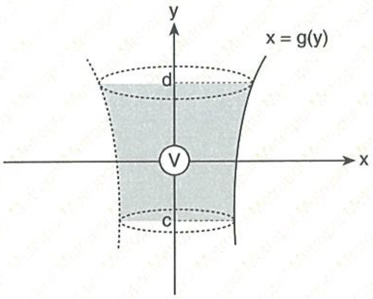
▼ The volume of a solid formed by a region bounded by the function y = f(x), lines x = a, x = b and x-axis rotating around x-axis is calculated as the following.



$$\Rightarrow V=\pi\cdot\int_{a}^{b} y^2dx$$

▼ x = g(y) fonksiyonu, y ekseni ve y = c, y = d doğruları arasında kalan bölgenin y–ekseni etrafında döndürülmesiyle oluşan cismin hacmi aşağıdaki gibi bulunur.

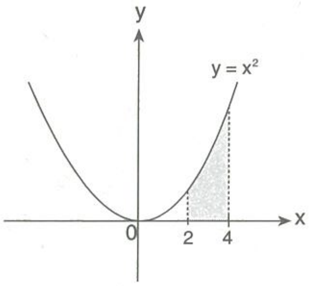
▼ The volume of a solid formed by a region bounded by the function x = g(y), lines y = c, y = d and y-axis rotating around y-axis is calculated as the following.



$$\Rightarrow V=\pi\cdot\int_{c}^{d} x^2dy$$

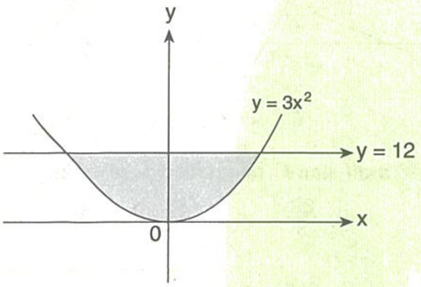
1. **Taralı alan /**

The shaded area = ? cm^2



A) $\frac{52}{3}$ B) $\frac{53}{3}$ C) 18 D) $\frac{55}{2}$ E) $\frac{56}{3}$

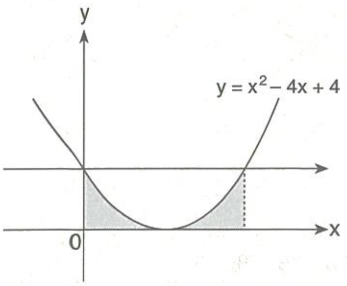
2.


y
y = 3x²
y = 12
x
0

$\Rightarrow$ **Taralı alan / The shaded area = ? cm^2**

A) 8 B) 12 C) 16 D) 24 E) 32

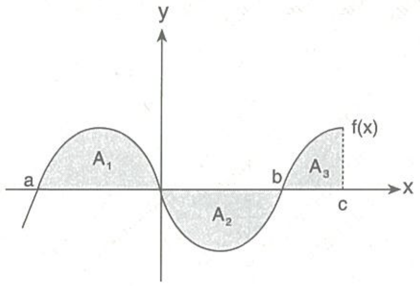
3.



$\Rightarrow$ **Taralı alan / The shaded area = ? cm^2**

A) 4 B) 5 C) $\frac{16}{3}$ D) $\frac{17}{3}$ E) 6

7.

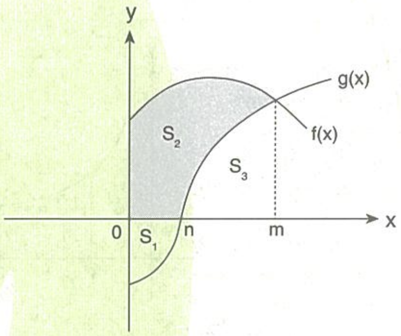

y
x
a
b
c
A₁
A₂
A₃
f(x)

$A_1 + A_2 + A_3 = 12$ br^2 (u^2),

$\int_a^c f(x)\,dx = 8 \Rightarrow \mathbf{A_2 = ?\,br^2}\,(u^2)$

A) 2 B) 3 C) 4 D) 5 E) 6

5.


y
g(x)
f(x)
S₂
S₃
S₁
0
n
m
x

$\int_0^m f(x) = 12, \quad \int_0^n g(x) + \int_n^m g(x) = 0$

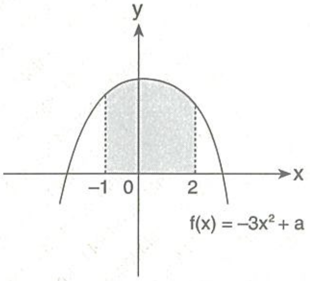
$S_1 = 4\,br^2(u^2) \Rightarrow \mathbf{S_2 = ?\,br^2}\,(u^2)$

A) 4 B) 6 C) 8 D) 9 E) 10

6. Taralı alan /

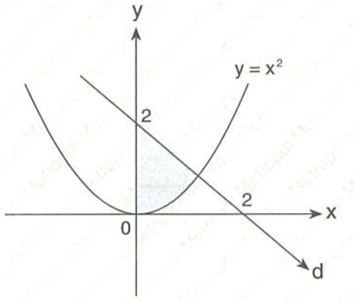
The shaded Area = 81 cm^2

$\Rightarrow$ **a = ?**


y
x
–1
0
2
f(x) = –3x² + a

A) 23 B) 24 C) 27 D) 30 E) 42

7.

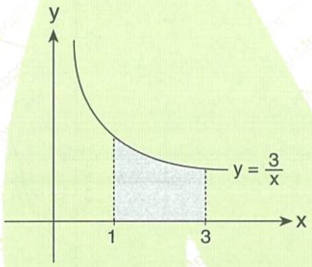


⇒ **Taralı alan / The shaded area = ? cm²**

A) $\frac{4}{3}$ B) $\frac{5}{2}$ C) $\frac{7}{6}$ D) $\frac{6}{5}$ E) $\frac{9}{4}$

8. **Taralı alan /**

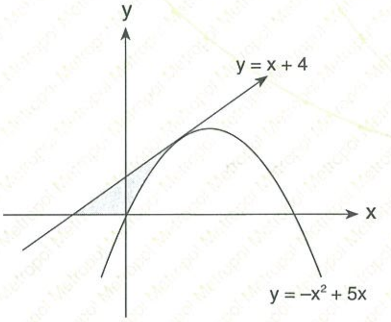
The shaded area = ? cm²


y
y = $\frac{3}{x}$
1
3
x

A) ln3 B) ln3 – 1 C) ln27 – 1

D) 2ln3 – 1 E) 3ln3

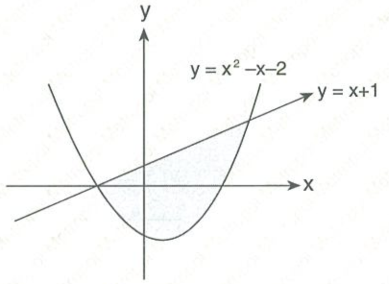
9.


y
y = x + 4
x
y = –x² + 5x

⇒ **Taralı alan / The shaded area = ? cm²**

A) 8 B) $\frac{32}{3}$ C) 12 D) $\frac{37}{3}$ E) 13

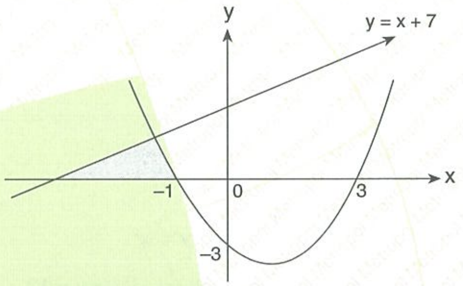
10.


y
y = x² –x–2
y = x+1
x

⇒ **Taralı alan / The shaded area = ? cm²**

A) $\frac{47}{3}$ B) $\frac{61}{3}$ C) $\frac{74}{3}$ D) $\frac{83}{3}$ E) $\frac{32}{3}$

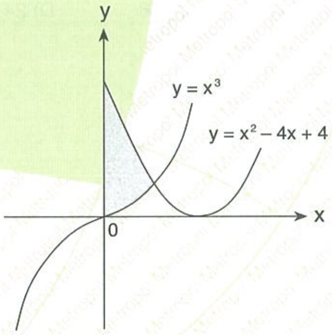
11.



⇒ **Taralı alan / The shaded area = ? cm²**

A) 27 B) $\frac{82}{3}$ C) $\frac{83}{3}$ D) 29 E) $\frac{89}{6}$

12.


y
y = x³
y = x² – 4x + 4
x
0

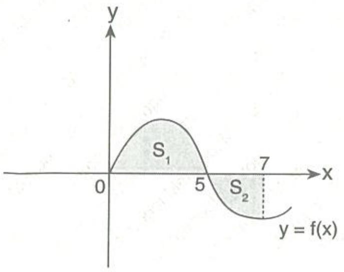
⇒ **Taralı alan / The shaded area = ? cm²**

A) $\frac{8}{3}$ B) $\frac{25}{12}$ C) $\frac{13}{6}$ D) $\frac{7}{6}$ E) $\frac{9}{4}$

YANITLAR / ANSWERS

1	2	3	4	5	6	7	8	9	10	11	12
E	E	C	A	C	D	C	E	B	E	E	B

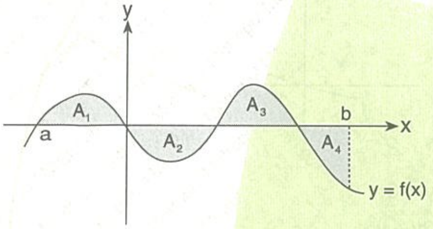
1.


y
S₁
7
x
0
5
S₂
y = f(x)

$\int_0^7 f(x)\,dx = 14, S_1 = 32\,cm^2 \Rightarrow$ **$S_2 = ?\,cm^2$**

A) 16 B) 18 C) 24 D) 28 E) 30

2.

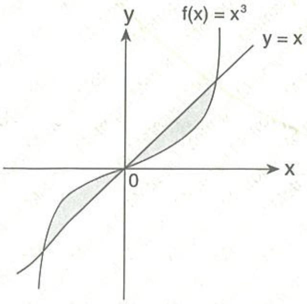

y
A₁
A₃
b
x
a
A₂
A₄
y = f(x)

$A_1 = 3\,cm^2, A_2 = 6\,cm^2, A_4 = 4\,cm^2,$

$\int_a^b f(x)\,dx = 9 \Rightarrow$ **$A_3 = ?\,cm^2$**

A) 8 B) 10 C) 12 D) 16 E) 18

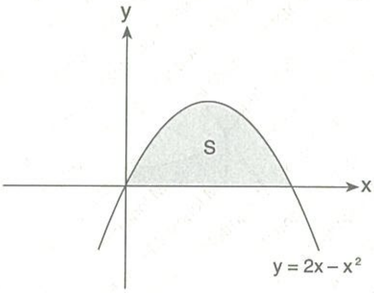
3.



$\Rightarrow$ **Taralı alan / The shaded area = ? cm²**

A) $\frac{3}{7}$ B) 1 C) $\frac{1}{2}$ D) $\frac{1}{4}$ E) $\frac{1}{8}$

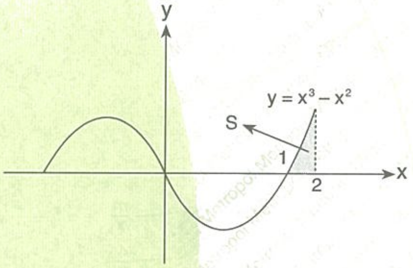
4.



$\Rightarrow$ **$S = ?\,cm^2$**

A) $\frac{2}{3}$ B) $\frac{4}{3}$ C) 1 D) $\frac{1}{2}$ E) $\frac{1}{4}$

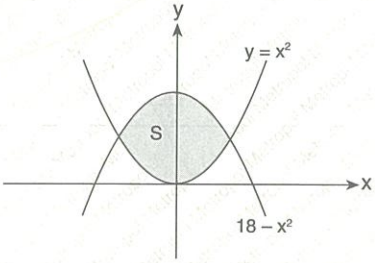
5.


y
y = x³ – x²
S
1
2
x

$\Rightarrow$ **$S = ?\,cm^2$**

A) 1 B) 2 C) $\frac{5}{2}$ D) $\frac{17}{12}$ E) $\frac{1}{4}$

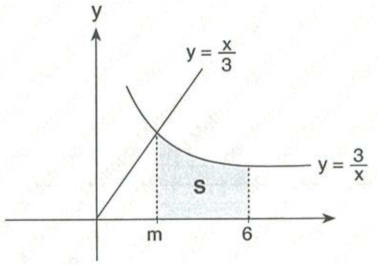
6.


y
y = x²
S
x
18 – x²

$\Rightarrow$ **$S = ?\,cm^2$**

A) 48 B) 54 C) 64 D) 72 E) 80

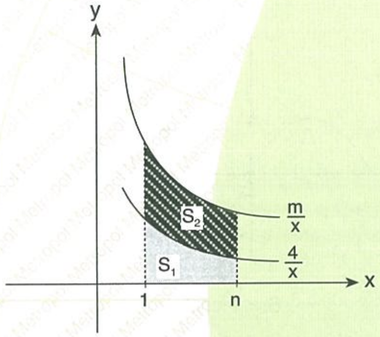
7.


y
$y = \frac{x}{3}$
$y = \frac{3}{x}$
S
m
6

$\Rightarrow$ **S = ? cm^2**

A) 1 – ln6 B) 3ln3 C) ln2 D) ln8 E) ln3 – ln2

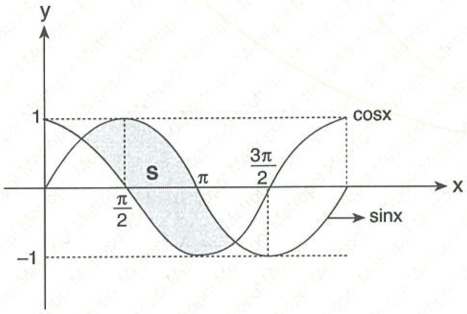
8.


y
S_2
S_1
$\frac{m}{x}$
$\frac{4}{x}$
x
1
n

$\left.\begin{array}{l} S_1 + S_2 = 48 \\ S_2 = 16 \end{array}\right\} \Rightarrow$ **m = ?**

A) 3 B) 4 C) 5 D) 6 E) 7

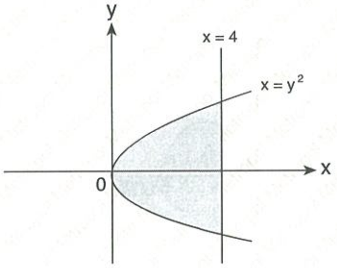
9.


y
1
cosx
S
π
$\frac{3\pi}{2}$
x
$\frac{\pi}{2}$
sinx
–1

$\Rightarrow$ **S = ? cm^2**

A) 1 B) $\sqrt{3}$ C) $2\sqrt{2}$ D) $2\sqrt{3}$ E) 2

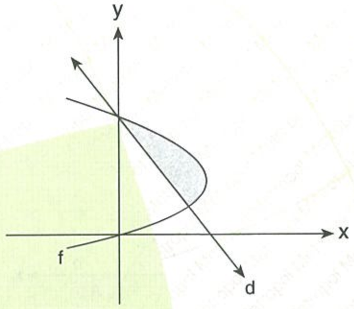
10.



$\Rightarrow$ **Taralı alan / The shaded area = ? cm^2**

A) $\frac{32}{3}$ B) 12 C) $\frac{27}{4}$ D) 8 E) $\frac{9}{2}$

11.



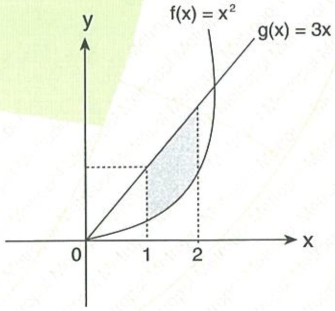
d: x + y = 3

f : $x = 3y - y^2$

$\Rightarrow$ **Taralı alan / The shaded area = ? cm^2**

A) $\frac{7}{2}$ B) 3 C) $\frac{3}{2}$ D) $\frac{4}{3}$ E) 2

12.


y
$f(x) = x^2$
$g(x) = 3x$
0
1
2
x

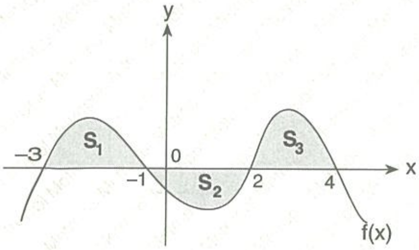
$\Rightarrow$ **Taralı alan / The shaded area = ? cm^2**

A) 2 B) $\frac{5}{2}$ C) $\frac{11}{3}$ D) $\frac{13}{6}$ E) 3

YANITLAR / ANSWERS

1	2	3	4	5	6	7	8	9	10	11	12
B	D	C	B	D	D	D	D	C	A	D	D

1.

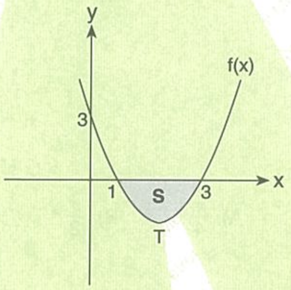

y
x
–3
0
–1
2
4
S_1
S_2
S_3
f(x)

$S_1 = 7\,\text{br}^2\,(u^2), S_2 = 9\,\text{br}^2\,(u^2), S_3 = 12\,\text{br}^2\,(u^2)$

$\Rightarrow \int_{-3}^{4} 3 \cdot f(x)\,dx = ?$

A) –30 B) –10 C) 10 D) 30 E) 34

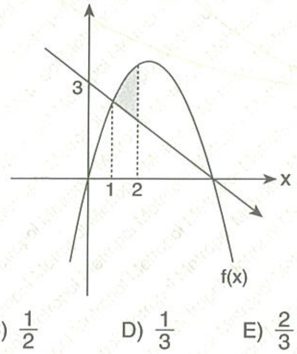
2. **TA / SA = ? br^2 (u^2)**



A) $\frac{1}{3}$ B) $\frac{2}{3}$ C) 1 D) $\frac{4}{3}$ E) $\frac{5}{3}$

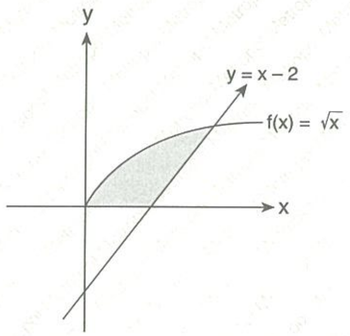
3. $f(x) = -x^2 + 3x$

$\Rightarrow$ **TA / SA = ? br^2 (u^2)**



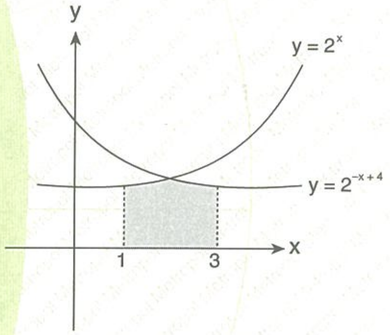
A) 2 B) 1 C) $\frac{1}{2}$ D) $\frac{1}{3}$ E) $\frac{2}{3}$

4. **TA / SA = ? br^2 (u^2)**



A) $\frac{4}{3}$ B) $\frac{8}{3}$ C) $\frac{10}{3}$ D) $\frac{14}{3}$ E) $\frac{16}{3}$

5. **TA / SA = ? br^2 (u^2)**

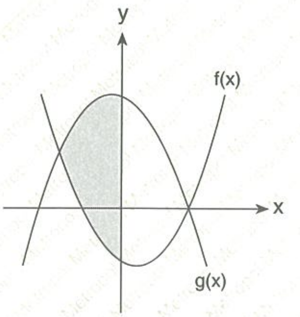

y
x
$y = 2^x$
$y = 2^{-x+4}$
1
3

A) $\frac{2}{\ln 2}$ B) $\frac{4}{\ln 2}$ C) ln2 D) 2ln2 E) 4ln2

6. $f(x) = x^2 - 2x - 3,$

$g(x) = -x^2 - x + 12.$

$\Rightarrow$ **TA / SA = ? br^2 (u^2)**

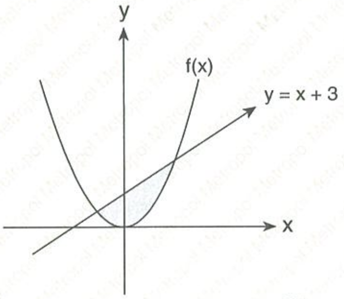


A) $\frac{125}{8}$ B) $\frac{125}{4}$ C) $\frac{575}{24}$

D) $\frac{575}{12}$ E) $\frac{625}{24}$

7. $f(x) = 2x^2$

$\Rightarrow$ **TA / SA = ? br^2 (u^2)**

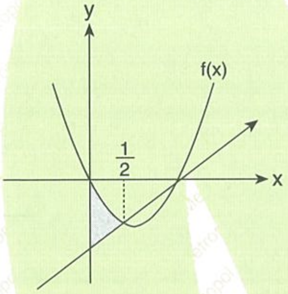


A) $\frac{10}{3}$ B) $\frac{25}{4}$ C) $\frac{75}{8}$

D) $\frac{125}{24}$ E) $\frac{15}{4}$

8. $f(x) = x^2 - x$

$\Rightarrow$ **TA / SA = ? br^2 (u^2)**



A) $\frac{5}{48}$ B) $\frac{11}{48}$ C) $\frac{13}{24}$

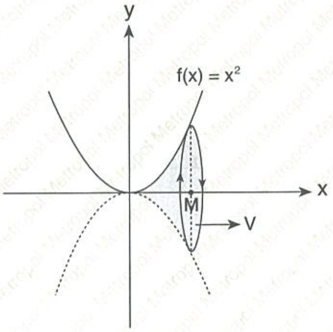
D) $\frac{13}{48}$ E) $\frac{5}{8}$

9. $\int_{0}^{\sqrt{2}} \left(\sqrt{4-x^2} - x\right) dx = ?$

A) $\frac{\pi}{8}$ B) $\frac{\pi}{6}$ C) $\frac{\pi}{4}$ D) $\frac{\pi}{2}$ E) π

10. M(3, 0)

$\Rightarrow$ **V = ? br^3 (u^3)**

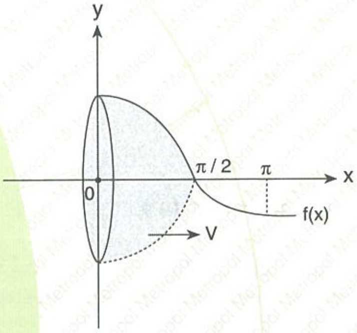


A) $\frac{81\pi}{4}$ B) $\frac{128\pi}{3}$ C) $\frac{144\pi}{5}$

D) $\frac{175\pi}{3}$ E) $\frac{243\pi}{5}$

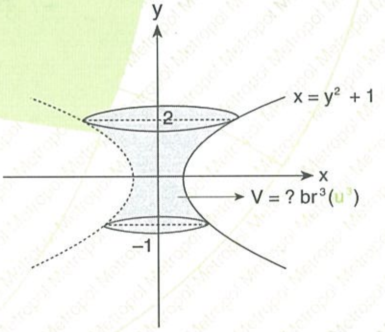
11. $f(x) = \cos x$

$\Rightarrow$ **V = ? br^3 (u^3)**



A) $\frac{\pi}{4}$ B) $\frac{\pi}{2}$ C) $\frac{\pi^2}{4}$ D) $\frac{\pi^2}{8}$ E) π^2

12.



A) $\frac{78\pi}{5}$ B) $\frac{81\pi}{5}$ C) $\frac{96\pi}{5}$

D) $\frac{296\pi}{15}$ E) $\frac{304\pi}{15}$

YANITLAR / ANSWERS

1	2	3	4	5	6	7	8	9	10	11	12
D	D	E	C	B	C	D	A	D	E	C	A

BÖLÜM
CHAPTER
33

TARİHSEL NOT / HISTORICAL NOTE

TERENCE TAO [1975 -]

Tao, 9 yaşında üniversite düzeyinde matematik derslerine devam eden, erken yaşlardan itibaren olağanüstü matematiksel yetenekler sergiledi. Tao, ilk on yaşında yarışan Uluslararası Matematik Olimpiyatları'nda bugüne kadarki en genç katılımcıydı; 1986, 1987 ve 1988'de bronz, gümüş ve altın madalya kazandı. Olympiat'ın tarihindeki üç madalyanın her birinin en genç kazananı oldu ve on üçüncü doğum gününden kısa bir süre sonra altın madalya kazandı. Matematik alanında, Tao, Oxford Üniversitesi'nden Ben J. Green ile yaptığı işbirliği ile bilinir; birlikte Green-Tao teoremini kanıtladılar. Onun işbirlikçi zihniyetiyle tanınan Tao, 2006 yılına kadar keşiflerinde 30'dan fazla kişi ile çalışmıştı. Tao tarafından 2016 yılında yaklaşık 300 araştırma yazısı ve 17 kitap yayınlanmıştır. 2 numaralı bir Erdös sayısına sahiptir.

Tao exhibited extraordinary mathematical abilities from an early age, attending university-level mathematics courses at the age of 9. Tao was the youngest participant to date in the International Mathematical Olympiad, first competing at the age of ten; in 1986, 1987, and 1988, he won a bronze, silver, and gold medal. He remains the youngest winner of each of the three medals in the Olympiad's history, winning the gold medal shortly after his thirteenth birthday. Within the field of mathematics, Tao is known for his collaboration with Ben J. Green of Oxford University; together they proved the Green–Tao theorem. Known for his collaborative mindset, by 2006 Tao had worked with over 30 others in his discoveries. By 2016 Tao had published about 300 research papers and 17 books. He has an Erd's number of 2.

MATRİS ve DETERMİNANT
MATRIX and DETERMINANT

Bu bölüm, 108 test sorusu içermektedir.
This chapter includes 108 test questions.

BÖLÜM / CHAPTER 33

MATRİS ve DETERMİNANT / MATRIX and DETERMINANT

- Matris ve Determinant / Matrix and Determinant .... 643 – 662

BÖLÜM
33
CHAPTER

MATRİS ve DETERMİNANT
MATRIX and DETERMINANT

MATRİS / MATRIX

▼ $\longrightarrow$ satır – row

$$A=\begin{bmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \cdots & a_{2n} \\ \vdots & \vdots & \vdots & \cdots & \vdots \\ a_{m1} & a_{m2} & a_{m3} & \cdots & a_{mn} \end{bmatrix}_{mxn} \downarrow \text{sütun – column}$$

Yukarıdaki n tane sütun, m tane satırdan oluşan dikdörtgen tabloya ***matris*** denir.

The rectangle table composed of n columns (vertical lines) and m rows (horizantal lines) is called a *matrix.*

$a_{ij} \in R \ (1 \le i \le m,\ 1 \le j \le n)$

m : Satır sayısı / Number of rows

n : Sütun sayısı / Number of columns

MATRİS ÇEŞİTLERİ / MATRIX TYPES

KARE MATRİS / SQUARE MATRIX

Tanım: Satır ve sütun sayısı aynı olan matrise ***kare matris*** denir.

Definition: A matrix having equal number of rows and columns is called a ***square matrix***.

$$A=\begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{bmatrix}_{4x4} \longrightarrow \text{esas köşegen}$$

main diagonal

▼ **Kare Matrislere Örnekler /** Examples for square matrices:

$$A=\begin{bmatrix} 2 & 3 & -4 & 1 \\ 0 & 5 & -1 & 5 \\ 0 & 0 & -3 & 1 \\ 0 & 0 & 0 & 4 \end{bmatrix}_{4x4} \longrightarrow \text{esas köşegen}$$

main diagonal

$$B=\begin{bmatrix} -1 & 0 & 0 \\ 3 & 4 & 0 \\ -1 & -7 & 2 \end{bmatrix}_{3x3} \longrightarrow \text{esas köşegen}$$

main diagonal

$$C=\begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 6 \end{bmatrix}_{4x4} \longrightarrow \text{esas köşegen}$$

main diagonal

BİRİM MATRİS / IDENTITY MATRIX

Tanım: Köşegen (esas köşegen) üzerindeki elemanları 1 ve diğer elemanları 0 olan kare matrise ***birim matris*** denir. I veya I_n ile gösterilir.

Definition: The n x n matrix that consists of 1's on its main diagonal and 0's elsewhere is called the *identity matrix* of order n and is denoted by I or I_n.

▼ **Birim Matrislere Örnekler / Examples for identity matrices:**

$$I_2=\begin{bmatrix}1&0\\0&1\end{bmatrix}_{2x2},\quad I_3=\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}_{3x3},\quad I_4=\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&1\end{bmatrix}_{4x4},\ \ldots$$

MATRİSLERİN EŞİTLİĞİ / EQUALITY of MATRICES

▼
$$\begin{bmatrix}x&y&z\\m&n&t\\d&e&f\end{bmatrix}=\begin{bmatrix}a_{11}&a_{12}&a_{13}\\a_{21}&a_{22}&a_{23}\\a_{31}&a_{32}&a_{33}\end{bmatrix}\Rightarrow\begin{cases}x=a_{11},\ y=a_{12},\ z=a_{13}\\m=a_{21},\ n=a_{22},\ t=a_{23}\\d=a_{31},\ e=a_{32},\ f=a_{33}\end{cases}$$

MATRİSLERİN TOPLAMI ve FARKI / SUM and DIFFERENCE of MATRICES

▼ $A=[a_{ij}]_{m\,x\,n},\ B=[b_{ij}]_{m\,x\,n}\Rightarrow A+B=[a_{ij}+b_{ij}]_{m\,x\,n},\quad A-B=[a_{ij}-b_{ij}]_{m\,x\,n}$

BİR MATRİSİN BİR SKALERLE ÇARPIMI / PRODUCT of a SCALAR and a MATRIX

▼ $\alpha\in R$,

$$\alpha\cdot\begin{bmatrix}a&b&c\\x&y&z\\m&n&t\end{bmatrix}=\begin{bmatrix}\alpha\cdot a&\alpha\cdot b&\alpha\cdot c\\\alpha\cdot x&\alpha\cdot y&\alpha\cdot z\\\alpha\cdot m&\alpha\cdot n&\alpha\cdot t\end{bmatrix}$$

Önemli / Important

- Tek satıra sahip olan matrise satır matrisi, tek sütuna sahip matrise ise sütun matrisi denir.
- A matrix that has only one row is called a row matris, and a matris that has only one column is called a column matrix.

MATRİSLERİN ÇARPIMI / PRODUCT of MATRICES

▼ İki matrisin çarpılabilmesi için; birinci matrisin sütun sayısı, ikinci matrisin satır sayısına eşit olmalıdır.

▼ For the product of two matrices to be feasible, the number of columns of the first matrix must equal to the number of rows of the second matrix.

$A = [a_{ij}]_{m \times n}$ ve $B = [b_{ij}]_{n \times p} \Rightarrow A \cdot B = C = [c_{ij}]_{mxp}$

$$A \cdot B = \begin{bmatrix} a_1 & a_2 & \dots & a_n \end{bmatrix}_{1xn} \cdot \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{bmatrix}_{nx1}$$

$$\Rightarrow A \cdot B = \begin{bmatrix} a_1 \cdot b_1 + a_2 \cdot b_2 + \dots + a_n \cdot b_n \end{bmatrix}_{1x1}$$

▼ **Örneğin / For example:**

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}_{2x2}, \quad B = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}_{2x2}$$

$$\Rightarrow A \cdot B = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \cdot \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} = \begin{bmatrix} a_{11}b_{11} + a_{12}b_{21} & a_{11}b_{12} + a_{12}b_{22} \\ a_{21}b_{11} + a_{22}b_{21} & a_{21}b_{12} + a_{22}b_{22} \end{bmatrix}_{2x2}$$

Önemli / Important

- A ve B matrislerinin çarpımı, A · B = AB biçiminde gösterilir.
- Multiplication of matrices A and B is denoted by A · B = AB.

Özellik / Property

★ $AB \neq BA$

★ $(AB) \cdot C = A \cdot (BC)$

★ $(B + C) \cdot A = BA + CA$

★ $k \cdot (AB) = (kA) \cdot B = A \cdot (kB) \quad (k \in R)$

★ $IA = AI = A$

MATRİSİN KUVVETİ / POWER of a MATRIX

▼ $A^2 = A \cdot A$

$A^3 = A \cdot A \cdot A = A^2 \cdot A$

$A^4 = A^2 \cdot A^2$

$\vdots$

$$A^n = \underbrace{A \cdot A \cdot A \cdot \dots \cdot A}_{n \text{ tane / times}}$$

Özellik / Property

★ $I_n^k = I_n$

MATRİSİN TRANSPOZU / TRANSPOSE of a MATRIX

▼ Bir A matrisinin transpozu A^T olarak gösterilir ve satırların yerine sütunların, sütunların yerine satırların yazılması ile elde edilir.

▼ The transpoze of a matrix A is denoted by A^T. To find the transpose of a matrix, rows of matrix are written instead of columns and columns of matrix are written instead of rows.

$$A=\begin{bmatrix} a & b & c \\ x & y & z \\ m & n & t \end{bmatrix} \Rightarrow A^T=\begin{bmatrix} a & x & m \\ b & y & n \\ c & z & t \end{bmatrix}$$

Özellik / Property

★ Matrislerin toplamının transpozu, transpozlarının toplamıdır.

★ The transpose of sum of matrices is the sum of transposes.

$$(A+B)^T = A^T + B^T$$

★ Matrislerin çarpımının transpozu, transpozlarının yer değiştirerek çarpımıdır.

★ The transpose of a product of matrices is the product of transposes in opposite order.

$$(A \cdot B)^T = B^T \cdot A^T$$

★ Bir matrisin transpozunun transpozu kendisidir.

★ The transpose of the transpose of a matrix is itself.

$$(A^T)^T = A$$

★ Bir matrisin bir sabitle çarpımının transpozu, transpozunun sabitle çarpımına eşittir.

★ The transpose of product of a matrix and a scalar is equal to product of the scalar and of the transpose of matrix.

$$(k \cdot A)^T = k \cdot A^T$$

MATRİSİN TERSİ / THE INVERSE of a MATRIX

▼ Bir matrisin tersi A^{-1} ile gösterilir.

▼ Inverse of a matrix is denoted by A^{-1}.

$$A=\begin{bmatrix} a & b \\ c & d \end{bmatrix} \Rightarrow A^{-1}=\frac{1}{ad-bc}\cdot\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

Özellik / Property

★ $A \cdot A^{-1} = I_n$

★ $(A^T)^{-1} = (A^{-1})^T$

★ $(A \cdot B)^{-1} = A^{-1} \cdot B^{-1}$

★ $(A^{-1})^{-1} = A$

DETERMİNANT / DETERMINANT

▼ f: $[a_{ij}] \rightarrow R$ olacak şekilde tanımlanan fonksiyonlara ***determinant*** denir.

▼ The functions defined as f: $[a_{ij}] \rightarrow R$ are called as the ***determinat***.

▼ A matrisinin determinantı **detA** ya da **IAI** ile gösterilir.

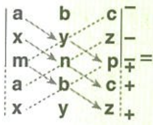
▼ The determinant of the matrix A is denoted by **detA** or **IAI**.

2x2'lik MATRİSİN DETERMİNANTI / THE DETERMINANT of a 2x2 MATRIX

▼ $A=\begin{bmatrix} a & b \\ c & d \end{bmatrix}_{2x2} \Rightarrow |A| = \det A = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = a\cdot d - b\cdot c$

3x3'lük MATRISIN DETERMINANTI (SARRUS KURALI) / THE DETERMINANT of a 3x3 MATRIX (RULE of SARRUS)

▼



$$A=\begin{bmatrix} a & b & c \\ x & y & z \\ m & n & p \end{bmatrix} \Rightarrow \det A = |A| = \begin{vmatrix} a & b & c \\ x & y & z \\ m & n & p \\ a & b & c \\ x & y & z \end{vmatrix} = (a\cdot y\cdot p + x\cdot n\cdot c + m\cdot b\cdot z) - (m\cdot y\cdot c + a\cdot n\cdot z + x\cdot b\cdot p)$$

nxn'lik MATRİSİN DETERMİNANTI / THE DETERMINANT of a nxn MATRIX

▼ **Minör:** A kare matrisinin i. satır ve j. sütununun silinmesiyle elde edilen matrisin determinantına ***matrisin küçüğü (minörü)*** denir. M_{ij} ile gösterilir.

▼ **Minor:** If A is a square matrix, the minor M_{ij} of the entry aij is the determinant of the matrix obtained by deleting the i^{th} row and j^{th} column of A.

▼ **Kofaktör:** Bir matrisin i. satır, j. sütun elemanının kofaktörü $A_{ij} = (-1)^{i+j}.M_{ij}$ ile gösterilir.

▼ **Cofactor:** The cafactor A_{ij} of the entry a_{ij} is given by $A_{ij} = (-1)^{i+j} M_{ij}$.

Özellik / Property

★ Bir matrisin determinantı, herhangi bir satır veya sütun elemanları ile kofaktörlerinin çarpımının toplamına eşittir.

★ The determinant of a matrix is sum of the productions of elements of any single raw or column with cofactors of those elements.

Özellik / Property

★ $|A| = |A^T|$

★ $|A \cdot B| = \det(A \cdot B) = \det A \cdot \det B = |A| \cdot |B|$

★ $|A^n| = \det(A^n) = [\det(A)]^n = |A|^n$

★ $|A^{-1}| = (\det A)^{-1} = \dfrac{1}{\det A} = \dfrac{1}{|A|}$

★ $A = A^{-1} \Rightarrow |A| = \det A = -1$

★ Bir kare matrisin herhangi iki satır veya sütunu yer değiştirirse, ***determinantın değeri işaret değiştirir.***

★ If any two rows or columns of a square matrix is interchanged, the sign of its determinant changes.

★ A kare matrisinin herhangi satır veya sütununun tamamı 0 ise, ***|A| = 0*** dır.

★ If the whole entries in any row or column of a square matrix A is 0, then $|A| = 0$.

★ A kare matrisinin herhangi iki satır veya sütunu aynı ise, ***determinantı 0*** dır.

★ If any two rows or columns of a square matrix A are the same, determinant will be 0.

★ Bir determinantın herhangi iki satır veya sütununun tamamı orantılı ise, ***determinantı sıfırdır.***

★ If the entries of any two rows or columns of a determinant are proportional, the determinant will be zero.

★ A kare matrisinin herhangi satır veya sütununun tamamı k reel sayısı ile çarpılırsa, matrisin determinantı ***k·|A|*** olur.

★ When the entries of any row or column of a square matrix A are multiplied by a real number k, the new determinant becomes $k \cdot |A|$.

★ 2 x 2'lik A matrisi ve $k \in R$ için, ***$|k \cdot A| = k^2 \cdot |A|$*** dir.

★ For a 2 x 2 matrix A and $k \in R$, $|kA| = k^2|A|$.

★ Bir matrisin herhangi satır veya sütunu k reel sayısı ile çarpılır ve başka sütun veya satıra eklenirse ***determinant değişmez.***

★ The value of a determinant does not change if its any row or column is multiplied by a real number k and then added to another row or column, respectively.

LİNEER DENKLEM SİSTEMLERİ / SYSTEMS of LINEAR EQUATIONS

▼ $A \cdot X = B \Rightarrow \boldsymbol{X = A^{-1} \cdot B}$

1. $\begin{bmatrix} a+1 & a+4 \\ 2 & 3a \end{bmatrix} = \begin{bmatrix} 3 & 3b \\ 2 & a+2b \end{bmatrix} \Rightarrow$ **$a+b=?$**

A) 2 B) 4 C) 6 D) 8 E) 16

2. $x \cdot \begin{bmatrix} -1 \\ 2 \end{bmatrix} - y \cdot \begin{bmatrix} -7 \\ 6 \end{bmatrix} = \begin{bmatrix} 13 \\ -10 \end{bmatrix} \Rightarrow$ **$x-y=?$**

A) –1 B) 0 C) 1 D) 2 E) 5

3. $A = \begin{bmatrix} x-y & 6 \\ -2 & x+y \end{bmatrix}$, $B = \begin{bmatrix} 8 & 6 \\ -2 & 12 \end{bmatrix}$

$A = B \Rightarrow$ **$x^2 - y^2 = ?$**

A) 96 B) 104 C) 108 D) 112 D) 116

4. $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$, $B = \begin{bmatrix} 2 & -1 & -3 \\ 4 & 5 & -7 \end{bmatrix} \Rightarrow$ **$2A - 3B = ?$**

A) $\begin{bmatrix} -4 & 7 & 15 \\ -4 & -5 & 33 \end{bmatrix}$ B) $\begin{bmatrix} 4 & 7 & 15 \\ -4 & -5 & 33 \end{bmatrix}$

C) $\begin{bmatrix} -4 & 7 & 15 \\ 4 & -5 & 33 \end{bmatrix}$ D) $\begin{bmatrix} 4 & 7 & 15 \\ 4 & -5 & 33 \end{bmatrix}$

E) $\begin{bmatrix} 4 & 7 & 15 \\ 4 & 5 & 33 \end{bmatrix}$

5. $A = \begin{bmatrix} 6 & -2 \\ 8 & 3 \end{bmatrix}$, $C = \begin{bmatrix} 2 & 4 \\ -6 & 5 \end{bmatrix}$

$C + 2B = A \Rightarrow$ **$B = ?$**

A) $\begin{bmatrix} -2 & 3 \\ 7 & 1 \end{bmatrix}$ B) $\begin{bmatrix} 2 & 3 \\ 7 & 1 \end{bmatrix}$ C) $\begin{bmatrix} 2 & 3 \\ 7 & -1 \end{bmatrix}$

D) $\begin{bmatrix} 2 & -3 \\ 7 & 1 \end{bmatrix}$ E) $\begin{bmatrix} 2 & -3 \\ 7 & -1 \end{bmatrix}$

6. $A = \begin{bmatrix} 3 & -1 \\ 2 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 4 & 0 \\ 2 & 3 \end{bmatrix} \Rightarrow$ **$A \cdot B = ?$**

A) $\begin{bmatrix} 10 & -3 \\ 16 & 12 \end{bmatrix}$ B) $\begin{bmatrix} 5 & -3 \\ 8 & 6 \end{bmatrix}$ C) $\begin{bmatrix} 5 & 3 \\ 4 & 3 \end{bmatrix}$

D) $\begin{bmatrix} 10 & -4 \\ 8 & 6 \end{bmatrix}$ E) $\begin{bmatrix} 10 & -8 \\ 16 & 10 \end{bmatrix}$

7. $A = \begin{bmatrix} 2 & 1 & -1 \end{bmatrix}$, $B = \begin{bmatrix} 1 & -1 \\ -1 & 1 \\ 1 & -1 \end{bmatrix} \Rightarrow$ **$A \cdot B = ?$**

A) $\begin{bmatrix} -1 & 0 & 1 \\ 1 & -1 & 0 \end{bmatrix}$ B) $\begin{bmatrix} 0 \\ -1 \end{bmatrix}$ C) $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$

D) $\begin{bmatrix} 0 & 0 \end{bmatrix}$ E) $\begin{bmatrix} 1 & 1 \end{bmatrix}$

8. $A=\begin{bmatrix}1 & 2\\ -2 & 3\end{bmatrix},\ B=\begin{bmatrix}-4 & 6\\ -6 & 2\end{bmatrix} \Rightarrow \mathbf{A^2-B=?}$

A) $\begin{bmatrix}-2 & -4\\ 6 & 1\end{bmatrix}$ B) $\begin{bmatrix}-1 & 2\\ -2 & -3\end{bmatrix}$ C) $\begin{bmatrix}1 & -2\\ 2 & 3\end{bmatrix}$

D) $\begin{bmatrix}1 & 2\\ -2 & 3\end{bmatrix}$ E) $\begin{bmatrix}1 & 3\\ 2 & -2\end{bmatrix}$

9. $A=\begin{bmatrix}2 & 3\\ -1 & 4\end{bmatrix},\ B=\begin{bmatrix}5 & -2\\ 1 & 6\end{bmatrix} \Rightarrow \mathbf{A\cdot B=?}$

A) $\begin{bmatrix}13 & 14\\ 1 & 26\end{bmatrix}$ B) $\begin{bmatrix}13 & 14\\ 1 & -26\end{bmatrix}$ C) $\begin{bmatrix}13 & 14\\ -1 & 26\end{bmatrix}$

D) $\begin{bmatrix}-13 & 14\\ 1 & 26\end{bmatrix}$ E) $\begin{bmatrix}13 & -14\\ 1 & 26\end{bmatrix}$

10. $A=\begin{bmatrix}-1 & 2 & 3\\ -2 & 4 & 5\end{bmatrix},\ B=\begin{bmatrix}-2 & 3\\ 6 & 7\\ -3 & 6\end{bmatrix} \Rightarrow \mathbf{A\cdot B=?}$

A) $\begin{bmatrix}29 & 5\\ 13 & 52\end{bmatrix}$ B) $\begin{bmatrix}5 & 29\\ 13 & 52\end{bmatrix}$ C) $\begin{bmatrix}5 & 29\\ 52 & 13\end{bmatrix}$

D) $\begin{bmatrix}52 & 29\\ 13 & 5\end{bmatrix}$ E) $\begin{bmatrix}5 & 13\\ 29 & 52\end{bmatrix}$

11. $A=\begin{bmatrix}3 & 2\\ 1 & 4\end{bmatrix},\ B=A^T-A \Rightarrow \mathbf{B=?}$

A) $\begin{bmatrix}0 & -1\\ 1 & 0\end{bmatrix}$ B) $\begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix}$ C) $\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$

D) $\begin{bmatrix}0 & -2\\ 2 & 0\end{bmatrix}$ E) $\begin{bmatrix}0 & -2\\ 1 & 0\end{bmatrix}$

12. $A-2\cdot\begin{bmatrix}2 & 3\\ 1 & 4\end{bmatrix}+3I=0 \Rightarrow \mathbf{A=?}$

A) $\begin{bmatrix}3\\ 4\end{bmatrix}$ B) $\begin{bmatrix}1 & 6\\ 2 & 5\end{bmatrix}$ C) $\begin{bmatrix}-1 & 3\\ 1 & 5\end{bmatrix}$

D) $\begin{bmatrix}-1 & 6\\ 2 & 5\end{bmatrix}$ E) $\begin{bmatrix}1 & 6\\ -2 & -5\end{bmatrix}$

13. $A=\begin{bmatrix}3 & -2\\ 1 & 4\end{bmatrix},$

$f(x) = 3x + 2I \Rightarrow \mathbf{f(A) = ?}$

A) $\begin{bmatrix}11 & -6\\ -4 & 5\end{bmatrix}$ B) $\begin{bmatrix}11 & -6\\ 3 & 14\end{bmatrix}$ C) $\begin{bmatrix}11 & 6\\ 4 & 5\end{bmatrix}$

D) $\begin{bmatrix}11 & -6\\ 4 & 5\end{bmatrix}$ E) $\begin{bmatrix}11 & -6\\ 4 & -5\end{bmatrix}$

14. $A=\begin{bmatrix}8 & 3\\ 3 & 6\end{bmatrix},$

$A\cdot A^T=\begin{bmatrix}8 & 3\\ 3 & 6\end{bmatrix} \Rightarrow \mathbf{a^2+b^2=?}$

A) 9 B) 10 C) 13 D) 19 E) 24

15. $A=\begin{bmatrix}a & 2\\ 3 & -1\end{bmatrix}\cdot\begin{bmatrix}2 & a\\ -1 & 3\end{bmatrix}=\begin{bmatrix}2 & 10\\ 7 & 3\end{bmatrix} \Rightarrow \mathbf{a=?}$

A) 2 B) 1 C) 0 D) −1 E) −2

YANITLAR / ANSWERS

1	2	3	4	5	6	7	8
B	A	A	A	E	A	D	D
9	**10**	**11**	**12**	**13**	**14**	**15**	
C	B	A	B	B	A	A	

1. $A=\begin{bmatrix} x & 3 \\ 4 & 2 \end{bmatrix}$, $B=\begin{bmatrix} 2 & y \\ -1 & 3 \end{bmatrix}$,

$3A+2B=\begin{bmatrix} -2 & 13 \\ 10 & 12 \end{bmatrix} \Rightarrow \mathbf{x \cdot y = ?}$

A) –8 B) –6 C) –4 D) 4 E) 6

2. $A=\begin{bmatrix} -3 & 4 \\ 0 & 3 \end{bmatrix} \Rightarrow \mathbf{A^{15} = ?}$

A) $3^{14}\cdot\begin{bmatrix} -3 & 4 \\ 0 & 3 \end{bmatrix}$ B) $3^{28}\cdot\begin{bmatrix} -3 & 4 \\ 0 & 3 \end{bmatrix}$ C) $3^{14}\cdot\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

D) $\begin{bmatrix} -3^{14} & 4^{14} \\ 0 & 3^{14} \end{bmatrix}$ E) $\begin{bmatrix} -3^{15} & 4^{15} \\ 0 & 3^{15} \end{bmatrix}$

3. $2\cdot\begin{bmatrix} 2 & 3 \\ -1 & 4 \\ 0 & 5 \end{bmatrix}+\begin{bmatrix} a & b \\ c & d \\ e & f \end{bmatrix}=3\cdot\begin{bmatrix} 1 & 0 \\ 0 & -3 \\ 0 & 2 \end{bmatrix}$

$\Rightarrow \mathbf{a+b+c+d+e+f=?}$

A) –28 B) –26 C) –24 D) –20 E) –18

4. $\begin{bmatrix} x+y & 3 \\ 5-x & 0 \end{bmatrix}=\begin{bmatrix} 4 & x \\ x-y & 0 \end{bmatrix} \Rightarrow \mathbf{3x+2y=?}$

A) 8 B) 9 C) 10 D) 11 E) 12

5. $\begin{bmatrix} \sin x & \cos x \\ \cos x & \sin x \end{bmatrix}\cdot\begin{bmatrix} \cos x \\ \sin x \end{bmatrix}=\begin{bmatrix} 1 \\ 1 \end{bmatrix} \Rightarrow \mathbf{x=?}$

A) 0° B) 30° C) 45° D) 60° E) 90°

6. $x > 0$,

$A=\begin{bmatrix} x & \frac{1}{2} \\ \frac{1}{4} & y \end{bmatrix}$, $A=A^{-1} \Rightarrow \mathbf{x=?}$

A) $\frac{\sqrt{7}}{2}$ B) $\frac{\sqrt{7}}{4}$ C) $\frac{\sqrt{14}}{2}$

D) $\frac{\sqrt{14}}{4}$ E) $\frac{\sqrt{21}}{4}$

7. $A=\begin{bmatrix} 1 & x \\ y & 3 \end{bmatrix}$, $B=\begin{bmatrix} -5 & m \\ 12 & n \end{bmatrix}$,

$A^2=B^T \Rightarrow \mathbf{x+y+m+n=?}$

A) –6 B) –5 C) –4 D) –2 E) –1

8. $A=\begin{bmatrix} 1 & 2 \\ 3 & 5 \end{bmatrix}\cdot\begin{bmatrix} 2 & 4 & 1 \\ 3 & 1 & 5 \end{bmatrix}=\begin{bmatrix} 8 & x & 11 \\ 21 & y & 28 \end{bmatrix} \Rightarrow \mathbf{x+y=?}$

A) 22 B) 23 C) 24 D) 25 E) 26

9. $A = \begin{bmatrix} 4 & 4 \\ x & 2 \end{bmatrix}$, $A = A^T$,

$A^{-1} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \Rightarrow$ **a+b+c+d= ?**

A) $\frac{1}{4}$ B) $\frac{1}{2}$ C) 1 D) 2 E) 4

10. $A = \begin{bmatrix} a^2 & b \\ 3 & 4a+4 \end{bmatrix}$, $A = A^{-1} \Rightarrow$ **a+b=?**

A) –9 B) –7 C) –5 D) –3 E) –1

11. $x^2 - 3x - 4 = 0$, ÇK(SS)$=\{x_1, x_2\}$

$\begin{bmatrix} x_1 & x_2 \\ x_1 & x_2 \end{bmatrix} \cdot \begin{bmatrix} x_1 & 0 \\ 0 & x_2 \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

$\Rightarrow$ **a + b + c + d = ?**

A) 5 B) 11 C) 17 D) 29 E) 34

12. $A = \begin{bmatrix} 1 & 1 \\ 2 & -1 \end{bmatrix}$, $A^{53} = x \cdot \begin{bmatrix} 1 & 1 \\ 2 & -1 \end{bmatrix} \Rightarrow$ **x=?**

A) -2^{26} B) -2^{36} C) 2^{26} D) 3^{26} E) 3^{27}

13. $f(x) = x^2 - 4x + 4$,

$A = \begin{bmatrix} -4 & 1 \\ 3 & 2 \end{bmatrix} \Rightarrow$ **f(A) = ?**

A) $\begin{bmatrix} 39 & -6 \\ -18 & 3 \end{bmatrix}$ B) $\begin{bmatrix} 33 & -9 \\ -21 & 3 \end{bmatrix}$ C) $\begin{bmatrix} 39 & -9 \\ 18 & 3 \end{bmatrix}$ D) $\begin{bmatrix} 33 & -18 \\ 9 & -3 \end{bmatrix}$ E) $\begin{bmatrix} 39 & -18 \\ -9 & -3 \end{bmatrix}$

14. $A - 2B = \begin{bmatrix} 3 & 0 \\ 4 & -5 \end{bmatrix}$, $A + 3B = \begin{bmatrix} -2 & 1 \\ 3 & 4 \end{bmatrix} \Rightarrow$ **A=?**

A) $\begin{bmatrix} 1 & \frac{2}{5} \\ \frac{3}{5} & \frac{13}{5} \end{bmatrix}$ B) $\begin{bmatrix} 1 & \frac{2}{5} \\ \frac{13}{5} & \frac{17}{5} \end{bmatrix}$ C) $\begin{bmatrix} 1 & \frac{2}{5} \\ \frac{18}{5} & -\frac{7}{5} \end{bmatrix}$ D) $\begin{bmatrix} \frac{3}{5} & \frac{2}{5} \\ \frac{14}{5} & \frac{21}{5} \end{bmatrix}$ E) $\begin{bmatrix} \frac{3}{5} & \frac{4}{5} \\ \frac{18}{5} & \frac{21}{5} \end{bmatrix}$

15. $\begin{bmatrix} 3 & -2 \\ -4 & 5 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 2 & 4 \\ -2 & 3 & -5 \end{bmatrix} = \begin{bmatrix} x & \cdot & \cdot \\ \cdot & y & \cdot \\ \cdot & \cdot & z \end{bmatrix}$

$\Rightarrow$ **x + y + z = ?**

A) 6 B) 7 C) 8 D) 9 E) 10

16. $A = \begin{bmatrix} 1 & -1 \\ 3 & 1 \end{bmatrix}$, $A^{12} = x \cdot I \Rightarrow$ **x=?**

A) -2^{12} B) -2^6 C) –2 D) 2^6 E) 2^{12}

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1	2	3	4	5	6	7	8
C	A	B	D	C	D	C	B
9	**10**	**11**	**12**	**13**	**14**	**15**	**16**
A	B	E	D	A	C	D	E

1. $A=\begin{bmatrix}2&1&0\\3&-1&2\\2&4&-3\end{bmatrix} \Rightarrow$ **det(A) = ?**

A) –23 B) –11 C) –3 D) 3 E) 23

2. $A=\begin{bmatrix}12&4\\8&3\end{bmatrix} \Rightarrow$ **det(A) = ?**

A) 2 B) 3 C) 4 D) 5 E) 6

3. $\begin{vmatrix}x+2&4\\2k&x\end{vmatrix}=0,\ x_1=4 \Rightarrow \mathbf{x_2=?}$

A) –6 B) –2 C) –1 D) 2 E) 3

4. $A=\begin{bmatrix}x-2&1\\1&2\end{bmatrix},\ B=\begin{bmatrix}1&0\\3x&1\end{bmatrix}$

detA = 1 ⇒ **B = ?**

A) $\begin{bmatrix}1&0\\9&1\end{bmatrix}$ B) $\begin{bmatrix}1&4\\33&0\end{bmatrix}$ C) $\begin{bmatrix}1&0\\50&1\end{bmatrix}$

D) $\begin{bmatrix}1\\60\end{bmatrix}$ E) $\begin{bmatrix}1&0\\68&1\end{bmatrix}$

5. $\det(A)=-2 \Rightarrow \mathbf{|A^2|=?}$

A) 4 B) 2 C) 0 D) –2 E) –4

6. $\begin{bmatrix}\mathbf{0}&\mathbf{3}\\\mathbf{1}&\mathbf{0}\\\mathbf{2}&\mathbf{1}\end{bmatrix}\cdot\begin{bmatrix}\mathbf{1}&\mathbf{0}\\\mathbf{0}&\mathbf{2}\\\mathbf{0}&\mathbf{1}\end{bmatrix}^T$ **= ?**

A) $\begin{bmatrix}0&6&3\\1&0&0\\2&3&1\end{bmatrix}$ B) $\begin{bmatrix}0&6&3\\1&0&0\\2&2&1\end{bmatrix}$ C) $\begin{bmatrix}2&3&1\\1&0&2\\1&1&0\end{bmatrix}$

D) $\begin{bmatrix}6&2&0\\3&0&1\\4&2&0\end{bmatrix}$ E) $\begin{bmatrix}0&2&1\\4&0&6\\5&2&1\end{bmatrix}$

7. $A=\begin{bmatrix}3&4\\2&4\end{bmatrix} \Rightarrow \mathbf{A^{-1}=?}$

A) $\begin{bmatrix}4&-5\\-2&3\end{bmatrix}$ B) $\begin{bmatrix}1&-1\\-\frac{1}{2}&\frac{3}{4}\end{bmatrix}$ C) $\begin{bmatrix}1&1\\2&-1\end{bmatrix}$

D) A E) $\frac{1}{2}A$

8. $A=\begin{bmatrix}2^{18}&2^{17}\\2^{14}&2^{15}\end{bmatrix},$

$\det A = 3\cdot 2^{m-3} \Rightarrow$ **m=?**

A) 32 B) 34 C) 36 D) 38 E) 40

9. $A = \begin{bmatrix} -2 & 3 \\ 4 & 5 \end{bmatrix}, B = \begin{bmatrix} 1 & 2 \\ -1 & 4 \end{bmatrix} \Rightarrow \mathbf{A^T + B^T = ?}$

A) $\begin{bmatrix} 3 & 1 \\ -5 & 6 \end{bmatrix}$ B) $\begin{bmatrix} -3 & 1 \\ 5 & 6 \end{bmatrix}$ C) $\begin{bmatrix} -1 & 3 \\ 5 & 9 \end{bmatrix}$

D) $\begin{bmatrix} -3 & 1 \\ -5 & 6 \end{bmatrix}$ E) $\begin{bmatrix} -3 & 1 \\ 5 & -6 \end{bmatrix}$

10. $A = \begin{bmatrix} a & -1 \\ b & 0 \end{bmatrix}, B = \begin{bmatrix} -3 & 1 \\ 2a & 4 \end{bmatrix}$

$A \cdot B = \begin{bmatrix} -5a & -b-3 \\ 2a-4 & a+3 \end{bmatrix} \Rightarrow \mathbf{a + b = ?}$

A) 0 B) 1 C) −1 D) −2 E) −3

11. $A = \begin{bmatrix} 3 & 2 \\ 0 & -3 \end{bmatrix} \Rightarrow \mathbf{A^{18} = ?}$

A) $3^{18}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ B) $3^{17}\begin{bmatrix} 3 & 2 \\ 0 & -3 \end{bmatrix}$ C) $3^{19}\begin{bmatrix} 3 & 2 \\ 0 & -3 \end{bmatrix}$

D) $3^{17}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ E) $3^{18}\begin{bmatrix} -3 & 2 \\ 0 & 3 \end{bmatrix}$

12. $A = \begin{bmatrix} 1 & 0 \\ 5 & 1 \end{bmatrix} \Rightarrow \mathbf{A^{14} = ?}$

A) $\begin{bmatrix} 1 & 0 \\ 70 & 1 \end{bmatrix}$ B) $\begin{bmatrix} 1 & 0 \\ 5^{14} & 1 \end{bmatrix}$ C) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

D) $5^{14}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ E) $5^{14}\begin{bmatrix} 1 & 0 \\ 5 & 1 \end{bmatrix}$

13. $A = \begin{bmatrix} 2 & 0 \\ 0 & 4 \end{bmatrix} \Rightarrow \mathbf{A^{12} = ?}$

A) $2^{13}\begin{bmatrix} 1 & 1 \\ 1 & 2^{12} \end{bmatrix}$ B) $2^{12}\begin{bmatrix} 1 & 0 \\ 0 & 2^{12} \end{bmatrix}$

C) $2^{11}\begin{bmatrix} 1 & 1 \\ 1 & 2^{12} \end{bmatrix}$ D) $2^{10}\begin{bmatrix} 1 & 1 \\ 1 & 2^{12} \end{bmatrix}$

E) $2^{9}\begin{bmatrix} 1 & 1 \\ 1 & 2^{12} \end{bmatrix}$

14. $A = \begin{bmatrix} 2 & -3 \\ 1 & -2 \end{bmatrix} \Rightarrow \mathbf{A^{25} = ?}$

A) $\begin{bmatrix} 2 & -3 \\ 1 & 2 \end{bmatrix}$ B) $\begin{bmatrix} 2 & -3 \\ 1 & -2 \end{bmatrix}$ C) $\begin{bmatrix} 2 & 3 \\ 1 & -2 \end{bmatrix}$

D) $\begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$ E) $\begin{bmatrix} 2 & -3 \\ -2 & 1 \end{bmatrix}$

15. $A = \begin{bmatrix} -4 & 0 \\ 0 & 4 \end{bmatrix} \Rightarrow \mathbf{A^{3} = ?}$

A) 16A B) 15A C) 12A D) 9A E) 16I

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1	2	3	4	5	6	7	8
D	C	A	A	A	B	B	B
9	10	11	12	13	14	15	
C	B	A	A	B	B	A	

1. $A=\begin{bmatrix}1 & -3\\ 3 & 4\end{bmatrix},\ B=\begin{bmatrix}-2 & 1\\ 0 & -1\end{bmatrix}$

$\Rightarrow$ **(A + B) · (A − B) = ?**

A) $\begin{bmatrix}-9 & -6\\ 18 & -3\end{bmatrix}$ B) $\begin{bmatrix}9 & 6\\ 18 & 3\end{bmatrix}$ C) $\begin{bmatrix}9 & -6\\ 18 & 3\end{bmatrix}$

D) $\begin{bmatrix}-9 & 6\\ 18 & 3\end{bmatrix}$ E) $\begin{bmatrix}-9 & -6\\ 18 & 3\end{bmatrix}$

2. $\begin{bmatrix}1 & 1 & 2\\ 0 & 1 & 3\\ 1 & 1 & 0\end{bmatrix}\cdot\begin{bmatrix}x\\ y\\ z\end{bmatrix}=\begin{bmatrix}6\\ 6\\ 2\end{bmatrix}\Rightarrow \mathbf{x+y+z=?}$

A) 10 B) 7 C) 5 D) 4 E) 3

3. $\begin{vmatrix}\mathbf{2008} & \mathbf{2004}\\ \mathbf{2004} & \mathbf{2000}\end{vmatrix}=?$

A) –4 B) –16 C) –2004

D) –4008 E) –4012

4. $A=\begin{bmatrix}73 & 42 & 12\\ 3 & 4 & 6\\ 6 & 8 & 12\end{bmatrix}\Rightarrow \mathbf{\det A=?}$

A) 3 B) 2 C) 1 D) 0 E) –2

5. $A=\begin{bmatrix}a & b & x\\ 3 & 4 & y\\ 2 & 2 & c\end{bmatrix},\ B=\begin{bmatrix}b & a & x\\ 4 & 3 & y\\ 2 & 2 & c\end{bmatrix}$

$3\cdot|A|-|B|=12\Rightarrow \mathbf{|A|=?}$

A) 3 B) 4 C) 6 D) 8 E) 12

6. $A=\begin{bmatrix}13 & 31\\ 5 & 12\end{bmatrix}\Rightarrow \mathbf{\det(A^{23})=?}$

A) 1243 B) 1080 C) 940

D) 256 E) 1

7. $A=\begin{bmatrix}\sin 12^\circ & \cos 12^\circ\\ \cos 108^\circ & \cos 18^\circ\end{bmatrix}\Rightarrow \mathbf{\det(A^2)=?}$

A) $\frac{1}{4}$ B) $\frac{1}{3}$ C) $\frac{1}{2}$ D) $\frac{2}{3}$ E) $\frac{\sqrt{3}}{2}$

8. $A=\begin{bmatrix}x & y\\ 10 & 1\end{bmatrix},\ A^{-1}=\begin{bmatrix}a & 4\\ 8 & b\end{bmatrix}\Rightarrow \mathbf{y=?}$

A) 1 B) 2 C) 3 D) 4 E) 5

9. $$\left.\begin{array}{l} A=\begin{bmatrix} 12 & 8 & 4 \\ 4 & 2 & 6 \\ 3 & 5 & 7 \end{bmatrix} \\ \\ B=\begin{bmatrix} 12 & 8 & 4 \\ 1 & -3 & -1 \\ 3 & 5 & 7 \end{bmatrix} \end{array}\right\} \Rightarrow \det A-\det B=?$$

A) –2 B) –1 C) 0 D) 1 E) 2

10. $A=\begin{bmatrix} a & 2 & 1 \\ 3 & a & 2 \\ 1 & 1 & 0 \end{bmatrix}$,

$\det A = 4 \Rightarrow$ **a = ?**

A) 3 B) 2 C) 1 D) 0 E) –2

11. $\begin{vmatrix} 1 & 2 & 0 \\ 2 & x & -1 \\ x & 1 & 0 \end{vmatrix}=3 \Rightarrow$ **x = ?**

A) –2 B) –1 C) 1 D) 2 E) 3

12. $A=\begin{bmatrix} \sin\alpha & \cos\alpha-1 \\ \cos\alpha+1 & -\sin\alpha \end{bmatrix} \Rightarrow \det(A)=?$

A) cosα B) sin2α C) 2

C) 0 D) –1

13. $\begin{vmatrix} a & b \\ c & d \end{vmatrix}=-6 \Rightarrow \begin{vmatrix} a & a-3b \\ c & c-3d \end{vmatrix}=?$

A) –18 B) –12 C) –6 D) 6 E) 18

14. $A=\begin{vmatrix} 3 & x & x^2 \\ 1 & 2 & 3 \\ 9 & 6 & 3 \end{vmatrix}=-72 \Rightarrow$ **ÇK(SS) = ?**

A) $\{1,2\}$ B) $\{2,3\}$ C) $\{1,3\}$

D) $\{-1,3\}$ E) $\{-3,1\}$

15. $\begin{vmatrix} 1 & 2 & 3 \\ a & b & c \\ 3 & 2 & 1 \end{vmatrix}=4$

$\begin{vmatrix} 2 & 4 & 6 \\ a & b & c \\ -9 & -6 & -3 \end{vmatrix}=x$

$\Rightarrow$ **x = ?**

A) 3 B) 4 C) –4 D) –16 E) –24

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1	2	3	4	5	6	7	8
E	D	B	D	A	E	A	E
9	10	11	12	13	14	15	
C	C	B	D	E	D	E	

1. $\begin{vmatrix} 1 & 2 & -1 \\ 0 & 1 & 3 \\ 2 & -1 & x \end{vmatrix} = 14 \Rightarrow \mathbf{x = ?}$

A) -3
B) -2
C) 0
D) 2
E) 3

2. $A = \begin{bmatrix} 3 & 1 \\ 2 & 1 \end{bmatrix}, B = \begin{bmatrix} 4 \\ 3 \end{bmatrix}, C = \begin{bmatrix} 2 & 3 \end{bmatrix},$

$A \cdot K = B \Rightarrow \mathbf{K \cdot C = ?}$

A) $\begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}$
B) $\begin{bmatrix} 2 & 3 \\ 2 & 3 \end{bmatrix}$
C) $\begin{bmatrix} 2 & 4 \\ 3 & 4 \end{bmatrix}$
D) $\begin{bmatrix} 2 \\ 3 \end{bmatrix}$
E) $\begin{bmatrix} 2 & 3 \end{bmatrix}$

3. $A = \begin{bmatrix} 3 & x \\ 4 & y \end{bmatrix}, A^{-1} = A \Rightarrow \mathbf{x \cdot y = ?}$

A) -6
B) -3
C) -1
D) 3
E) 6

4. $\begin{vmatrix} x+1 & 2 & 3 \\ 1 & x+2 & 3 \\ 1 & 2 & x+3 \end{vmatrix} = 0 \Rightarrow \textbf{ÇK(SS) = ?}$

A) $\{-6, -3\}$
B) $\{-6, -1\}$
C) $\{-6, 0\}$
D) $\{-3, -1\}$
E) $\{-3, 0\}$

5. $\begin{vmatrix} \sin 3x & -\frac{1}{2} \\ 2\sin x & 1 \end{vmatrix} = \sin 2x,\ 0 \le x < \pi \Rightarrow \textbf{ÇK(SS) = ?}$

A) $\left\{\frac{\pi}{3}, \frac{2\pi}{3}\right\}$
B) $\left\{0, \frac{\pi}{3}, \frac{\pi}{2}\right\}$
C) $\left\{\frac{\pi}{3}, \frac{\pi}{2}\right\}$
D) $\left\{\frac{\pi}{2}, \frac{2\pi}{3}\right\}$
E) $\left\{\frac{\pi}{2}, \frac{4\pi}{3}\right\}$

6. $A = \begin{bmatrix} x+1 & 2 \\ 2 & x+2 \end{bmatrix}, B = \begin{bmatrix} -1 & 6 \\ 6 & 2 \end{bmatrix},$

$3A - B = I \Rightarrow \mathbf{x = ?}$

A) -2
B) -1
C) 0
D) 1
E) 2

7. $x \in (0, 2\pi)$

$\begin{vmatrix} \sin x & \tan x \\ \cot x & \cos x \end{vmatrix} = -\frac{1}{2} \Rightarrow \mathbf{x = ?}$

A) $\frac{\pi}{8}$
B) $\frac{\pi}{6}$
C) $\frac{\pi}{4}$
D) $\frac{\pi}{3}$
E) $\frac{\pi}{2}$

8. $i = \sqrt{-1}$

$A = \begin{bmatrix} i & i+1 \\ 1+i & i \end{bmatrix} \Rightarrow \mathbf{|A| = ?}$

A) $1 - i$
B) $1 + i$
C) $2 - i$
D) $-1 - 2i$
E) $1 + 2i$

9. $f(x) = x^2 - 2x + 3$

$A = \begin{bmatrix} -1 & 2 \\ 3 & 4 \end{bmatrix} \Rightarrow \mathbf{|f(A)| = ?}$

A) 148 B) 164 C) 172 D) 186 E) 198

10. $\begin{vmatrix} 1 & 2 & -1 \\ 3 & 1 & 4 \\ 2 & 0 & 1 \end{vmatrix} = \begin{vmatrix} x & 1 \\ 2 & 5 \end{vmatrix} \Rightarrow \mathbf{x = ?}$

A) 1 B) 2 C) 3 D) 4 E) 5

11. $x + 3y - 2z = 1,$

$2x - 4y + z = -3,$

$3x - 2y - 3z = -10 \Rightarrow$ **ÇK(SS) = ?**

A) {(1, –3, –4)} B) {(1, 2, –3)} C) {(1, 2, 3)} D) {(1, 3, 4)} E) {(1, 3, 3)}

12. $2x - 3y + z = -3,$

$x + y + 3z = 6,$

$-x - 2y + 2z = 3 \Rightarrow$ **ÇK(SS) = ?**

A) {(–1, 1, 2)} B) {(–1, –1, 2)} C) {(–1, –1, –2)} D) {(1, 1, 2)} E) {(1, 2, 2)}

13. $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \Rightarrow A^{1975} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

$\Rightarrow \mathbf{a + b + c + d = ?}$

A) 1972 B) 1973 C) 1975 D) 1976 E) 1977

14. $\begin{vmatrix} \mathbf{2013} & \mathbf{2012} \\ \mathbf{2011} & \mathbf{2010} \end{vmatrix} = \mathbf{?}$

A) –4 B) –2 C) 2 D) 2013 E) 4026

15. $f(x) = \begin{vmatrix} x^2 & x & 1 \\ -2 & 3 & 4 \\ x & 2x & 1 \end{vmatrix} \Rightarrow \mathbf{f'\left(\frac{1}{2}\right) = ?}$

A) –8 B) –6 C) –4 D) 4 E) 8

16. $A = \begin{bmatrix} \sin x & -\cos x \\ \cos x & \sin x \end{bmatrix} \Rightarrow \mathbf{A^{-1} = ?}$

A) $\begin{bmatrix} -\sin x & \cos x \\ -\cos x & -\sin x \end{bmatrix}$ B) $\begin{bmatrix} \sin x & \cos x \\ \cos x & -\sin x \end{bmatrix}$

C) $\begin{bmatrix} \sin x & -\cos x \\ \cos x & \sin x \end{bmatrix}$ D) $\begin{bmatrix} \cos x & -\sin x \\ \sin x & \cos x \end{bmatrix}$

E) $\begin{bmatrix} \sin x & \cos x \\ -\cos x & \sin x \end{bmatrix}$

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1	2	3	4	5	6	7	8
A	B	E	C	B	B	C	D
9	**10**	**11**	**12**	**13**	**14**	**15**	**16**
E	C	C	A	E	B	C	E

1. $\begin{bmatrix} 2 & 3 \\ 2 & 4 \end{bmatrix} \cdot x = \begin{bmatrix} 1 & 2 \\ 2 & 0 \end{bmatrix} \Rightarrow$ **x = ?**

A) $\begin{bmatrix} \frac{1}{2} & 1 \\ -\frac{1}{2} & 1 \end{bmatrix}$ B) $\begin{bmatrix} -\frac{1}{2} & 2 \\ \frac{1}{2} & -1 \end{bmatrix}$ C) $\begin{bmatrix} -2 & 4 \\ -1 & 2 \end{bmatrix}$

D) $\begin{bmatrix} -1 & 4 \\ 1 & -2 \end{bmatrix}$ E) $\begin{bmatrix} -\frac{1}{4} & 1 \\ \frac{1}{4} & -\frac{1}{2} \end{bmatrix}$

2. $A = \begin{bmatrix} 1 & 3 \\ 2 & 8 \end{bmatrix}$, $B = \begin{bmatrix} 0 & 1 \\ 3 & 4 \end{bmatrix} \Rightarrow$ **$(A \cdot B)^T = ?$**

A) $\begin{bmatrix} 9 & 13 \\ 24 & 34 \end{bmatrix}$ B) $\begin{bmatrix} 9 & 24 \\ 13 & 34 \end{bmatrix}$ C) $\begin{bmatrix} 3 & 12 \\ 11 & 17 \end{bmatrix}$

D) $\begin{bmatrix} 24 & 34 \\ 9 & 13 \end{bmatrix}$ E) $\begin{bmatrix} 6 & 18 \\ 4 & 32 \end{bmatrix}$

3. $A = \begin{bmatrix} 2 & 3 & 1 \\ -1 & 0 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 1 \\ 0 & 1 \\ 1 & 1 \end{bmatrix} \Rightarrow$ **$(AB)^T = ?$**

A) $\begin{bmatrix} 3 & 6 \\ 1 & 1 \end{bmatrix}$ B) $\begin{bmatrix} 4 & 8 \\ 3 & 2 \end{bmatrix}$ C) $\begin{bmatrix} 4 & 3 \\ 8 & 2 \end{bmatrix}$

D) $\begin{bmatrix} 1 & 6 \\ 1 & 3 \end{bmatrix}$ E) $\begin{bmatrix} 3 & 1 \\ 6 & 1 \end{bmatrix}$

4. $A \cdot \begin{bmatrix} 3 & 1 \\ -2 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 7 \\ -5 & 9 \end{bmatrix} \Rightarrow$ **A = ?**

A) $\begin{bmatrix} 6 & 2 \\ 3 & 1 \end{bmatrix}$ B) $\begin{bmatrix} 3 & 1 \\ 2 & 4 \end{bmatrix}$ C) $\begin{bmatrix} 4 & 2 \\ 1 & 3 \end{bmatrix}$

D) $\begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix}$ E) $\begin{bmatrix} 5 & 3 \\ 2 & 6 \end{bmatrix}$

5. $A = \begin{bmatrix} 2 & 1 \\ -1 & 2 \\ -2 & 3 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 2 & -1 \\ -1 & 3 & 1 \end{bmatrix}$

$\Rightarrow |AB| = ?$

A) –10 B) –2 C) 0 D) 6 E) 12

6. $A = \begin{bmatrix} 9879 & 9880 \\ 9877 & 9878 \end{bmatrix} \Rightarrow$ **$|A| = ?$**

A) 4 B) 3 C) 2 D) 1 E) –2

7. $A = \begin{bmatrix} 1 & -2 & 3 \\ 2 & 1 & -1 \\ -2 & -1 & 2 \end{bmatrix} \Rightarrow$ **$|A| = ?$**

A) 5 B) 7 C) 11 D) 12 E) 15

8. $\begin{bmatrix}a & c\\ c & a\end{bmatrix}+\begin{bmatrix}b & d\\ -d & -b\end{bmatrix}=2\begin{bmatrix}2 & 3\\ 5 & 1\end{bmatrix}$

$\Rightarrow \mathbf{a+b+c+d=?}$

A) 6 B) 8 C) 10 D) 12 E) 14

9. $A=\begin{bmatrix}1 & 3\\ 0 & 1\end{bmatrix} \Rightarrow \mathbf{A^{27}=?}$

A) $3^{26}\begin{bmatrix}1 & 1\\ 0 & 1\end{bmatrix}$ B) $\begin{bmatrix}1 & 3^{27}\\ 0 & 1\end{bmatrix}$ C) $\begin{bmatrix}1 & 81\\ 0 & 1\end{bmatrix}$

D) $\begin{bmatrix}27 & 81\\ 0 & 27\end{bmatrix}$ E) $\begin{bmatrix}27 & 81\\ 24 & 24\end{bmatrix}$

10. $\begin{bmatrix}1 & 1 & 1\\ 1 & 2 & 3\\ 0 & 1 & 1\end{bmatrix}\begin{bmatrix}x\\ y\\ z\end{bmatrix}=\begin{bmatrix}2\\ 3\\ -4\end{bmatrix} \Rightarrow \mathbf{z=?}$

A) 9 B) 8 C) 7 D) 6 E) 5

11. $\begin{bmatrix}1 & 2 & 3\\ 1 & 1 & 2\\ 0 & 1 & 2\end{bmatrix}\begin{bmatrix}x\\ y\\ z\end{bmatrix}=\begin{bmatrix}3\\ 4\\ 2\end{bmatrix} \Rightarrow \mathbf{x-y-z=?}$

A) 0 B) 1 C) 2 D) 3 E) 4

12. $A=\begin{bmatrix}2 & 1 & -1\\ 2 & -3 & -1\\ 1 & 3 & 2\end{bmatrix} \Rightarrow \mathbf{|A|=?}$

A) 0 B) –20 C) –24 D) –48 E) –72

13. $\begin{bmatrix}x-1 & -1 & -2\\ 0 & x-2 & 2\\ 0 & 0 & x-3\end{bmatrix}=0 \Rightarrow$ **ÇK(SS) = ?**

A) $\{1,2,3\}$ B) $\{1,2\}$ C) $\{2,1\}$

D) $\{2,3,4\}$ E) $\{2,3\}$

14. $A=\begin{bmatrix}1 & -2 & 3\\ -2 & 3 & 1\\ 0 & 1 & 0\end{bmatrix},\ B=\begin{bmatrix}1 & 0 & 2\\ 3 & -2 & 5\\ 2 & 1 & 3\end{bmatrix}$

$\Rightarrow$ **det(A·B) = ?**

A) –21 B) –18 C) 24 D) 27 E) 28

15. $\begin{vmatrix}x-2 & 2\\ 3 & x-3\end{vmatrix}=0 \Rightarrow$ **ÇK(SS) = ?**

A) $\{2,3\}$ B) $\{1,5\}$ C) $\{0,5\}$

D) $\{2,-5\}$ E) $\{0,-5\}$

YANITLAR / ANSWERS

1	2	3	4	5	6	7	8
D	B	E	D	C	C	A	C
9	**10**	**11**	**12**	**13**	**14**	**15**	
C	E	D	B	A	A	C	